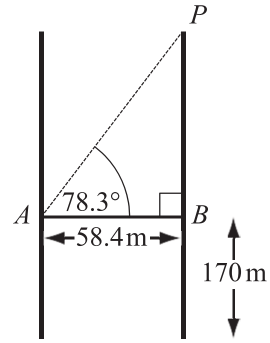
2D Pythagoras & SOHCAHTOA

Model Answer

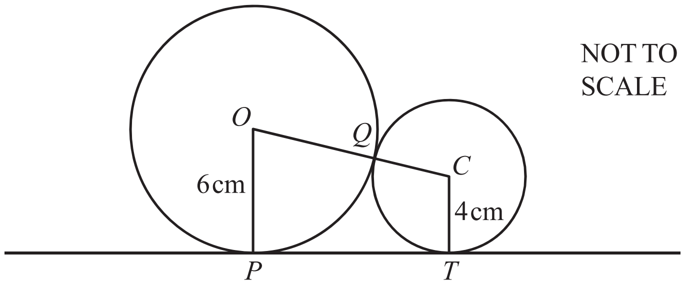
EXAM PAPERS PRACTICE



The line AB represents the glass walkway between the Petronas Towers in Kuala Lumpur.
The walkway is 58.4 metres long and is 170 metres above the ground.
The angle of elevation of the point P from A is 78.3°.

Calculate the height of P above the ground. [3]

$$\tan(78.3) = x/58.4$$
$$x = 282.00$$
$$282 + 170$$
$$= 452$$



Two circles, centres O and C, of radius 6 cm and 4 cm respectively, touch at Q.
PT is a tangent to both circles.

(a) Write down the distance OC. [1]

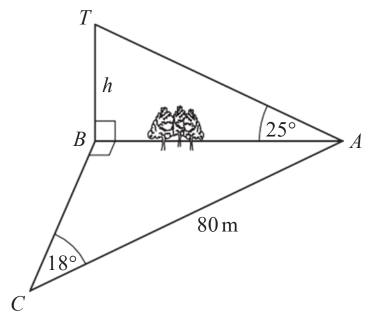
$$\begin{aligned} OC &= OP + CT \\ &= 6 \text{ cm} + 4 \text{ cm} \\ &= 10 \text{ cm} \end{aligned}$$

(b) Calculate the distance PT. [3]

$$\begin{aligned} OA &= OP - CT \\ &= 6 - 4 \\ &= 2 \text{ cm} \end{aligned}$$

using pythagoras formula

$$OC^2 = OA^2 + AC^2$$
$$AC^2 = OC^2 - OA^2$$
$$AC^2 = 10^2 - 2^2$$
$$\begin{aligned} AC &= \sqrt{100 - 4} \\ &= \sqrt{96} \\ &= 9.8 \text{ cm} \end{aligned}$$
$$PT = 9.8 \text{ cm}$$



Mahmoud is working out the height, h metres, of a tower BT which stands on level ground.
He measures the angle TAB as 25°.
He cannot measure the distance AB and so he walks 80 m from A to C, where angle ACB = 18° and angle ABC = 90°.

Calculate

(a) the distance AB, [2]

Horizontal distance AB can be obtained from $\triangle ABC$

From trigonometry :

$Sin18^\circ =$ opposite / hypotenus

$Sin18^\circ = AB/80$

$AB = 0.3090169 * 80$

$AB = 24.72$ m

(b) the height of the tower, BT. [2]

The height BT, of tower from $\triangle$ABT

From trigonometry :

$Tan25^\circ =$ opposite / Adjacent

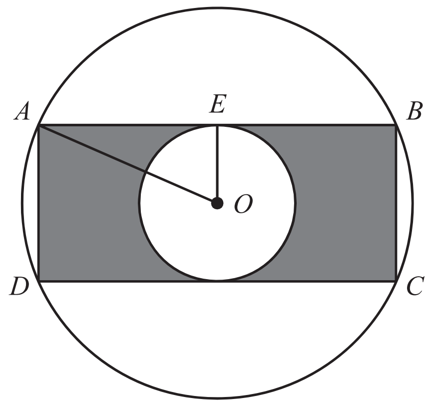
$Tan25^\circ =$ h/24.72 m

h $= Tan25 * 24.72$

h = 11.527

h = 11.53 m

Hence, height of tower BT = 11.53 m



NOT TO SCALE

A, B, C and D lie on a circle, centre O, radius 8 cm.
AB and CD are tangents to a circle, centre O, radius 4 cm.
$ABCD$ is a rectangle.
(a) Calculate the distance AE.

[2]

Considering right triangle AOE, apply the Pythagorean Theorem to find AE, given that,

$EO = 4$ cm

$AO = 8$ cm

Thus:

$AE = \sqrt{(AO^2 - EO^2)}$

$AE = \sqrt{(8^2 - 4^2)}$

$\mathbf{AE} = 6.9$ cm

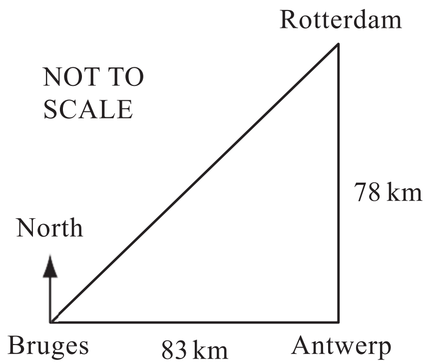
(b) Calculate the shaded area.

[3]

$\text{Area of rectangle} = \text{length} \times \text{width} = 2(6.9) \times 2(4) = 110.4 \text{ sq. cm}$

$Area of circle = \pi(4)^2 = 50.27 sq. cm$

$Shaded Area = 110.4 - 50.27 = 60.13\ sq.\ cm$



Antwerp is 78 km due South of Rotterdam and 83 km due East of Bruges, as shown in the diagram.

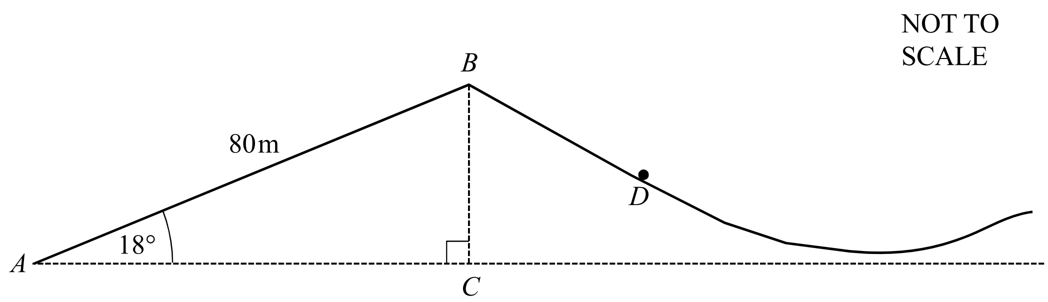
Calculate

(a) the distance between Bruges and Rotterdam, [2]

The distance between Bruges and Rotterdam is 78 kilometers

(b) the bearing of Rotterdam from Bruges, correct to the nearest degree. [3]

The bearing of Rotterdam from Bruges, correct to the nearest degree, is 178 degrees.



The diagram shows the start of a roller-coaster ride at a fairground.
A car rises from A to B along a straight track.

(a) $AB = 80$ metres and angle $BAC = 18°$.
Calculate the vertical height of B above A. [2]

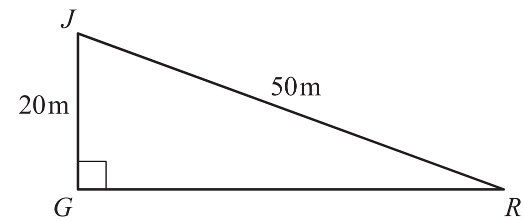
$$\sin 18^\circ = \frac{BC}{AB}$$
$$BC = AB \times \sin 18^\circ$$
$$BC = 80 \text{ m} \times 0.3$$
$$BC = 24 \text{ m}$$

(b) The car runs down the slope from B to D, a distance of s metres.
Use the formula $s = t(p + qt)$ to find the value of s, given that $p = 4$, $t = 3$ and $q = 3.8$.

[2]

$$s = t(p + qt)$$
$$s = 3(4 + 3.8 \times 3)$$
$$s = 46.2$$

Question 7



JGR is a right-angled triangle. *JR* = 50m and *JG* = 20m.
Calculate angle *JRG*. [2]

Given: Right angled triangle

To find: LR

Solution:

opposite angle be x

the side opposite to angle x

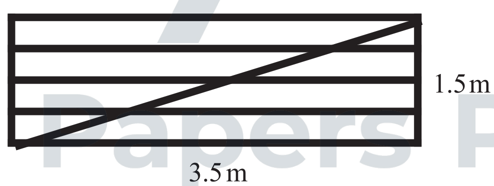
$= 20 \text{ m}; \text{ hyp } = 50 \text{ m}$

$\sin(x) = \frac{0pp}{hyp} = \frac{20}{50} = 0.4$

$x = \sin^{-1}(0.4)$

$= 23.58 = 24^\circ$

Question 8



The diagram represents a rectangular gate measuring 1.5m by 3.5m.
It is made from eight lengths of wood.

Calculate the total length of wood needed to make the gate. [3]

There are five horizontal wooden pieces

$\Rightarrow 5 \times 3.5 \text{ m} = 17.5 \text{ m}$

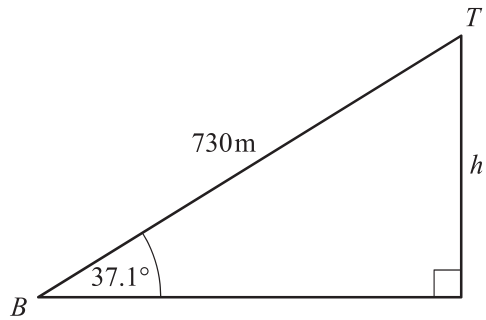
There are 2 vertical wooden pieces

$\Longrightarrow 2 \times 1.5 \text{ m} = 3 \text{ m}$

There is one diagonal wooden piece $\Rightarrow \sqrt{(3.5)^2 + (1.5)^2} = 3.8$ m approx

Total $= 17.5 + 3 + 3.8 = 24.3$ m

The diagram represents the ski lift in Queenstown New Zealand.



NOT TO SCALE

(a) The length of the cable from the bottom, B, to the top, T, is 730 metres.

The angle of elevation of T from B is 37.1°.

Calculate the change in altitude, h metres, from the bottom to the top. [2]

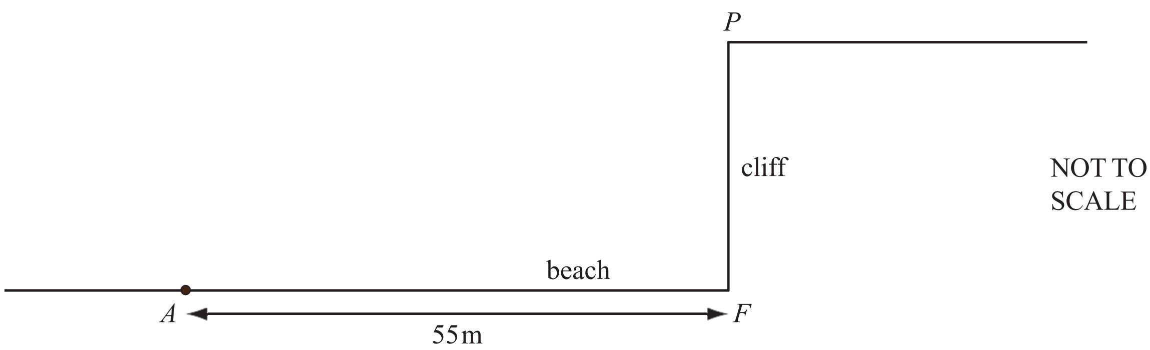
$$\begin{aligned} \sin(37.1^\circ) &= \frac{h}{730^\circ} \\ h &= 730 \sin(37.1^\circ) \\ &= 730 \times 0.6032 \\ &= 440.34. \end{aligned}$$

(b) The lift travels along the cable at 3.65 metres per second.

Calculate how long it takes to travel from B to T.

Give your answer in minutes and seconds. [2]

$$\begin{aligned} \text{Time taken } &= \frac{730}{3.65} \text{ seconds} \\ &= 200 \text{ seconds} \\ \text{Time taken } &= 3 \text{ min } 20 \text{ sec} \end{aligned}$$



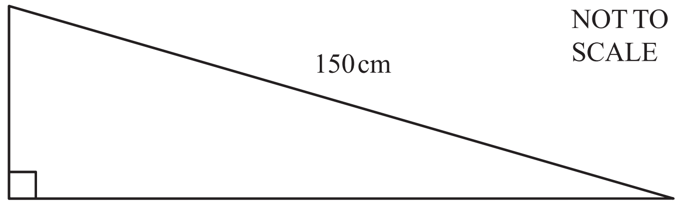
The diagram shows a point P at the top of a cliff.
The point F is on the beach and vertically below P.
The point A is 55m from F, along the horizontal beach.
The angle of elevation of P from A is 17°.

Calculate PF, the height of the cliff. [3]

$$\tan \angle PAF = \tan 17^\circ$$

$$\tan 17^\circ = \frac{PF}{AF}$$

$$\begin{aligned} PF &= AF \tan 17^\circ \\ &= 55 \tan 17^\circ \\ &= 16.815 \\ &= 17 \text{ m} \end{aligned}$$



The right-angled triangle in the diagram has sides of length $7x$ cm, $24x$ cm and 150 cm.

(a) Show that $x^2 = 36$. [2]

In right Δ $H^2 = P^2 + B^2 \Rightarrow (150)^2 = (7x)^2 + (24x)^2$

$\Rightarrow 150 = \sqrt{49x^2 + 576x^2}$

$\Rightarrow \quad 150 = \sqrt{625x^2}$

$x = 150/25$

$x = 6$

(b) Calculate the perimeter of the triangle. [1]

Perimeter of

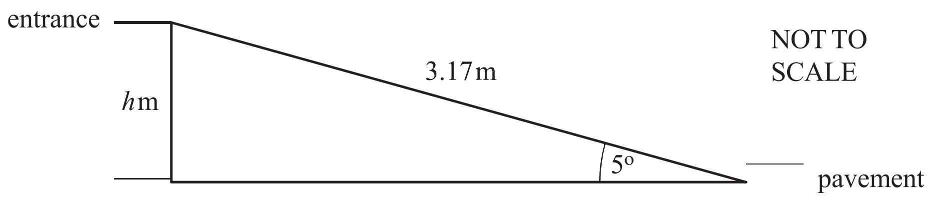
$\Delta \quad = 150 + 7x + 24x$

$x \quad 150 + 7 \times 6 + 24 \times 6$

$x \quad 150 + 42 + 144.$

$= 336$

Question 12



A shop has a wheelchair ramp to its entrance from the pavement.
The ramp is 3.17 metres long and is inclined at 5° to the horizontal.
Calculate the height, h metres, of the entrance above the pavement.
Show all your working.

[2]

Solution:

$\sin 5^{\circ} = \frac{h}{3.17}$

$h = 3.17 \times \sin 5^{\circ}$

$h = 0.2762$ meter

Hence, final answer is $= 0.2762$ meter

Question 13

Calculate the value of $(\cos 40^{\circ})^2 + (\sin 40^{\circ})^2$.

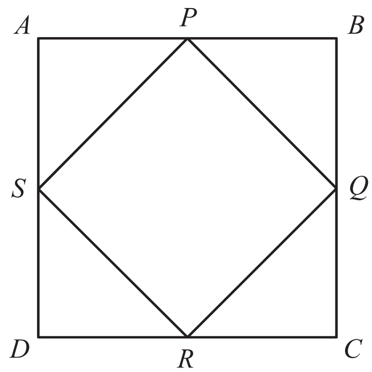
[2]

$\cos^2\theta + \sin^2\theta = 1$

In this case, when θ is 40 degrees, the expression evaluates to 1. Therefore:

$(\cos 40^{\circ})^2 + (\sin 40^{\circ})^2 = 1$

A square $ABCD$, of side 8 cm, has another square, $PQRS$, drawn inside it.
P, Q, R and S are at the midpoints of each side of the square $ABCD$, as shown in the diagram.



NOT TO SCALE

(a) Calculate the length of PQ. [2]

In square $ABCD$, $AB = BC$, $\angle ABC = 90°$ And P, Q are at the midpoint of AB and BC.

So $PB = \frac{1}{2}AB = \frac{1}{2}BC = BQ = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$

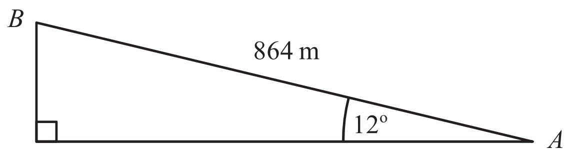
$In\ triangle\ PBQ,\ PQ = \sqrt{PB^2 + BQ^2} = \sqrt{(4 \text{ cm})^2 + (4 \text{ cm})^2} = 4\sqrt{2} \text{ cm}$

(b) Calculate the area of the square $PQRS$. [1]

The area of the square $PQRS = PQ^2 = (4\sqrt{2} \text{ cm})^2 = 32\text{sq} \cdot \text{cm}$

Question 15

A mountain railway AB is of length 864 m and rises at an angle of 12° to the horizontal.
A train is 586 m above sea level when it is at A.
Calculate the height above sea level of the train when it reaches B. [3]



NOT TO SCALE

$$\sin 12^{\circ} = \frac{h}{864}$$

$$h = 864 \sin 12^{\circ}$$

So,

When train is at B

its height above the sea level

$= (586 + h)$

$= 568 + 864 \sin 12^{\circ}$

$= [568 + (864 \times 0.201)]$

$= 568 + 178.848$

$= 746.848$ m