Pure Mathematics:
Geometric Sequences and Series

Name: _____

Class: _____

Date: _____

Time:

Total marks available:

Total marks achieved: _____

Edexcel IAL AS and A Levels Mathematics
Topic : Pure Mathematics
Sub Topic : Sequences and Series
Type : Topic Questions

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Edexcel International Advanced Subsidiary in Mathematics (XMA01)
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First examination June

To be used by all students preparing for Edexcel IAL AS and A Levels Mathematics

Students of other boards may also find this useful

Q1.

Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

(a) According to model *A*, determine Kim's annual salary in year 2.

(3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

(b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10

(3)

(c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10

(3)

(Total for question = 9 marks)

Q2.

In a geometric sequence u_1, u_2, u_3, ...

- the common ratio is *r*
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that *r* satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

(Total for question = 8 marks)

Q3.

A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \dfrac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u^k is the kth term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

(Total for question = 7 marks)

Q4.

The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

(Total for question = 6 marks)

Q5.

The first term of a geometric series is 20 and the common ratio is $\dfrac{7}{8}$. The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which $S_\infty - S_N < 0.5$

(4)

(Total for question = 8 marks)

Q6.

A geometric series has first term a and common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(3)

The second term of a geometric series is -320 and the fifth term is $\dfrac{512}{25}$

(b) Find the value of the common ratio.

(2)

(c) Hence find the sum of the first 13 terms of the series, giving your answer to 2 decimal places.

(3)

(Total for question = 8 marks)

Q7.

In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan *A*, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan *A*.

(3)

In training plan *B*, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan *B*. Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, *x* km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total for question = 11 marks)

Q8.

Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she
has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

(Total for question = 7 marks)

Q9.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by *p* % each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of *p*, giving your answer to 2 decimal places.

(3)

According to the model, at the end of *N* years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of *N*.

(5)

(Total for question = 8 marks)

Q10.

A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio

r

- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of *r*, giving your answer to 3 decimal places.

(2)

After a total of *N* hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of *N*.

(4)

(Total for question = 9 marks)