Mark Scheme (Results)

Summer 2026

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 1 Pure Mathematics

Edexcel and BTEC Qualifications

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General Marking Guidance

- Allcandidates mustreceive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCEMATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp - decimal places
- sf - significant figures
- - The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(NB specific markschemes maysometimesoverridethesegeneralprinciples)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=\cdots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\cdots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, q \neq 0$ leading to $x=\cdots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	
1(a)	$(-2, 20)$	B1 B1 **(2)**	1.1b 1.1b
(b)	$(-7, 3)$	B1 B1 **(2)**	1.1b 1.1b
			(4 marks)

Notes

General guidelines for all parts:

Remember tocheck answers written against the questions.
If there is any contradiction, mark the answers given in the body of the scripts.
If there is no labelling, mark the responses in the order given.
The coordinates need to be values not just a calculation e.g. **not** −4−3 for −7

Points can be written as a coordinate pair or separately as $x = ..., y = ...$

Do not allow coordinates written the wrong way round but isw if necessary
e.g. x=−2,y=20→(20,−2) scores B1B1 and isw

Condone missing brackets (one or both) e.g. x=−7,y=3 **or** (−7,3 **or** −7,3 for (−7,3)

Condone a missing comma e.g. (−73) for (−7,3)

Condone use of a semi-colon e.g. (−7;3) for (−7,3)

Condone vector notation e.g. $\begin{pmatrix}-2\\20\end{pmatrix}$ for (−2,20) and condone $\left(\frac{-2}{20}\right)$

Note that B0B1 is not a possible mark profile in this question

Notein both parts, some candidates show theirthinkingbytransforming the point piecewise e.g. (−4,5)→(−2,5)→(−2,20)

In such cases, mark their final **pair** of coordinates

(a)

B1: One correct coordinate, condone poor notation. See above.

B1: Both coordinates correct, condone poor notation. See above.

(b)

B1: One correct coordinate, condone poor notation. See above.

B1: Both coordinates correct, condone poor notation. See above.

Question	Scheme	Marks	AOs
2(a)	$f(1.1) = -0.379\ldots \quad f(1.2) = 0.655\ldots$	M1	1.1b
	Change of sign and function is continuous in the interval $\Rightarrow$ root in [1.1, 1.2] *	A1*	2.4
		(2)	
(b)(i)	$(x_2 =)\dfrac{1+\ln(7-3(1.1))}{2} = \ldots$	M1	1.1b
	$(x_2 =)$ awrt 1.1542	A1	1.1b
(ii)	$(\alpha =)\,1.1384$	B1cao	2.2a
		(3)	
			(5 marks)

Notes

(a)

M1: Attempts both f(1.1) and f(1.2) and obtains at least one correct to 1 significant figure or allow truncated to one decimal place. May use a narrower interval which you will need to check the value(s) on your calculator.

A1*: Completes the argument
Requires

- f(1.1) and f(1.2) correct to 1 significant figure or allow truncated to one decimal place (or their two allowable values if a narrower interval is used)
- a full reason including "change of sign" oe **and** "continuity" oe. Accept equivalent statements for sign change e.g. $f(1.1) \times f(1.2) < 0$ or e.g. $f(1.1) < 0,\ f(1.2) > 0$
- a minimal conclusion e.g. root, tick, α lies in the interval oe

Accept as a minimum, "change of sign, continuous, root" and mark positively their explanation.

Condone "the graph is continuous" and condone reference to x instead of α
Do not condone "change of sign, therefore continuous" or other incorrect statements such as e.g. "because x is continuous", or "the interval is continuous"

(b)(i) Check by the question. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

M1: Attempts to use the iterative formula with $x_1 = 1.1$ The value embedded in the formula is sufficient. May be implied by awrt 1.15 or awrt 1.132 provided no incorrect working is seen.
Candidates sometimes find x_3 (or possibly subsequent terms) rather than x_2 in which case the M1 can be implied. (See table on the next page for the first few iterations)

A1: awrt 1.1542 M1A1 is scored for sight of 1.1542 so we can ignore any labelling.

(ii)

B1cao:($\alpha =$) 1.1384 (4dp) Must be this value and **not** awrt 1.1384 but isw if they then round to 3sf. If the candidate lists their iterations but does not select an answer then take the final value, which still requires the answer to be given to 4dp.

Note that awrt 1.1384 does not imply M1

For reference:

x_1	1.1
x_2 .	1.15416641
x_3 .	1.1317101024
x_4 .	1.141142755
…	…
α	1.138365696

Question	Scheme	Marks	AOs
3(a)	$y = x^3 - 2x^2 + 5x - 7$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 3x^2 - 4x + 5$	M1 A1	1.1b 1.1b
		(2)	
(b)	y coordinate at P is 3	B1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 3(2)^2 - 4(2) + 5 = 9$ $y - 3 = 9(x - 2)$ or $y = 9x + c, 3 = 9 \times 2 + c \Rightarrow c = ...$	M1	1.1b
	$y = 9x - 15$	A1	1.1b
		(3)	
			(5 marks)

Notes

Mark (a) and (b) together

(a)

M1: For $x^n \to x^{n-1}$ seen at least once $x^3 \to x^2$ or $x^2 \to x$ or $5x \to 5$ or $-7 \to 0$

Indices may be unprocessed e.g. $x^3 \to x^{3-1 \text{ or } 2}$ $x^2 \to x^{2-1}$ or $5x \to 5x^0$

Do not withhold the mark if other powers on other terms are increased by 1

A1: Correct simplified expression with indices processed $3x^2 - 4x + 5$

Do not allow x^1 for x or $5x^0$ for 5

Apply isw if necessary, once a correct answer is seen.

The "$\frac{\mathrm{d}y}{\mathrm{d}x}$=" is **not** required. $3x^2 - 4x + 5 + c$ scores A0

(b)

B1: Correct y coordinate **which must come from a correct method if seen**. May be seen embedded in the correct position in an attempt of the equation of the tangent or seen as (2, 3)

M1: A complete method for the tangent that requires **both** of the following:

- substitutes $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find a value (must have scored M1 in part (a))

  Look for sight of $x = 2$ embedded followed by an answer. Condone slips.

  If no calculation is seen to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$ then the value of their gradient must be correct for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ from part (a).

- uses their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their y at $x = 2$ correctly to find the required equation.

  Look for y–"3"="9"(x–2) Do not condone sign slips for this mark.

  If they use $y = mx + c$ they must proceed as far as $c = ...$

  Do not allow this mark if they use a perpendicular gradient.

	This mark cannot be scored from a made up gradient and their y coordinate cannot be from an incorrect method i.e. from using their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (even if labelled as y) If a y coordinate is stated or implied with no working it must be 3
A1:	$y=9x-15$ (in this form) and following correct differentiation seen. isw

Question	Scheme	Marks	AOs
4(a)	$f(x) \geq -2$	B1	1.1b
		(1)	
(b)	e.g. $\text{gf}(2) = \text{g}\left(3(2)^2 - 2\right) = \text{g}(10) = \dfrac{5(10)-1}{10-4} = \ldots$ or e.g. $\text{gf}(x) = \dfrac{5(3x^2-2)-1}{3x^2-2-4} = \dfrac{5(3(2)^2-2)-1}{3(2)^2-2-4} = \ldots$	M1	1.1b
	$\dfrac{49}{6}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{5x-1}{x-4} \Rightarrow y(x-4) = 5x-1 \Rightarrow yx - 5x = 4y - 1$ $\Rightarrow x(y-5) = 4y-1$	M1	1.1b
	$\left(\text{g}^{-1}(x) =\right) \dfrac{4x-1}{x-5}$ or e.g. $\dfrac{1-4x}{5-x}$	A1	1.1b
		(2)	
Alt(c)	$y = \dfrac{5x-20+19}{x-4} \Rightarrow y = 5 + \dfrac{19}{x-4} \Rightarrow y - 5 = \dfrac{19}{x-4}$	M1	1.1b
	$\left(\text{g}^{-1}(x) =\right) 4 + \dfrac{19}{x-5}$ or e.g. $4 - \dfrac{19}{5-x}$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)

B1: Correct range.

f (x) or f or y and allow e.g. $-2 \leqslant y < \infty$ or $\{y \in \mathbb{R} : y \geqslant -2\}$ or $[-2, \infty)$.

Do not allow $-2 \leq x < \infty$, -2 [illegible] y [illegible]

(b)

M1: A complete method to find gf(2)

EITHER

- substitutes x=2 into f and substitutes the result into g to find a value for gf(2).

OR

- attempts gf(x) and substitutes $x = 2$ condoning slips. They must proceed to find a value.

e.g. gf(x)= $\dfrac{5(3x^2-2)-1}{3x^2-2-4} = \dfrac{5(3(2)^2-2)-1}{3(2)^2-2-4} = \ldots$

In both approaches condone arithmetical slips and bracket errors/omissions provided the order of the functions used is correct. May be implied by awrt 8.2

A1: $\frac{49}{6}$ or exact equivalent. e.g. $8\frac{1}{6}$ or $8.1\dot{6}$ but not 8.1666 or $8.166\ldots$.

isw once a correct answer is seen.

Note: May interchange x and y at the start or work towards $x = \ldots$
Ignore any attempts at the domain

(c)

M1: Correct attempt to multiply up by (x – 4), multiply out and isolate x by taking a factor of x out or equivalent work. e.g. $x(y-5)=4y-1$

Condone sign slips in their rearrangement and allow invisible brackets to be implied by further work.

A1: $\frac{4x-1}{x-5}$ oe e.g. $\frac{1-4x}{5-x}$ (does not need to be simplified)

A correct answer with no incorrect working seen scores M1A1. Do not be concerned by the labelling ofthe function so allow once rearranged to e.g. $y=\frac{4x-1}{x-5}$ or even $f^{-1}(x)=\frac{4x-1}{x-5}$

Alt(c)

M1: Writes $y=\frac{5x-1}{x-4}$ .in the form $y=5+\frac{k}{x-4}$ $k\neq 0$ and rearranges to isolate y and 5 on one sideoftheirequation.

Condonesignslipsin their rearrangement and allow invisible brackets to be implied by furtherwork.

A1: $4+\frac{19}{x-5}$ oee.g. $4-\frac{19}{5-x}$ (does not need to be simplified)

A correctanswerwith no incorrect working seen scores M1A1. Do not be concerned by the labellingofthefunction so allow once rearranged to e.g. $y=4+\frac{19}{x-5}$ or even $f^{-1}(x)=4+\frac{19}{x-5}$

Question	Scheme	Marks	AOs
5(a)	10 (m)	B1	2.2a
		(1)	
(b)	$y = 5 \Rightarrow 10\sin\left(\frac{t^2}{4}\right) = 5 \Rightarrow \sin\left(\frac{t^2}{4}\right) = \frac{1}{2} \Rightarrow \frac{t^2}{4} = \ldots$	M1	3.1b
	$\Rightarrow (t =)$ awrt 3.2 or $\sqrt{\frac{10\pi}{3}}$ (or condone $10\sqrt{6}$ or awrt 24.5 in degrees)	A1	1.1b
	$x = 100\sqrt{t} = 100\sqrt{"\sqrt{\frac{10\pi}{3}}"} = \ldots$	dM1	2.1
	$(x =)$ awrt 180 (m) (or awrt 495 (m) (see notes))	A1	3.2a
		(4)	
Alt(b)	$x = 100\sqrt{t} \Rightarrow \sqrt{t} = \frac{x}{100} \Rightarrow t = \left(\frac{x}{100}\right)^2 \Rightarrow y = \ldots$ or $y = 10\sin\left(\frac{t^2}{4}\right) \Rightarrow t = 2\sqrt{\sin^{-1}\left(\frac{y}{10}\right)} \Rightarrow x = \ldots$	M1	3.1b
	$y = 10\sin\left(\frac{1}{4}\left(\frac{x}{100}\right)^4\right)$ or $x = 100\sqrt{2\sqrt{\sin^{-1}\left(\frac{y}{10}\right)}}$	A1	1.1b
	$10\sin\left(\frac{1}{4}\left(\frac{x}{100}\right)^4\right) = 5 \Rightarrow \sin\left(\frac{1}{4}\left(\frac{x}{100}\right)^4\right) = \frac{1}{2} \Rightarrow \frac{1}{4}\left(\frac{x}{100}\right)^4 = \frac{5\pi}{6} \Rightarrow x = \ldots$ or $x = 100\sqrt{2\sqrt{\sin^{-1}\left(\frac{5}{10}\right)}} \Rightarrow x = \ldots$	dM1	2.1
	$(x =)$ awrt 180 (m) (or awrt 495 (m) (see notes))	A1	3.2a
		(4)	
			(5 marks)

Notes

(a)

B1: 10 (m) condone lack of units but if given they must be metres oe (condone $y = \ldots$)

(b) **Note the angle unit is radians in the question but condone assuming the angle unit is degrees to score all available marks.**

M1: Sets $y = 5$ and uses the parametric equation for y to reach at least $\frac{t^2}{4} = ...$ (expression or value - you do not need to be concerned by the angle it is set equal to including if it is degrees or radians). May be implied by awrt 1.4, awrt 3.2 or awrt 11.0, awrt 24.5. If just a value for t is stated then you may need to check $y = 5$ was substituted in.

A1: awrt 3.2 or $\sqrt{\frac{10\pi}{3}}$ which may be seen as $\frac{\sqrt{30\pi}}{3}$. May be seen within their working. Condone if other values are seen.
If assuming the angle unit is degrees then condone $10\sqrt{6}$ or awrt $24.5(^\circ)$ Do not withhold this mark if they proceed to use a different value for t

dM1: Correct use of their t value to find a value for x. It is dependent on the previous method mark. May be implied by awrt 120 or awrt 180 (or awrt 331 or awrt 495 if they have assumed the angle unit was degrees). If just a value is stated then you may need to check this is correct for their value of t.

A1: awrt 180 (m) or awrt 495 (m) condone lack of units but if given they must be metres (providing all previous marks have been scored)
As a minimum sight of awrt 3.2 (or awrt 24.5) and awrt 180 (m) (or awrt 495 (m)) can score M1A1dM1A1
Note if more than one answer is given score for awrt 180 (m) (awrt 495 (m)) and isw

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Alt(b)Findingthe cartesian equation

M1: Uses the x parameter to eliminate t and find y in terms of x so that $t=Bx^2$ $y=C\sin(Dx^4)$ oe where $B,C,D \neq 0$ May just see the cartesian equation of the required form.
Alternatively uses the y parameter to eliminate t and find x in terms of y so that $x=P\sqrt{\sqrt{Q\sin^{-1}(Ry)}}$ where $P,Q,R \neq 0$
The $y =$ or $x =$ may be implied by further work

A1: $y=10\sin\left(\frac{1}{4}\left(\frac{x}{100}\right)^4\right)$ oe or $x=100\sqrt{\sqrt{2\sin^{-1}\left(\frac{y}{10}\right)}}$ oe (allow unprocessed indices)
The $y =$ or $x =$ may be implied by further work

dM1: Sets their "$10\sin\left(\frac{1}{4}\left(\frac{x}{100}\right)^4\right)$"$=5$, and proceeds to find a value for x. It is dependent on the previous method mark. You do not need to be concerned by the mechanics of the rearrangement. Alternatively they may substitute $y = 5$ into $x=$"$100\sqrt{\sqrt{2\sin^{-1}\left(\frac{y}{10}\right)}}$" and proceed to find a value for x.
May be implied by awrt 120 or awrt 180 (or awrt 331 or awrt 495 if they have assumed the angle unit was degrees).

A1: awrt 180 (m) or awrt 495 (m) condone lack of units but if given they must be metres (providing all previous marks have been scored)
Note if more than one answer is given score for awrt 180 (m) (awrt 495 (m)) and isw

Question	Scheme	Marks	AOs
6(a)	$\int \frac{3}{7x+1}\,dx = \frac{3}{7}\ln(7x+1)+c$	M1 A1	3.1a 1.1b
		(2)	
(b)	$\int_2^p \frac{3}{7x+1}\,dx = \frac{6}{7} \Rightarrow \frac{3}{7}\ln(7p+1) - \frac{3}{7}\ln(15) = \frac{6}{7}$	M1	1.1b
	$\Rightarrow \ln\frac{7p+1}{15} = 2 \Rightarrow \frac{7p+1}{15} = e^2$ oe	dM1	1.1b
	$\Rightarrow (p =)\frac{15e^2-1}{7}$	A1ft	2.1
		(3)	
			(5 marks)

Notes

Mark (a) and (b) together

(a)

M1: Integrates to a form $A\ln(7x+1)$ $A \neq 0$ Condone missing brackets or modulus signs.
If the substitution $u=7x+1$ is attempted, the mark can be awarded for $k\ln u$ $k \neq 0$
Please note that $A\ln(7ax+a)$ or $A\ln a(7x+1)$ where a is a positive constant is also correct.

A1: $\frac{3}{7}\ln(7x+1)+c$ including "+ c" Brackets or modulus signs must be present. Isw after a correct answer.
Also accept $\frac{3}{7}\ln(7ax+a)+c$ where a is also a positive constant.
Withhold this mark for spurious notation e.g. an integral sign or dx in their final expression.

(b)

M1: Substitutes $x = p$ and $x = 2$ into an expression of the form $A\ln(7x+1)$ $A \neq 0$, (or $A\ln a(7x+1)$) subtracts either way round and sets equal to $\frac{6}{7}$. Condone invisible brackets and arithmetical slips. May be implied by further work.
Likely to be rare but may change the limits if they are working in terms of u so they would be substituting in e.g. 15 and $7p+1$ into an expression of the form $k\ln u$ $k \neq 0$, subtracting either way round and sets equal to $\frac{6}{7}$

dM1: Proceeds correctly to an equation not involving logarithms e.g. $\frac{7p+1}{\text{"15"}} = e^{\frac{6}{7}/A}$ oe
Do not allow this mark to be scored if they evaluate ln15 as a decimal.
Condone $e^{\frac{6}{7}/A}$ to be a decimal which you may need to check on your calculator.
Do not allow e.g. $p = \frac{e^{2+\ln 15}-1}{7}$ unless they correctly deal with ln15

A1ft: $(p=)\dfrac{15\mathrm{e}^2-1}{7}$ oe which must be simplified e.g. $(p=)\dfrac{15\mathrm{e}^2}{7}-\dfrac{1}{7}$ isw You may ft on their $\dfrac{3}{7}$ from part (a) so score for $(p=)\dfrac{15\mathrm{e}^{\frac{\frac{6}{7}}{"\frac{3}{7}"}}-1}{7}$

Question	Scheme	Marks	AOs
7(a)	(Area $R_1 =$) $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ oe OR (Area $R_2 =$) $\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)$ oe e.g. $\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin\theta$	B1	2.2a
	(Area $R_1 =$) $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ oe and (Area $R_2 =$) $\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)$ oe	B1	2.2a
	$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = 2\left(\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)\right)$ oe	M1	1.1b
	$(\theta - \sin\theta = 2\pi - 2\theta - 2\sin\theta)$ $\sin\theta + 3\theta - 2\pi = 0$	A1	2.1
		(4)	
(b)	$f'(\theta) = \cos\theta + 3$	B1ft	1.1b
	$\theta_1 = 1.3 \Rightarrow \theta_2 = 1.3 - \frac{f(1.3)}{f'(1.3)} = 1.3 - \frac{\sin(1.3) + 3(1.3) - 2\pi}{\cos(1.3) + 3}$	M1	1.1b
	$(\theta_2 =)$ awrt 1.734	A1	1.1b
		(3)	
		(7 marks)	

Notes

(a) **Note in the expressions for R_1 or R_2 either of $\sin(\pi-\theta)$ or $\sin\theta$ may be used as they are equivalent. Condone missing trailing brackets throughout.**
Do not penalise the use of or switching between $\sin(180-\theta)$ and $\sin(\pi-\theta)$

B1: A correct expression for **one** of the segments seen at least once. May be seen within an equation. Look for equivalent expressions e.g.
(Area $R_1 =$) $\frac{1}{2}r^2(\theta - \sin(\pi-\theta))$ or (Area $R_2 =$) $\frac{1}{2}r^2(\pi-\theta-\sin\theta)$

If converting the angle in radians to degrees for the sector(s) look for e.g.
(Area $R_1 =$) $\frac{\left(\theta\times\frac{180}{\pi}\right)}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta$ (Area $R_2 =$) $\frac{\left(180 - \frac{180\theta}{\pi}\right)}{360}\pi r^2 - \frac{1}{2}r^2\sin(\pi-\theta)$

B1: Correct expressions for **both** segments seen at least once. May see equivalent correct expressions. See note for 1st B1. May be seen within an equation. **Note they cannot just find R_2 (or R_1) using $R_1 = 2R_2$ to score this mark.**

M1:	Correctly forms an equation using $R_1 = 2R_2$ where their areas are of the form (Area R_1 =) $\alpha r^2(\theta \pm \sin\theta)$ oe and (Area R_2 =) $\beta r^2(\pi - \theta \pm \sin\theta)$ oe
	Note they may have already cancelled through by r^2 when they form the equation. Do not condone slips if they do not multiply through by 2 correctly oe for each term (or other similar methods). Invisible brackets would need to be implied by further work to ensure the correct equation was formed for their areas.
	May alternatively form the equation $R_1 = \frac{2}{3}(R_1 + R_2)$ oe
	e.g. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{2}{3}\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta + \frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)\right)$ oe
A1:	$\sin\theta + 3\theta - 2\pi = 0$ oe with all terms on one side of the equation and not seen to come from an incorrect method. Must see =0. All previous marks must have been scored.

Alt(a) Using the full semi-circle

B1:	See main scheme notes
B1:	Correct area of semi-circle $\frac{\pi r^2}{2}$ **and** the combined area of both triangles (may be separate) $\frac{1}{2}r^2\sin\theta + \frac{1}{2}r^2\sin(\pi-\theta)$ oe e.g. $r^2\sin\theta$
M1:	Uses $R_1 = 2R_2$ to correctly form the equation $R_1 + R_2 = 3R_2$ oe **OR** $R_1 + R_2 = \frac{3}{2}R_1$ oe where their individual areas are of the form (Area R_1 =) $\alpha r^2(\theta \pm \sin\theta)$ oe or (Area R_2 =) $\beta r^2(\pi - \theta \pm \sin\theta)$ and $R_1 + R_2 = \frac{\pi r^2}{2} \pm \gamma r^2\left(\sin\theta \pm \sin(\pi-\theta)\right)$ oe
	Using R_1 e.g. $\frac{\pi r^2}{2} - \frac{1}{2}r^2\sin\theta - \frac{1}{2}r^2\sin(\pi-\theta) = \frac{3}{2}\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$ oe
	Using R_2 e.g. $\frac{\pi r^2}{2} - \frac{1}{2}r^2\sin\theta - \frac{1}{2}r^2\sin(\pi-\theta) = 3\left(\frac{1}{2}r^2(\pi-\theta) - \frac{1}{2}r^2\sin(\pi-\theta)\right)$ oe
	Note they may have already cancelled through by r^2 when they form the equation. Condone slips in simplifying their expressions for the areas before forming the equation.
A1:	See main scheme notes

(b)	**Allow new values for p and q to be stated or used in part (b) if neither are found in part (a)**
B1ft:	$f'(\theta) = \cos\theta + 3$ oe which may be unsimplified. e.g. $f'(\theta) = -\cos\theta - 2\cos(\theta + \pi) + 3$
	May be embedded in the N-R formula. Follow through on their p and q to achieve $p\cos\theta + q$ oe which may be unsimplified and allow if still in terms of p and q
M1:	Attempts $1.3 - \frac{p\sin 1.3 + 1.3q - 2\pi}{\alpha\cos 1.3 + \beta}$ oe using their p and q (both non-zero) and $\alpha, \beta \neq 0$
	The value 1.3 embedded in the correct places is sufficient for this mark and allow this mark to be scored if still in terms of p and q i.e. $\alpha = p$ and $\beta = q$

	Note: Just stating $1.3-\frac{f(1.3)}{f'(1.3)}=...$ is M0 unless implied by further work e.g. $f(1.3)$ and $f'(1.3)$ e.g. $1.3-\frac{"-1.419627122"}{"3.267498829"}=...$**or** by their final answer Must be a correct N-R formula used. You may need to check their values with accuracy of at least 3sf or allow truncated to 2dp using their *p* and *q*. Allow if attempted in degrees. $f(1.3)=-2.360497974$ and $f'(1.3)=3.999742609$ and gives $\alpha=1.890162469$ (again look for 3sf or allow truncated to 2dp) Note that the full N-R calculator display is 1.734469053 The value of α is approximately 1.7674890… and scores M0A0 without evidence of using the N-R formula to imply the M mark
A1:	awrt 1.734 Ignore any subsequent iterations. Correct ans. with no incorrect working M1A1 Accept 1.7345 (may have rounded to 4dp) isw

Question	Scheme	Marks	AOs
8(a)	$\lim_{\delta x\to 0}\sum_{x=0}^{3} x e^{2x}\,\delta x = \int_0^3 x e^{2x}\,\mathrm{d}x$	B1	1.2
		(1)	
(b)	$\int x e^{2x}\,\mathrm{d}x = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,\mathrm{d}x$	M1	3.1a
	$\int x e^{2x}\,\mathrm{d}x = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}(+c)$	A1	1.1b
	$\int_0^3 x e^{2x}\,\mathrm{d}x = \left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^3 = \frac{3}{2}e^6 - \frac{1}{4}e^6 - \left(-\frac{1}{4}\right)$	dM1	2.1
	$\frac{5}{4}e^6 + \frac{1}{4}$	A1	2.2a
		(4)	
			(5 marks)

Notes

Mark (a) and (b) together

(a)

B1: States $\int_0^3 xe^{2x}\,\mathrm{d}x$ or equivalent but must include the limits and the dx. If not scored in part (a) check part (b) to see if B1 can be scored.
Condone dx ↔ δx as it is very difficult to tell one from another sometimes.
Withhold this mark if there is spurious notation e.g. "lim" or $+c$

(b)

M1: Obtains an expression of the form $\alpha xe^{2x} - \beta\int e^{2x}(\mathrm{d}x)$ oe where $\alpha, \beta > 0$ You do not need to be concerned about how they arrive at this. M1 may be scored via the DI method for the expression of the form $\alpha xe^{2x} - \beta e^{2x}$ where $\alpha, \beta > 0$ (see below notes for A1)

A1: $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}(+c)$ oe Allow this to be unsimplified.

Watch out for the DI method which some will do:

	D	I
+	x	e^{2x}
−	1	$\frac{e^{2x}}{2}$
+	0	$\frac{e^{2x}}{4}$

	Giving correct integration $\int_0^3 x\mathrm{e}^{2x}\,\mathrm{d}x = \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}(+c)$ Score M1 for the expression of the form $\alpha x\mathrm{e}^{2x} - \beta\mathrm{e}^{2x}$ where $\alpha, \beta \neq 0$ and A1 for a correct expression
dM1:	Applies the limits 0 and 3 to an expression of the form $\alpha x\mathrm{e}^{2x} - \beta\mathrm{e}^{2x}$ oe where $\alpha, \beta \neq 0$ and subtracts either way round, achieving the expression of the form $A\mathrm{e}^{6} + B$ where $A, B \neq 0$ and which may be unsimplified i.e. terms do not need to be collected. It is dependent on the previous method mark. An expression with the limits substituted in is sufficient to score this mark or may be implied by $\frac{5}{4}\mathrm{e}^{6} + \frac{1}{4}$ oe following correct integration. Do not allow this mark to be scored if they have assumed substituting in 0 into the expression gives a value of 0 unless it was embedded into the expression. However, condone the slip for $\left[\text{"}\frac{1}{2}\text{"}x\mathrm{e}^{2x} - \text{"}\frac{1}{4}\text{"}\mathrm{e}^{2x}\right]_0^3 = \text{"}\frac{5}{4}\text{"}\mathrm{e}^{6}\text{"}+\frac{1}{4}\text{"}$ to score dM1A0
A1:	$\frac{5}{4}\mathrm{e}^{6} + \frac{1}{4}$ oe in the required form and all previous marks in (b) must have been scored. Fractions do not need to be in their lowest terms. Condone missing/poor notation e.g. missing the dx throughout. If they subtract the wrong way round then withhold this mark if they just omit the negative signs. Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{5}{4}\mathrm{e}^{6} + \frac{1}{4} + c\,\mathrm{d}x$ is M1A1dM1A0

Question	Scheme	Marks	AOs
9(a)	e.g. LHS: $(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta) = \frac{\cos\theta}{\sin\theta}-1+1-\frac{\sin\theta}{\cos\theta}$	M1	1.1b
	$=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}=\frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$ OR $=\frac{1}{\tan\theta}-\tan\theta=\frac{1-\tan^2\theta}{\tan\theta}$	dM1	3.1a
	$=2\cot 2\theta$	A1	2.1
		(3)	
Alt(a)	e.g. RHS: $k\cot 2\theta=\frac{k}{\tan 2\theta}=\frac{k\cos 2\theta}{\sin 2\theta}=\frac{k(\cos^2\theta-\sin^2\theta)}{2\sin\theta\cos\theta}$	M1	1.1b
	$=\frac{k}{2}\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}=\frac{k}{2}(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta)$	dM1	3.1a
	$\Rightarrow k=2$	A1	2.1
		(3)	
(b)	$5(\cos x+\sin x)(\operatorname{cosec} x-\sec x)=4\operatorname{cosec}^2 2x$ $\Rightarrow 5\times 2\cot 2x=4\operatorname{cosec}^2 2x$	B1ft	1.1b
	e.g. $10\cot 2x=4\operatorname{cosec}^2 2x \Rightarrow 10\cot 2x=4(1+\cot^2 2x)$ $2\cot^2 2x-5\cot 2x+2=0 \Rightarrow \cot 2x=2, \frac{1}{2}$ or e.g. $10\cot 2x=4\operatorname{cosec}^2 2x \Rightarrow \frac{10\cos 2x}{\sin 2x}=\frac{4}{\sin^2 2x}$ $\Rightarrow 5\cos 2x\sin^2 2x=2\sin 2x \Rightarrow \sin 2x(5\sin 2x\cos 2x-2)=0$ $\Rightarrow 5\sin 2x\cos 2x=2 \Rightarrow \sin 4x=0.8$	M1	3.1a
	$\cot 2x=2,\frac{1}{2}\Rightarrow x=\ \ldots$ or e.g. $\sin 4x=0.8 \Rightarrow x=\ \ldots$	dM1	2.1
	$(x=)$ awrt -76.7, awrt -58.3, awrt 13.3, awrt 31.7	A1A1	1.1b 1.1b
		(5)	
			(8 marks)

Notes

Condone a complete proof entirely in x (or another variable) instead of θ

Condone notational errors and missing variables in their working provided the intention is clear and only penalise other errors on the final A mark.

(a) **e.g. working from the LHS**

M1: Attempts to expand the brackets and uses both $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$

Condone sign slips.

May see $\operatorname{cosec}\theta-\sec\theta=\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta} \Rightarrow (\cos\theta+\sin\theta)\frac{(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$

Also allow this to be implied by e.g. $(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta)= \ \cot\theta \ \ \tan\theta$

dM1: **EITHER** uses both $\cos^2\theta - \sin^2\theta \quad \cos 2\theta$ and $\sin 2\theta \quad 2\sin\theta\cos\theta$

OR uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and simplifies to a single fraction in $\tan\theta$ Dep. on previous M

A1: Achieves $2\cot 2\theta$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

Alt(a) e.g. working from the RHS

M1: Either uses both $\cos^2\theta - \sin^2\theta \quad \cos 2\theta$ and $\sin 2\theta \quad 2\sin\theta\cos\theta$ **or** uses $\tan 2\theta \quad \dfrac{2\tan\theta}{1-\tan^2\theta}$

dM1: Uses the difference of two squares, and both $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$

It is dependent on the previous method mark. Condone sign slips.

A1: Proceeds from $\dfrac{k}{2}(\cos\theta+\sin\theta)(\operatorname{cosec}\theta-\sec\theta)$ to deduce that $k=2$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

Note if they work from both sides they are likely to score the two method marks from each of M1 in the main scheme notes and alternative method notes. The final A will be scored for reaching a point where they can deduce that $k=2$

Score in the order which maximises the marks.

e.g.

M1: LHS: $\operatorname{cosec}\theta-\sec\theta = \dfrac{\cos\theta-\sin\theta}{\sin\theta\cos\theta} \Rightarrow (\cos\theta+\sin\theta)\dfrac{(\cos\theta-\sin\theta)}{\sin\theta\cos\theta} = \cot\theta - \tan\theta$

(main scheme 1st M1)

dM1: RHS: $k\cot 2\theta = k\dfrac{1-\tan^2\theta}{2\tan\theta} = \dfrac{k}{2}\cot\theta - \dfrac{k}{2}\tan\theta$ (alternative method 1st M1)

A1: Achieves two similar statements from which $1=\dfrac{k}{2} \quad k=2$ with all previous marks scored and no genuine errors including signs, manipulation and mixed variables.

(b) May work in a different variable which is acceptable.

Condone mixed variables provided it is recovered in later work

B1ft: Deduces the correct equation using the result from part (a). Follow through their k and allow if in terms of k. i.e. $5 \quad k\cot 2x = 4\operatorname{cosec}^2 2x$. Allow a restart but they would need to reach the same stage.

M1: Requires:

- EITHER uses $1 \quad \cot^2 2x = \quad \operatorname{cosec}^2 2x$ to obtain **and solve** a 3TQ in $\cot 2x$. They can solve the 3TQ by factorising (may factorise for an equivalent quadratic equation which is acceptable), completing the square, formula (usual rules apply) or via a calculator. If just the roots are stated then they must **both** be correct to 2sf for their 3TQ. You may need to check this. This mark cannot be implied by proceeding from a 3TQ to an angle with no intermediate working. May multiply through by $\tan^2 2x$ and solve a 3TQ in $\tan 2x$.

•	OR e.g. converts to $\sin 2x$ and $\cos 2x$ and uses $\sin 4x = 2\sin 2x\cos 2x$ to obtain an equation of the form $\sin 4x = p$ oe where p $\lvert 1 \rvert$ Do not be concerned by cancelling both sides by $\sin 2x$ and losing the equation $\sin 2x = 0$ (0 is not a valid solution)
dM1:	Correct order of operations for their equation to obtain **at least one value for *x***. It is dependent on the previous method mark. e.g. $\sin 4x = p \quad x = \frac{1}{4}\sin^{-1} p = \ldots$ If just a value is stated you may need to check this on your calculator.
A1:	For any 2 of awrt-77 ,awrt-58 ,awrt13 ,awrt32 Also allow in radians (awrt-1.3,awrt-1.0,awrt 0.23, awrt 0.55)
A1:	awrt-76.7 ,awrt-58.3 ,awrt13.3 ,awrt31.7 and no extras in the given range (ignore any found outside of the range). Note the common additional solution of 0 is A0. The degrees symbol is not required. Answers in radians score A0

Question	Scheme	Marks	AOs
10(a)	$x=2 \Rightarrow y=\frac{14}{\sqrt{12+4}}=\frac{7}{2}$	B1	1.1b
	$y=\frac{7x}{\sqrt{3x^2+4}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{7\sqrt{3x^2+4}-7x\times\frac{1}{2}(3x^2+4)^{-\frac{1}{2}}\times 6x}{3x^2+4}$	M1 A1	2.1 1.1b
	$\frac{7\sqrt{3(2)^2+4}-7(2)\times\frac{1}{2}(3(2)^2+4)^{-\frac{1}{2}}\times 6(2)}{3(2)^2+4}=\frac{7}{16}$ $\Rightarrow y-\frac{7}{2}=-\frac{16}{7}(x-2)$	M1	2.1
	$\Rightarrow 32x+14y-113=0$	A1	1.1b
		(5)	
(b)	$y=\frac{113}{14}$	B1ft **(1)**	1.1b
		B1ft	
(c)	Trapezium area is $\frac{1}{2}\left(\frac{113}{14}+\frac{7}{2}\right)\times 2=\frac{81}{7}$		2.2a
	$\int\frac{7x}{\sqrt{3x^2+4}}\,\mathrm{d}x=\frac{7}{3}(3x^2+4)^{\frac{1}{2}}(+c)$	M1 A1	1.1b 1.1b
	Area is $\frac{81}{7}-\left[\frac{7}{3}(3x^2+4)^{\frac{1}{2}}\right]_0^2=\frac{81}{7}-\left(\frac{7}{3}\times 4-\frac{7}{3}\times 2\right)=\frac{81}{7}-\frac{14}{3}$	M1	3.1a
	$=\frac{145}{21}$	A1	1.1b
		(5)	
			(11 marks)

Notes

Check the diagram for all parts. If there is a contradiction between answers, the answer in the main body of the script takes precedence.

(a)

B1: $y=\frac{7}{2}$ or may see e.g. $\left(2,\frac{7}{2}\right)$ oe

M1: Differentiates to achieve the form (which may be unsimplified)

Quotient rule: $\frac{A\sqrt{3x^2+4}-Bx^2(3x^2+4)^{-\frac{1}{2}}}{3x^2+4}$ oe A,B 0

Product rule: $C(3x^2+4)^{-\frac{1}{2}}-Dx^2(3x^2+4)^{-\frac{3}{2}}$ oe C,D 0

Implicit: $E(3x^2+4)^{\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x}+Fxy(3x^2+4)^{-\frac{1}{2}}=G$ oe E,F,G 0 (May see alternatives)

A1: $\dfrac{7\sqrt{3x^2+4} - 7x \times \frac{1}{2}(3x^2+4)^{-\frac{1}{2}} \times 6x}{3x^2+4}$ oe which may be unsimplified or simplified

e.g. $7(3x^2+4)^{-\frac{1}{2}} - 21x^2(3x^2+4)^{-\frac{3}{2}}$ or $28(3x^2+4)^{-\frac{3}{2}}$ or $(3x^2+4)^{\frac{1}{2}}\dfrac{dy}{dx} + 3xy(3x^2+4)^{-\frac{1}{2}} = 7$

Whilst not strictly dependent on the first M, there must have been some attempt to differentiate i.e. they cannot get a gradient from substituting x=2 into the original equation (it must be a changed function) and using the negative reciprocal of this as their m. If no method is shown to find the y coordinate then it must be $\frac{7}{2}$

M1: A complete method for the equation of the normal:

- attempts to find the gradient at P using their $\dfrac{dy}{dx}$. Condone arithmetical slips and miscopying provided the intention is clear. If just a value is stated you may need to check this is correct for their $\dfrac{dy}{dx}$. (Accept e.g. x=2, $\dfrac{dy}{dx}$ =... without needing to check)
- uses the negative reciprocal to find the gradient of the normal at P (note they may have already manipulated their $\dfrac{dy}{dx}$ to $-\dfrac{dx}{dy}$. $m \to -\dfrac{1}{m}$) This must be a correct process in the method
- uses their gradient of the normal with x = 2 and their y coordinate to find the equation of the normal i.e. $y-$"$\frac{7}{2}$"$=-\dfrac{1}{\text{"}\frac{7}{16}\text{"}}(x-2)$ Condone a sign slip on the $y-$ "$\frac{7}{2}$"

If they use $y=mx+c$ they must proceed as far as c = …

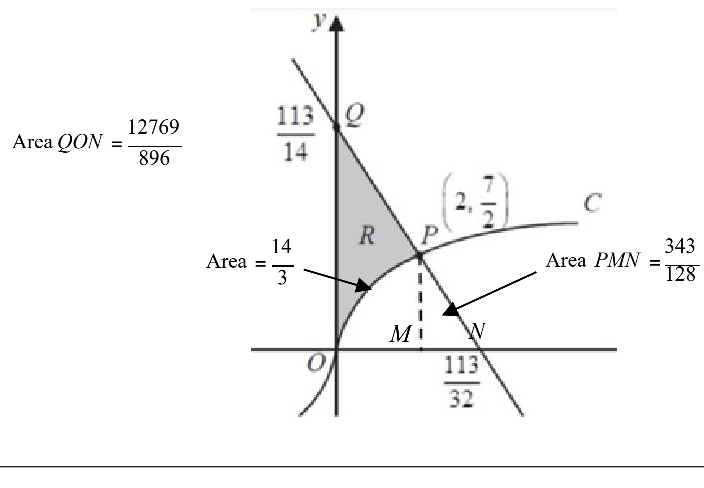
A1: $32x+14y-113=0$ Allow any integer multiple of this equation.

(b)

B1ft: $\dfrac{113}{14}$ or exact ft on their normal equation so allow for $-\dfrac{\text{their } c}{\text{their } b}$

Accept e.g. $\left(0, \text{"}\dfrac{113}{14}\text{"}\right)$ oe Also accept $-\dfrac{c}{b}$ or $\left(0, -\dfrac{\text{their } c}{\text{their } b}\right)$ if no equation was found in (a)

(c)



B1ft: Area of trapezium: $\frac{1}{2}\times\left("\frac{113}{14}"+"\frac{7}{2}"\right)\times 2$ oe e.g. $\frac{1}{2}\times"\frac{113}{14}"\times"\frac{113}{32}"-\frac{1}{2}\times\left("\frac{113}{32}"-2\right)\times"\frac{7}{2}"$

Follow through their part (b) and their y coordinate of P.
The expression is sufficient. The x coordinate of point N see the diagram above must be greater than 2 if used and the y coordinate of point Q must be greater than 0

Also allow if in terms of a, b and c where appropriate
e.g. $\frac{1}{2}\left(-\frac{c}{b}+"\frac{7}{2}"\right)\times 2$ oe e.g. $\frac{1}{2}\times\left(-\frac{c}{b}\right)\times\left(-\frac{c}{a}\right)-\frac{1}{2}\times\left(-\frac{c}{a}-2\right)\times"\frac{7}{2}"$ if no equation was found in (a)

Alternatively, may integrate the equation of l so look for

$$\int_0^2 "-\frac{16}{7}x+\frac{113}{14}"\,dx=\left["-\frac{8}{7}x^2+\frac{113}{14}x"\right]_0^2="-\frac{8}{7}"\times 2^2+"\frac{113}{14}"\times 2="\frac{81}{7}"$$

You would need to follow through on their l and they would need to have substituted into the correctly integrated expression.
isw if a correct expression is seen but they evaluate incorrectly.
If just a value is stated you may need to check that it is a correct follow through.

M1: Integrates via any method to obtain $D(3x^2+4)^{\frac{1}{2}}(+c)$ $D\neq 0$ May see attempts at the substitution method or even integration by parts but you just need to look for the correct form for their integrated expression to see that it is equivalent.
Note they may attempt

$$\int_0^2\left("-\frac{16}{7}x+\frac{113}{14}"-\frac{7x}{\sqrt{3x^2+4}}\right)dx$$ but ignore the integration on the straight line for this mark.

A1: $\frac{7}{3}(3x^2+4)^{\frac{1}{2}}(+c)$ oe which may be unsimplified. Look for the equivalent correct answer depending on their substitution e.g. using $u=3x^2+4$ $\frac{7}{3}\sqrt{u}(+c)$.

M1: Complete and correct strategy for the area of R. **Whilst not strictly dependent, there must have been some attempt to integrate to find the area under the curve, however poorly carried out.**

Look for area of their trapezium – area under the curve between $x=0$ and $x=2$ following an attempt to integrate where both areas are positive. Condone subtracting either way round for this mark.

e.g. $"\frac{81}{7}"-\left["\frac{7}{3}\left(3x^2+4\right)^{\frac{1}{2}}"\right]_0^2="\frac{81}{7}"-\left("\frac{7}{3}"\times 4-\frac{7}{3}\times 2"\right)$

Look out for other longer methods incorporating additional areas which then need subtracting. The overall strategy needs to be correct but condone arithmetical slips provided the intention is clear.

Any areas which required integration should have had **both** limits substituted in and subtracted either way round (may be implied). If they have used the substitution method then the limits may be different for their integrated expression. e.g. using $u=3x^2+4$ would require substituting in $u = 4$ and $u = 16$

A1: $\frac{145}{21}$ or exact equivalent e.g. $6\frac{19}{21}$ but do not allow a decimal

Question	Scheme	Marks	AOs
11(a)	$(R=)\sqrt{185}$	B1	1.1b
	$\tan\alpha=\frac{13}{4} \Rightarrow \alpha= \ldots$	M1	1.1b
	$\alpha=$ awrt 1.272 or $\sqrt{185}\cos(x+1.272)$	A1	1.1b
		(3)	
(b)(i)	$30-\sqrt{185}$ or awrt 16.4	B1ft	2.2a
(b)(ii)	$\cos\left(\frac{\pi t}{12}+1.272+0.2\right)=-1 \Rightarrow \frac{\pi t}{12}+1.272+0.2=\pi \Rightarrow t=\ldots$	M1	3.1b
	$(t=)$ awrt 6.4	A1	1.1b
	6:22 or 6:23oe (seenotes)	B1ft	3.2a
		(4)	
(c)	$D=30+\sqrt{185}\cos\left(\frac{\pi t}{12}+1.272+0.2\right)$ $\Rightarrow \left(\frac{\mathrm{d}D}{\mathrm{d}t}=\right)-\frac{\pi\sqrt{185}}{12}\sin\left(\frac{\pi t}{12}+1.272+0.2\right)$	M1	3.4
	3pm $\rightarrow$ $t=15$ $\Rightarrow \left(\frac{\mathrm{d}D}{\mathrm{d}t}=\right)-\frac{\pi\sqrt{185}}{12}\sin\left(\frac{15\pi}{12}+1.272+0.2\right)$	dM1	3.1b
	$\Rightarrow (k=)$ awrt2.75	A1	1.1b
		(3)	
(d)	Examples: • the demand for electricity is not the same every day • during cold weather the demand will be greater • in the summer, demand will be less	B1	3.5b
		(1)	
			(11 marks)

Notes

(a) **Note that B0M1A1 is possible**

B1: $\sqrt{185}$

M1: Awardfor $\tan\alpha=\pm\frac{13}{4} \Rightarrow \alpha=\ldots$, $\tan\alpha=\pm\frac{4}{13} \Rightarrow \alpha=\ldots$,

$\sin\alpha=\pm\frac{13}{"\sqrt{185}"} \Rightarrow \alpha=\ldots$ or $\cos\alpha=\pm\frac{4}{"\sqrt{185}"} \Rightarrow \alpha=\ldots$

May be implied by ($\pm$) any of awrt 1.27 or awrt 72.9° or awrt 0.298 or awrt 17.1° or awrt 1.87 or awrt 107°

A1: $\alpha=$awrt1.272 or $\sqrt{185}\cos(x+1.272)$

Note that $\sqrt{185}\cos(x+1.272)$ scores B1M1A1

Mark (b)(i) and(ii) together

(b)(i)

B1ft: $(30-\sqrt{185})$ (GW) **or** awrt 16. 4 (GW) Condone lack of / ignore units and follow through on their R to 3sf

(ii)

M1: Sets $\frac{\pi t}{12}$+"1.272"+0.2=$\pi \Rightarrow t$=.. May be implied by a correct value for t for their "1.272". If just a value is stated you may need to check this on your calculator.

May set "$30+\sqrt{185}\cos\left(\frac{\pi t}{12}+1.272+0.2\right)$"="$30-\sqrt{185}$" $\Rightarrow t$=... but condone if they have rounded $30-$"$\sqrt{185}$" to 3sf or better.

A1: awrt 6.4 and no others in the range $0 \leqslant t \leqslant 24$ unless rejected. This can be implied by 6:22 or 6:23 oe and no additional times given.
If no value for t is given then allow to be implied by 6:22 or 6:23 oe

B1ft 6:22 or 6:23 oe or ft on their final t to nearest min or truncated **provided** $0 \leqslant$ "t" $\leqslant 24$ e.g. 0622 or 6 hours 23 minutes after midnight. Condone various notation or forms provided there is evidence of the correct time. E.g. 6:23am 6.22 or 6,23 or 6;23 or 6/22. If using e.g. am or pm it must be consistent with their value of t.

(c) Notethat nodifferentiation seen scoresM0dM0A0.

M1: Attempts to differentiate their D=30+"$\sqrt{185}$"$\cos\left(\frac{\pi t}{12}+q\right)$ to obtain ...$\sin\left(\frac{\pi t}{12}+q\right)$ where q="1.272"+0.2oe or condone if 0.2 is missing

Condone also differentiating $D = 30+4\cos\left(\frac{\pi t}{12}+0.2\right)-13\sin\left(\frac{\pi t}{12}+0.2\right)$ to obtain ... $\sin\left(\frac{\pi t}{12}+0.2\right)$ +... $\cos\left(\frac{\pi t}{12}+0.2\right)$

NB correct derivative is $-\frac{4\pi}{12}\sin\left(\frac{\pi t}{12}+0.2\right)-\frac{13\pi}{12}\cos\left(\frac{\pi t}{12}+0.2\right)$

dM1: Substitutes t = 15 into their derivative and obtains a value. It is dependent on the previous method mark. 15 embedded in their derivative followed by a value or stating when t = 15, $\frac{\mathrm{d}D}{\mathrm{d}t}$ =... is sufficient but if just a value is stated with no working then you may need to check this value is correct for their $\frac{\mathrm{d}D}{\mathrm{d}t}$ (condone mislabelling)

A1: awrt 2.75

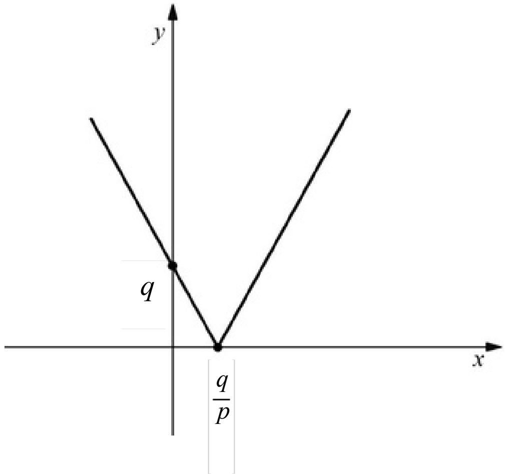
(d)

B1: for a valid reason why it would not be sensible to use the same equation to model the demand for electricity in the UK on **every day of the year.** Mark positively.

Look for any reason that suggests the demand is not the same **every day** or it is **different to this particular day.** They either need to:

- suggest the demand is not the same every day such as
  - it varies at different times of the year / the length of day(light) varies
  - this was only for a particular day
- suggest a possible scenario where the demand might be different such as
  - it is different at weekends
  - may have dinner at a different time of day on a Sunday
  - might introduce an incentive scheme
  - the price cap might affect how much people use
  - people might go abroad in school holidays

Do not allow “we do not have enough data” (too vague)

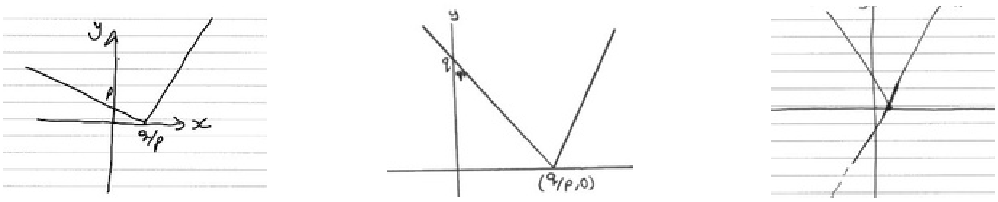
Question	Scheme	Marks	AOs
12(a)		B1 B1 B1	1.1b 1.1b 1.1b
		(3)	
(b)	$\frac{k}{x} = -px + q \Rightarrow px^2 - qx + k = 0$	M1 A1	2.1 1.1b
	$b^2 - 4ac < 0 \Rightarrow q^2 - 4pk < 0$	M1	3.1a
	$0 < k < \frac{q^2}{4p}$	A1	2.2a
		(4)	
			(7 marks)

Notes

(a) **Do not penalise if they have working for part (b) on the graph as well**

B1: A V shaped curve sitting on the positive x-axis and appearing in at least quadrant 1. Do not be too concerned about the graph being non-symmetrical.
Condone/give BOD if their right arm of the V extends as part of their working.

Examples of graphs which score B1



Note any labelling on the graph takes precedence over coordinates stated in the main body of the text.

B1: The vertex is on the positive x-axis and is marked $\left(\frac{q}{p},0\right)$ or allow $x = \frac{q}{p}$ which may be seen as the intercept on the graph. May be scored from an incorrect shape for the graph, but must have a single vertex on the x-axis. Condone the coordinates written the wrong way round provided point is in the correct position.

B1:	The graph **passes through the positive *y*-axis** at $(0, q)$ or allow $(y =)\, q$ which may be seen as the intercept on the graph. Note a graph which just meets at $(0, q)$ and does not appear in both quadrants 1 and 2 is B0 Condone the coordinates written the wrong way round provided point is in the correct position.
(b)	**Note some candidates also attempt $\frac{k}{x} = px - q$ Withhold the final A mark only when the range is not fully correct. E.g. they would have to reject the range of values $k \leqslant 0$ or give an inequality which is $k < \frac{q^2}{4p}$ oe only**
M1:	Sets $\frac{k}{x} = -px + q$ and rearranges to a 3TQ equation in x with terms collected on one side. Condone sign slips in their rearrangement only. Note setting $\frac{k}{x} = -px - q$ is incorrect and scores M0
A1:	$px^2 - qx + k = 0$ oe with all terms on the same side. May be implied by further work e.g. attempting the discriminant with correct values for a, b and c for their 3TQ equation in x
M1:	Attempts $b^2 - 4ac > 0$ for their 3TQ in x with a, b and c in terms of p, q and k. This does not need to be from setting $\frac{k}{x} = -px + q$. The constants p, q and k embedded in the correct positions in the inequality is sufficient for this mark. Condone use of … for this mark and if "=" seen then check further work to see if a valid inequality is formed.
A1:	$k < \frac{q^2}{4p}$ oe e.g. $k < 0.25q^{2}p^{-1}$ Does not require a lower bound for k but if given it must be $0 < k < \frac{q^2}{4p}$ oe May use alternative notation e.g. $\left\{k : k < \frac{q^2}{4p}\right\}$ or $\left(0, \frac{q^2}{4p}\right)$ Do not accept e.g. $\left[0, \frac{q^2}{4p}\right]$ or $k \leq \frac{q^2}{4p}$ Only isw if a correct range is given but they then attempt to use set/interval notation incorrectly.

Note: Alternatively, it is unlikely to be successful but they may attempt to square both sides and multiply out both sides, making little further progress. However, it they attempt the difference of two squares **and** reach a 3TQ equation in x with all terms collected on one side this may score M1.

Question	Scheme	Marks	AOs
13(a)	$\frac{\sqrt{2}\cos\theta}{\sin\theta}=\frac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta}$ or e.g. $\left(\sqrt{2}\cos\theta\right)^2=\sqrt{3}\cot\theta\sin\theta$	M1	3.1a
	$2\cos^2\theta=\sqrt{3}\cos\theta \Rightarrow \cos\theta=...\left(=\frac{\sqrt{3}}{2}\right)$	dM1	1.1b
	$r=\frac{\sqrt{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or e.g. $\frac{\sqrt{2}\cos"\frac{\pi}{6}"}{\sin"\frac{\pi}{6}"}$ or e.g. $\frac{\sqrt{3}\cot"\frac{\pi}{6}"}{\sqrt{2}\cos"\frac{\pi}{6}"}$	ddM1	2.1
	$r=\sqrt{6}$	A1	2.2a
		(4)	
(b)	e.g. $(u_1=)\frac{1}{\left(\sqrt{6}\right)^2}\sin\left(\frac{\pi}{6}\right)=\frac{1}{12}$ **OR** e.g. $(u_2=)\frac{\sin"\frac{\pi}{6}"}{"\sqrt{6}"}$	M1	3.1a
	$S_3=\frac{1}{12}+\frac{\sqrt{6}}{12}+\frac{6}{12}$ or $S_3=\frac{a\left(1-r^3\right)}{1-r}=\frac{1}{12}\left(\frac{1-\sqrt{6}^3}{1-\sqrt{6}}\right)$	dM1	2.1
	$=\frac{7+\sqrt{6}}{12}$ oe e.g. $\frac{7}{12}+\frac{\sqrt{6}}{12}$	A1	2.2a
		(3)	
Alt(b)	$u_3+u_4+u_5=\frac{1}{2}+\sqrt{2}\times\frac{\sqrt{3}}{2}+\sqrt{3}\times\frac{3}{\sqrt{3}}\left(=\frac{7+\sqrt{6}}{2}\right)$	M1	3.1a
	$\Rightarrow S_3=\frac{u_3+u_4+u_5}{r^2}=\frac{"\frac{7+\sqrt{6}}{2}"}{\left("\sqrt{6}"\right)^2}$	dM1	2.1
	$=\frac{7+\sqrt{6}}{12}$ oe e.g. $\frac{7}{12}+\frac{\sqrt{6}}{12}$	A1	2.2a
		(3)	
			(7 marks)

Notes

Mark (a) and (b) together and condone notational errors provided the intention is clear.
May work in degrees or radians – either is acceptable.

(a)

M1: Uses the 3 terms correctly to set up an equation in θ. Condone slips but the trigonometric functions should be in the correct places. Give BOD where cos and cot may look similar when written.

$$\frac{\sqrt{2}\cos\theta}{\sin\theta} = \frac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta} \text{ oe e.g. } \frac{\sqrt{2}\cos\theta}{\sin\theta} = \frac{\sqrt{3}\cos\theta}{\sqrt{2}\sin\theta\cos\theta}$$

May see e.g. $\sin\theta \times \frac{\sqrt{3}\cot\theta}{\sqrt{2}\cos\theta} = \sqrt{2}\cos\theta$

Condone a correct equation coming from $a = \sin\theta$, $ar = \sqrt{2}\cos\theta$, $ar^2 = \sqrt{3}\cot\theta$

dM1: Proceeds from their equation and rearranges to find an **exact** value for $\cos\theta$ where $-1 \leqslant \cos\theta \leqslant 1$ and $\cos\theta \neq 0$

It is dependent on the previous method mark. May be implied by further work such as an **exact** value for θ in radians ($\frac{\pi}{6}$ (radians)) or in degrees ($30(°)$) but not just the final answer $\sqrt{6}$ Do not be concerned by the mechanics of the rearrangement.

Note for any further marks they must only use exact values

ddM1: Attempts e.g. $\frac{u_4}{u_3} = \frac{\sqrt{2}\cos\theta}{\sin\theta}$ using their exact value for $\cos\theta$ and the corresponding value for $\sin\theta = \sqrt{1 - \left(\text{"}\frac{\sqrt{3}}{2}\text{"}\right)^2}$ and proceeds to find an **exact value** for r. It is dependent on both of the previous method marks.

Condone working with an exact value for $\cos\theta$ which would not give an exact value for θ provided they work with the exact values for $\cos\theta$ and $\sin\theta$ only. If they proceed to find an inexact value for θ and use this then this mark cannot be scored.

They must be attempting the division the correct way round.

If just an **exact** value of r is stated then you may need to check it is correct for their $\cos\theta$ (or θ if they have proceeded to find an **exact** θ and substituted in to find r)

A1: $\sqrt{6}$ cao (from a fully correct method)

Requires
- sight of a correct equation involving the 3 terms
- sight of either a correct value for $\cos\theta$ or θ

e.g. $\frac{\sqrt{2}\cos\theta}{\sin\theta} = \frac{\sqrt{3}}{\sqrt{2}\sin\theta} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow (r =)\frac{\sqrt{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{6}$ scores M1dM1ddM1A1

(b)

M1: A correct expression for the first term or second term which may be in terms of trigonometric functions (using u_3, u_4 or u_5) and/or uses their **exact** value for r from (a).

Note r may be expressed in terms of trigonometric functions but do not allow $r = \theta$ and r cannot clearly be an angle in radians or degrees e.g. $\frac{\pi}{6}$

e.g. $(u_1 =)\frac{\sin\left(\frac{\pi}{6}\right)}{\left(\sqrt{6}\right)^2}$ or $(u_1 =)\frac{\sin\theta}{\left(\frac{\sqrt{2}\cos\theta}{\sin\theta}\right)^2}$ or $(u_2 =)\frac{\sin\left(\frac{\pi}{6}\right)}{\sqrt{6}}$

May be implied by a correct **exact** value for a for their **exact** value for r
Condone if an exact value for r is not found in (a) and they just state a value for r then provided it is not clearly an angle in degrees or radians allow e.g. $r = 4$ $a = \frac{\sin\theta}{16}$ but not $r = \frac{\pi}{6}$ so $a = \frac{\sin\theta}{\left(\frac{\pi}{6}\right)^2}$ Note it is unlikely they will score any further marks unless they have chosen an exact value for r of either $\sqrt{6}, \sqrt{2}$ or $\frac{\sqrt{6}}{3}$

dM1: Attempts to find the sum of the first 3 terms using their exact value for r from (a). It is dependent on the previous method mark.
The expression is sufficient which must be numerical and exact. **All trigonometric functions must have been dealt with for this mark.**

e.g. $(S_3 =)\frac{1}{12} + \frac{\sqrt{6}}{12} + \frac{6}{12}$ or $\left(S_3 = \frac{a(1-r^3)}{1-r} = \right)\frac{1}{12}\left(\frac{1-\sqrt{6}^3}{1-\sqrt{6}}\right)$

A1: $\frac{7+\sqrt{6}}{12}$ or exact equivalent in simplest form e.g. $\frac{7}{12} + \frac{\sqrt{6}}{12}$ (from a fully correct method)

Alt(b) $S_3 = \frac{u_3 + u_4 + u_5}{r^2}$

M1: Adds the three terms in the question using their exact value for $\cos\theta$ where $-1 < \cos\theta < 1$ and $\cos\theta \neq 0$ and the corresponding value for $\sin\theta = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$.

May be implied by $\frac{7+\sqrt{6}}{2}$ oe Condone if they just state a value for $\cos\theta$ if they did not achieve one in part (a) provided $-1 < \cos\theta < 1$ and $\cos\theta \neq 0$
Alternatively they may sum the three terms using their **exact value** for θ correctly (θ cannot be r). e.g. $\sin\left(\frac{\pi}{6}\right) + \sqrt{2}\cos\left(\frac{\pi}{6}\right) + \sqrt{3}\cot\left(\frac{\pi}{6}\right)$
Note they may use the sum formula using exact values for $\sin\theta$ and r
If the method does not show where these values come from then they need to be consistent with their exact value for $\cos\theta$

The expression is sufficient which must be numerical and **exact**.

dM1: Attempts to divide their sum of three terms by their exact value for r^2. It is dependent on the previous method mark. The expression is sufficient which must be numerical and **exact**. **All trigonometric functions must have been dealt with for this mark.**
If they did not achieve an exact value for r in (a) it is unlikely that their chosen value for $\cos\theta$ will lead to an exact value for r.
Note they may attempt $S_3 = \dfrac{u_3 + u_4 + u_5}{r}$ **or use the summation formula with** $r = \dfrac{1}{"\sqrt{6}"}$ **which would score M1dM1 at the same time. The numerical and exact expression would be sufficient to score both marks.**

A1: $\dfrac{7+\sqrt{6}}{12}$ or exact equivalent in simplest form e.g. $\dfrac{7}{12}+\dfrac{\sqrt{6}}{12}$ (from a fully correct method)

Beware of incorrect values for r leading to $\dfrac{7+\sqrt{6}}{2}$ or $\dfrac{7+\sqrt{6}}{12}$

e.g. $r = \dfrac{\sqrt{6}}{6} \quad u_3 + u_4 + u_5 \Rightarrow \dfrac{7+\sqrt{6}}{12}$ (M1dM0A0 via Alt method)

e.g. $r = \dfrac{\sqrt{6}}{6} \Rightarrow a = \dfrac{\sin\left(\frac{\pi}{6}\right)}{r^2} \quad S_3 = 3 + 3\dfrac{\sqrt{6}}{6} + 3\left(\dfrac{\sqrt{6}}{6}\right)^2 = \dfrac{7+\sqrt{6}}{2}$ (M1dM1A0 via Main scheme)

Question	Scheme	Marks	AOs
14(a)	$\frac{dx}{dt} = x(A-t) \Rightarrow \int \frac{1}{x}\,dx = \int (A-t)\,dt$ $\Rightarrow \ln x = At - \frac{t^2}{2}\ (+c)$ oe	M1 A1	3.1a 1.1b
	$t = 0, x = 0.3 \Rightarrow \ln 0.3 = c$ $t = 4, x = 21 \Rightarrow \ln 21 = 4A - 8 + \ln 0.3$ $\Rightarrow A = \frac{8+\ln 70}{4}\ (= 3.06212...) \approx 3.06$	M1	3.1b
	e.g. $\ln x = 3.06t - \frac{t^2}{2} + \ln 0.3 \Rightarrow x = 0.3e^{3.06t - \frac{t^2}{2}}$ or e.g. $x = 0.3e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2}}$ or e.g. $x = 0.3e^{\text{awrt } 3.06t - \frac{t^2}{2}}$	dM1 A1	2.1 3.3
		(5)	
(b)	Maximum occurs when $\frac{dx}{dt} = 0 \Rightarrow t = A$	B1ft	2.2a
	$x = 0.3e^{3.06 \times 3.06 - \frac{3.06^2}{2}} = ...$	M1	3.4
	$(x =)$ awrt 32.6 (mg L^{-1}) or awrt 32.4 (mg L^{-1})	A1	3.2a
		(3)	
(c)	$0.1 = 0.3e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2}} \Rightarrow \frac{1}{3} = e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2}} \Rightarrow \left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2} = -\ln 3$ $\frac{t^2}{2} - 3.06t - \ln 3 = 0$	M1	3.1b
	$\frac{t^2}{2} - 3.06t - \ln 3 = 0 \Rightarrow t = ...$	dM1	3.4
	$(T =)$ awrt 6.46	A1	2.3
		(3)	
		(11 marks)	

Notes

(a)

M1: Separates the variables and integrates to the form $\ln x = At \pm \text{"}\frac{1}{2}\text{"}t^2\ (+c)$ oe

e.g. $\ln x = \left(A \pm \text{"}\frac{1}{2}\text{"}t\right)t\,(+c)$ or $\ln x = \pm\text{"}\frac{1}{2}\text{"}(A-t)^2\ (+c)$

A1: $\ln x = At - \frac{t^2}{2}\ (+c)$ oe

Note if there is no constant of integration in their expression then no further marks

can be scored in this part

M1: Requires:

- states or uses $t = 0$ and $x = 0.3$ to find their constant of integration.
- states or uses $t = 4$ and $x = 21$ to find A ($\neq 0$)

Both constants may be simplified or unsimplified. May be implied by their constants so may need to be checked. If working is seen they must be using their equation in x and t.

dM1: Proceeds from $\ln x = At - \frac{1}{2}t^2 + c$ oe to $x = e^{At - \frac{1}{2}t^2 + c}$ oe where A and c in their equation may be numerical if already found.

It is dependent on the <u>first</u> method mark only.

Condone sign slips and isw once a correct form is seen. Condone miscopying slips provided the intention is clear. Note that they may have done some rearranging in earlier work so you need to check the processing from when they have first integrated. Allow invisible brackets to be recovered or implied by further work.

A1: $x = 0.3e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2}}$ oe in the form $x = f(t)$ e.g. $x = \frac{3}{10}e^{2t + t\ln 70^{\frac{1}{4}} - \frac{t^2}{2}}$ or $x = 0.3e^{\left(\frac{8+\ln 70 - 2t}{4}\right)t}$

Allow e.g. $x = 0.3e^{\text{awrt}\,3.06t - \frac{t^2}{2}}$ or $x = \frac{3}{10}e^{2t + \text{awrt}\,1.06t - \frac{t^2}{2}}$ or $x = e^{\left(\frac{8+\ln 70}{4}\right)t - \frac{t^2}{2} + \ln 0.3}$

Must be $x =$…. and constants must be to 3sf or better where necessary. Condone the t appearing after the ln70 if seen and condone $-1.2(0)$ for ln0.3) Isw once a correct answer is seen.

(b)

B1ft: Deduces that the maximum occurs when $t = A$ or allow their numerical value for A to at least 3sf. Condone A 0

M1: Uses their equation of the model to find a numerical value for x when $\frac{dx}{dt} = 0$

Condone if they do not deduce $t = A$ and attempt to differentiate their equation from part (a) and allowing them to state $\frac{dx}{dt} = 0, t = ...$ and use this value in their equation of the model to find a numerical value for x. In either approach condone the value of t to be negative. If the value for t is not seen in their equation of the model and just a value for x is stated you may need to check this on your calculator (or allow $t = "3.06" \Rightarrow x = ...$)

Note they may alternatively attempt to complete the square on

$x = "0.3e^{-\frac{t^2}{2} + 3.062t}" \Rightarrow x = "0.3e^{-\frac{1}{2}(t-3.062)^2 + 4.6879..}"$ and deduce the maximum is when $t = "3.062"$ and proceeds to $x = "0.3e^{0 + 4.6879..}" = ...$

A1: awrt 32.6 (mgL^{-1}) ignore units. Condone awrt 32.4 for use of $A = 3.06$

(c) **Allow use of T or t (or another letter)**

M1: Sets their $f(t) = 0.1$ where $f(t)$ is of the form $\alpha e^{\beta t^2 + \gamma t + c}$ oe $\alpha, \beta, \gamma \neq 0$ (but c could be zero) and achieves a 3TQ equation in t which may be unsimplified. The = 0 may be

	implied by further work. May start with an earlier form of the equation from part (a) or proceed directly to this e.g. $\ln 0.1 = "3.06"t - \frac{t^2}{2} + \ln 0.3$ which scores M1 **You do not need to be concerned by the mechanics of the rearrangement and condone reverting back from an incorrect form equation back to the required form.**
dM1:	Solves their 3TQ in t (or T) by any correct method including a calculator. If just the roots are stated with no working seen then you may need to check that the root(s) is/are correct for their 3TQ to 2sf. It is dependent on the previous method mark.
A1:	awrt 6.46 with no other values and a correct 3TQ equation is seen which may be unsimplified. Condone use of rounded values provided an answer of awrt 6.46 is reached e.g. $\frac{t^2}{2} - "3.06"t + \ln 0.1 - \ln 0.3 (=0)$

Question	Scheme	Marks	AOs
15	Assume that a and b are both odd so that $a=2m+1$ and $b=2n+1$ where $m,n\in\mathbb{N}, m\neq n$		
	$a^2+b^2=(2m+1)^2+(2n+1)^2$ e.g. $=4m^2+4m+4n^2+4n+2$ or e.g. $=2(2m^2+2m+2n^2+2n+1)$ $\Rightarrow$ which is even	M1 A1	1.1b 2.1
	e.g. So c^2 is even $\Rightarrow$ c is even $c=2k$ say $\Rightarrow 2(2m^2+2m+2n^2+2n+1)=(2k)^2=4k^2$ $\Rightarrow 2m^2+2m+2n^2+2n+1=2k^2$	dM1	2.1
	As e.g. $2m^2+2m+2n^2+2n+1$ is odd and $2k^2$ is even there is a contradiction. Hence a and b cannot both be odd. *	A1*	2.4
		(4)	
			(4 marks)

Notes

M1: Substitutes the given algebraically distinct odd numbers into a^2+b^2, and expands. Terms do not need to be collected. Condone slips multiplying out including $(2m+1)+(2n+1)=4m^2+1+4n^2+1$

Condone coefficient slips but they should have an expression of the form $Am^2+Bm+Cn^2+Dn+E$ where $A,C,E \neq 0$

May be implied by their factorised form.

A1: Requires **both** a correct expression that is clearly even **AND** they either need to state that the expression is even oe or deduce that c^2 is even

e.g. $4m^2+4m+4n^2+4n+2$ which is even

e.g. $2(2m^2+2m+2n^2+2n+1)$ hence c^2 is even (or condone so c is even)

e.g. $4(m^2+m+n^2+n)+2$ which is not odd

Condone dividing through by 2 to show that there are no fractions

Do not accept e.g. "$4m^2+4m+4n^2+4n+1+1$ which is even" as the expression is not in a form where it clearly is even – they would need +2 not +1+1 or e.g. factorised

dM1: A full argument leading to a contradiction

e.g. recognises that c^2 must be of the form $4k^2$ and then divides through by 2 (can use any letter other than a, b, c, m or n for k)

A1*: Fully correct proof with appropriate reasoning/explanation and conclusion.
Requires
- all previous marks scored
- deduces they have a contradiction because $2m^2+2m+2n^2+2n+1$ oe is **odd** and $2k^2$ is **even** oe i.e. they need to explain what the contradiction is between the two statements
- minimal conclusion e.g. hence proven, tick, QED

Do not penalise partial rephrasing of the statement that they were asked to prove in their conclusion. e.g. omitting "both" provided their statement does not clearly contradict the proof.

Note there are many alternativewaysofproceedingtoacontradictionwhichshouldbe marked in the same way.

e.g. Considering multiples of 4

M1: See main scheme notes See

A1: main scheme notes

dM1: c^2 is even $\Rightarrow$ c is even $c=2k$

The expression for a^2+b^2 must be written as $4\left(m^2+m+n^2+n\right)+2$ oe with the factor of 4 taken out and c^2 written as $4k^2$ or $4 \times k^2$

Alternatively, they may divide through by 4

e.g. $4m^2+4m+4n^2+4n+2=(2k)=4k^2$ $^2m^2+m+n^2+n+\frac{1}{2}=k^2$

(They can use any letter other than a, b, c, m or n for k)

A1*: Fully correct proof with appropriate reasoning/explanation and conclusion.
Requires
- all previous marks scored
- deduces they have a contradiction because $4\left(m^2+m+n^2+n\right)+2$ oe is two more than a multiple of 4 oe e.g. the right-hand side is divisible by 4 but the left-hand side is not i.e. they need to explain what the contradiction is between the two statements.
- minimal conclusion e.g. hence proven, tick, QED

Do not penalise partial rephrasing of the statement that they were asked to prove in their conclusion. e.g. omitting "both" provided their statement does not clearly contradict the proof.