



Mark Scheme (Results)

Summer 2026

Pearson Edexcel GCE
In AS Mathematics (8MA0)
Paper 01 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b, and c)

3. Completing the square

Solving $x^2 + bx + c = 0 : (x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks	AOs
1 (a)	$4(x-3) > 8-2x \Rightarrow 6x > 20$	M1	1.1b
	$\Rightarrow x > \frac{10}{3}$ o.e.	A1	1.1b
		(2)	
(b)	$(2x+5)(x-8) < 0 \Rightarrow$ Critical values $-\frac{5}{2}, 8$	M1	1.1b
	$-\frac{5}{2} < x < 8$	A1	1.1b
		(2)	
(c)	$\frac{10}{3} < x < 8$	B1	1.1b
		(1)	
(5 marks)			
Notes:			

(a)

M1: Expands the bracket on LHS and simplifies to reach a form $ax \dots b$ or $x \dots c$ where $\dots$ is any equality or inequality.

A1: $x > \frac{10}{3}$ o.e. in terms of x but requires some intermediate line, e.g. $4x-12 > 8-2x$

Allow recovery if they use equality throughout and achieve the correct answer.

Answer only scores no marks. Do not ISW if they go on to state e.g. $x = 4$

(b)

M1: Achieves at least one of the critical values of $-\frac{5}{2}$ and/or 8 which may be seen embedded in an equation or inequality.

If they decide to expand the brackets and then attempt to solve from here, they must still find at least one of the critical values $-\frac{5}{2}$ and/or 8 i.e. this mark cannot be scored for an attempt to solve their quadratic.

A1: $-\frac{5}{2} < x < 8$ o.e. such as $x \in \left(-\frac{5}{2}, 8\right)$ Must be seen in (b) and not clearly their answer to (c).

Not scored for just $x < 8$, $x > -\frac{5}{2}$, but condone $x > -\frac{5}{2}$ **and** $x < 8$ or e.g. $x > -\frac{5}{2} \cap x < 8$

Answer only scores both marks.

Must be in terms of x but do not penalise a second time.

(c)

B1: $\frac{10}{3} < x < 8$ o.e. such as $x \in \left(\frac{20}{6}, 8\right)$

Not scored for just $x < 8$, $x > \frac{10}{3}$, but condone $x > \frac{10}{3}$ **and** $x < 8$ or e.g. $x > \frac{10}{3} \cap x < 8$

Note that this is not a follow through mark but allow it to be scored provided it follows from their answers in parts (a) and (b) (the A marks need not have been scored).

e.g. a) $k > \frac{10}{3}$, b) $x < 8$, $x < -\frac{5}{2}$ leading to c) $\frac{10}{3} < x < 8$ scores M1A0 M1A0 B1

e.g. a) $x > \frac{10}{3}$, b) $x = 8$, $x = -\frac{5}{2}$ (with no selection of inequalities) leading to

c) $\frac{10}{3} < x < 8$ scores M1A1 M1A0 B1 with the B1 condoned.

Note that e.g. a) $k > \frac{10}{3}$, b) $x > 8$, $x < -\frac{5}{2}$ leading to c) $\frac{10}{3} < x < 8$ scores M1A0 M1A0 B0 as the answer to part (c) does not follow.

Must be in terms of x but do not penalise a second time.

Question Number	Scheme	Marks	AOs
2 (a)	$(2-3x)^6$		
	Constant term = 2^6	B1	1.1b
	Correct form for any of terms 2, 3 or 4 ${}^6C_1 2^5 (\pm 3x)$ or ${}^6C_2 2^4 (\pm 3x)^2$ or ${}^6C_3 2^3 (\pm 3x)^3$	M1	1.1b
	Two correct and simplified terms from $-576x + 2160x^2 - 4320x^3$	A1	1.1b
	$64 - 576x + 2160x^2 - 4320x^3$	A1	1.1b
		(4)	
(b)	$\{x=\} -0.01$	B1	2.2a
		(1)	
(5 marks)			
Notes:			

(a)

- B1:** Constant term of 2^6 (or 64) which need not be simplified. Not scored for e.g. ${}^6C_0 2^6 (\pm 3x)^0$
Not scored if they have other constant terms in their expression, but condone if they go on to divide by e.g. 2 or 8
- M1:** Correct form for any of terms 2, 3 or 4 which may be implied by a correct simplified term.
Look for ${}^6C_1 2^5 (\pm 3x)$, ${}^6C_2 2^4 (\pm 3x)^2$ or ${}^6C_3 2^3 (\pm 3x)^3$ but condone lack of bracketing on $(\pm 3x)$
Alternatives to e.g. 6C_2 include $\binom{6}{2}$, $\frac{6!}{2!4!}$ or just 15 (FYI Pascal's triangle: 1, 6, 15, 20)
- A1:** Two correct and simplified terms from terms 2, 3 and 4: $-576x + 2160x^2 - 4320x^3$
Ignore subsequent work for this mark only.
- A1:** Fully correct and simplified $64 - 576x + 2160x^2 - 4320x^3$ and ignore attempts at x^4 etc.
Terms may be listed or on different lines. Accept e.g. $64 + -576x + 2160x^2 + -4320x^3$
Do not ignore subsequent work if they e.g. divide through by e.g. 2 or 8.

Alt (a)

- B1:** Correctly takes out a factor and proceeds to $2^6 \left(1 - \frac{3}{2}x\right)^6$ which may be unsimplified.
May be implied.
- M1:** Expands $(1 \pm kx)^6$, $k \neq \pm 1$ to give the correct structure for the 3rd or 4th term
 $\frac{(6)(5)}{2!}(kx)^2$ or $\frac{(6)(5)(4)}{3!}(kx)^3$ with or without the brackets around kx

A1: A correct simplified or unsimplified form of the expansion of $\{64\}\left(1 - \frac{3}{2}x\right)^6$

e.g. $\{64\}\left(1 + (6)\left(-\frac{3}{2}x\right) + \frac{(6)(5)}{2!}\left(\pm\frac{3}{2}x\right)^2 + \frac{(6)(5)(4)}{3!}\left(-\frac{3}{2}x\right)^3 + \dots\right)$ or e.g.

$$\{2^6\}\left(1 - 9x + \frac{135}{4}x^2 - \frac{135}{2}x^3\right)$$

May be implied by their final answer.

Allow terms to be listed, which may be on different lines.

Ignore subsequent work for this mark only.

A1: Fully correct and simplified $64 - 576x + 2160x^2 - 4320x^3$ and ignore attempts at x^4 etc.

Terms may be listed or on different lines. Accept e.g. $64 + -576x + 2160x^2 + -4320x^3$

Do not ignore subsequent work if they e.g. divide through by e.g. 2 or 8.

(b)

B1: Deduces that $x = -0.01$ or e.g. $-\frac{1}{100}$

Ignore any attempt to substitute a value for x in.

Question Number	Scheme	Marks	AOs
3 (a)	$8x^3 + 31x^2 - 4x = x(8x^2 + 31x - 4) = x(ax + b)(cx + d)$ with $ac = \pm 8, bd = \pm 4$	M1	1.1b
	$= x(8x - 1)(x + 4)$	A1	1.1b
		(2)	
(b)	Deduces $4^y = \frac{1}{8}$	M1	1.1b
	$y = -\frac{3}{2}$ only	A1	1.1b
		(2)	
(4 marks)			
Notes:			

(a) Condone any spurious = 0 throughout part (a).

Ignore any attempts to solve the cubic = 0, including attempts by completing the square, the quadratic formula or simply writing the roots down.

M1: Attempts to factorise by taking out/cancelling a factor of x or e.g. $8x$ and factorises the remaining quadratic factor using the usual rules to achieve $(ax + b)(cx + d)$ with

$ac = \pm 8, bd = \pm 4$ so allow e.g. $8x^3 + 31x^2 - 4x \rightarrow (8x^2 \pm 31x \pm 4) \rightarrow (4x + 1)(2x - 4)$

or $ac = \pm 1, bd = \pm \frac{1}{2}$ so allow e.g. $8x^3 + 31x^2 - 4x \rightarrow \left(x^2 \pm \frac{31}{8}x \pm \frac{1}{2}\right) \rightarrow (x + 2)\left(x - \frac{1}{4}\right)$

Condone incorrect work that is recovered, e.g.

$8x^2 + 31x - 4 \rightarrow (8x \pm 1)(8x \pm 32) \rightarrow (8x \pm 1)(x \pm 4)$

Alternatively, divides the cubic by either $(x \pm 4)$ or $(8x \pm 1)$ or $\left(x \pm \frac{1}{8}\right)$ and then takes out a factor of x or $8x$ from the remaining quadratic.

Condone e.g. $(x + 0)(8x - 1)(x + 4)$ for this mark.

A1: cao $x(8x - 1)(x + 4)$ or $8x\left(x - \frac{1}{8}\right)(x + 4)$ or e.g. $4x\left(2x - \frac{1}{4}\right)(x + 4)$ appearing on one line.

Correct answer only scores both marks. Note that e.g. $(x + 0)(8x - 1)(x + 4)$ scores A0.

Note that if they divide by x or $8x$ and then recover it for the final answer then they may score full marks.

$8x^3 + 31x^2 - 4x \rightarrow 8x^2 + 31x - 4 \rightarrow (8x - 1)(8x + 32) \rightarrow x(8x - 1)(x + 4)$ scores M1A1.

Apply ISW if they factorise correctly but then go on to solve and write down any roots.

(b)

M1: Deduces that 4^y equals any positive non-zero root from their $x(8x-1)(x+4)=0$

Alternatively, they may deduce e.g. $16^y = \left(\frac{1}{8}\right)^2$ or $64^y = \left(\frac{1}{8}\right)^3$

A1: $y = -\frac{3}{2}$ only o.e. with logs removed. Must follow $(8x-1)$ seen as a factor.

Any other roots for y seen must be rejected or $y = -\frac{3}{2}$ clearly selected.

Correct answer only (with no method) scores M0A0.

A minimally acceptable solution is $4^y = \frac{1}{8} \Rightarrow y = -\frac{3}{2}$

Does not need to be labelled as y but must clearly be their answer.

May come from e.g. $y = \frac{\log\left(\frac{1}{8}\right)}{\log 4} = -\frac{3}{2}$ without further work and/or evidence of use of base 2,

or it may come directly from $\log_4\left(\frac{1}{8}\right) = -\frac{3}{2}$

Question Number	Scheme	Marks	AOs
4 (a)	$t = 4, V = 50 \Rightarrow 50 = a + 2b$ or $t = 9, V = 20 \Rightarrow 20 = a + 3b$	M1	3.1b
	Attempts both and solves to find values for a and b	dM1	1.1b
	$V = 110 - 30\sqrt{t}$	A1	3.3
		(3)	
(b)	220 litres	B1ft	3.4
		(1)	
(c)	Sets their $110 - 30\sqrt{t} = 0 \Rightarrow t = \dots$	M1	3.4
	$\{0 \leq t \leq \frac{121}{9}\}$	A1ft	3.5b
		(2)	
(6 marks)			
Notes:			

(a)

M1: Forms at least one equation using $t = 4, V = 50$ or $t = 9, V = 20$ the correct way round in the given model, e.g., $50 = a + b\sqrt{4}$

dM1: Uses both pieces of information the correct way round and solves their equations to find values for **a and b**. Dependent on the previous method mark.

Condone a slip in substitution when creating the equations, e.g., failing to square root the 9 or 4, or miscopying 20 as 30, and do not be concerned about their processing to solve the simultaneous equations.

A1: cao $V = 110 - 30\sqrt{t}$ o.e. in any form e.g. $V = 110 - 30t^{\frac{1}{2}}$ or $V + 30\sqrt{t} = 110$

Condone e.g. $V = 110 + -30\sqrt{t}$

If just $a = 110$ and $b = -30$ seen in part (a) this mark may be awarded for a complete model seen in part (b) or part (c).

(b)

B1ft: 220 litres including units but follow through on twice their value for **a** provided it is positive.

May come from other models e.g. $V = a + bt$ for part (b).

(c) **Their model must be of the correct form for this part.**

M1: Realises that the model breaks down when $V = 0$ so solves their $110 - 30\sqrt{t} = 0 \Rightarrow t = \dots$

Do not be concerned about their processing to solve the equation for this mark.

May be implied by e.g. $t < \text{awrt } 13.4$ or by their $t \leq \dots$ following through on their value to 3sf.

A1ft: Finds the limitation to the model $\{0 \leq t \leq \frac{121}{9}\}$ There is no requirement to see the lower limit.

Condone solutions such as $t \leq 13.\dot{4}$ and $t < \frac{121}{9}$ or an equivalent statement in words, e.g. “the model is only valid for values of t up to “ $\frac{121}{9}$ ” or “ $13.\dot{4}$ is the last possible value...”.

Must be exact, so not e.g. $t \leq 13.4$ regardless of whether the exact critical value has previously been stated.

Follow through on their a and b so look for $t \leq \frac{a^2}{b^2}$ or $t < \frac{a^2}{b^2}$ but a and b must be numerical.

Ignore any other spurious incorrect statements such as t cannot be 0, provided they do not clearly affect the upper limit, and ignore any incorrect units.

Question Number	Scheme	Marks	AOs
5 (a)	Attempts correct cosine rule $\cos \theta^\circ = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5}$	M1	1.1b
	$\cos \theta^\circ = \left\{ \frac{-8}{40} \right\} = -\frac{1}{5} *$	A1*	1.1b
		(2)	
(b)	Uses e.g. $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta + \frac{1}{25} = 1 \Rightarrow \sin^2 \theta = \dots$	M1	3.1a
	$\sin \theta^\circ = \sqrt{\frac{24}{25}} = \left(\frac{2\sqrt{6}}{5} \right)$	A1	1.1b
	Attempts to find the area $= \frac{1}{2} \times 4 \times 5 \times \sqrt{\frac{24}{25}}$	M1	2.1
	$= 4\sqrt{6} \text{ (cm}^2\text{)}$	A1	1.1b
		(4)	
(6 marks)			
Notes:			

(a)

M1: Attempts the cosine rule the correct way round $\cos \theta^\circ = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5}$ o.e. such as

$$7^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos \theta^\circ$$

The formula they are substituting into should be correct, with values in the correct place, but condone e.g. a sign slip when they substitute provided a correct formula has been seen.

Allow any argument to be used (that is not clearly incorrect e.g. C).

A1*: Arrives at the given answer through correct work and no errors.

There is no need to see the degrees symbol at any point. Should be using θ but condone A or BAC in the final line. It is acceptable for other arguments to be used in their previous work but recovered in their final line to either θ , A or BAC provided they are not clearly incorrect e.g. C

Condone e.g. -0.2 or $\frac{-1}{5}$ or $\frac{1}{-5}$ for $-\frac{1}{5}$ and there is no need to see $-\frac{8}{40}$

(b)

M1: Uses $\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$ o.e. and proceeds to find an exact value for $\sin \theta$ or $\sin^2 \theta$ which may be implied by later work.

Alternatively, uses a right-angled triangle with sides ± 1 , ± 5 and $\pm \sqrt{24}$ in the correct places to find an exact value for $\sin \theta$ or $\sin^2 \theta$. May be implied (and you may not see the triangle).

Not scored for use of $\sin \left(\cos^{-1} \left(-\frac{1}{5} \right) \right)$

You may see attempts splitting triangle ABC into two right-angled triangles and then use of Pythagoras to find the height and lengths of these right-angled triangles. See the alternate mark scheme at the end or use review if unsure.

A1: Correct exact value for $\sin \theta$ which may be unsimplified. Condone inclusion of $\pm$

$\sin \theta = \sqrt{\frac{24}{25}} \left(\text{or } \frac{2\sqrt{6}}{5} \right)$ following sight of 101.5... and without a method seen does **not** imply

M1A1. However, $\sin \theta = \sqrt{\frac{24}{25}} \left(\text{or } \frac{2\sqrt{6}}{5} \right)$ without sight of 101.5... implies M1A1.

M1: Complete method to find the area of the triangle, which may be inexact.

While this mark is not dependent on the previous method mark, it does require a correct method to find a value for $\sin \theta$ or $\sin^2 \theta$.

Note that they may use the sine rule to find one of the other angles and then attempt

e.g. $\frac{1}{2} \times 7 \times 4 \times \sin ABC$

Condone, for this mark only, attempts that use $\sin \left(\cos^{-1} \left(-\frac{1}{5} \right) \right)$ or e.g. $\sin(101.5...)$

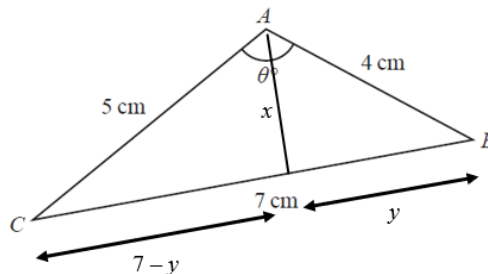
A1: $4\sqrt{6} \text{ cm}^2$ must be simplified and must follow award of the previous 3 marks.

Note that some are using $\theta = 101.5...$, finding area = awrt 9.80 and then realising this is

$4\sqrt{6}$ (or $\sqrt{96}$) and score only the second M mark without an otherwise correct method.

Ignore incorrect units.

Alt (b): Splitting triangle ABC and using Pythagoras (condoned)



M1: Complete method to split triangle ABC into two right-angled triangles and use Pythagoras and simultaneous equations to find the perpendicular height of the two right-angled triangles. e.g.

$$(7-y)^2 + x^2 = 5^2 \text{ and } y^2 + x^2 = 4^2 \Rightarrow 49 - 14y = 25 - 16 \Rightarrow y = \dots \left\{ \frac{20}{7} \right\} \Rightarrow x = \dots \left\{ \frac{8\sqrt{6}}{7} \right\}$$

A1: Correct exact perpendicular height of the right-angled triangles.

Note the perpendicular height to BC is $\frac{8\sqrt{6}}{7}$, to AC is $\frac{8\sqrt{6}}{5}$ and to AB is $2\sqrt{6}$

M1: Complete method to find the area of triangle ABC , which may be inexact.

e.g. $\frac{1}{2} \times 7 \times \frac{8\sqrt{6}}{7}$ or $\frac{1}{2} \times 5 \times \frac{8\sqrt{6}}{5}$ or $\frac{1}{2} \times 4 \times 2\sqrt{6}$

or even e.g. $\frac{1}{2} \times \frac{20}{7} \times \frac{8\sqrt{6}}{7} + \frac{1}{2} \times \left(7 - \frac{20}{7} \right) \times \frac{8\sqrt{6}}{7}$

While this mark is not dependent on the previous method mark, it does require a correct method to find a value for the perpendicular height of the triangle.

A1: As main scheme.

Question Number	Scheme	Marks	AOs
6 (a)	Attempts a scale factor $\frac{10}{4}, \frac{4}{10}$ o.e.	M1	3.1a
	$p = -15$	A1	1.1b
		(2)	
(b)	Attempts either $ \mathbf{a} + \mathbf{c} = \left \begin{pmatrix} 4+q \\ -3 \end{pmatrix} \right = \sqrt{(4+q)^2 + 9}$ or $ \mathbf{d} = \sqrt{30^2 + 6^2} \quad \{ = \sqrt{936} = 6\sqrt{26} \}$	M1	1.1b
	$2 \mathbf{a} + \mathbf{c} = \mathbf{d} \Rightarrow 4((4+q)^2 + 9) = 30^2 + 6^2 \quad \{ = 936 \}$	dM1	1.1b
	$(4+q)^2 = 225 \Rightarrow 4+q = \pm 15 \Rightarrow q = \dots$ or $q^2 + 8q - 209 = 0 \Rightarrow q = \dots$	ddM1	2.1
	$\Rightarrow q = 11, -19$	A1	1.1b
		(4)	
(6 marks)			
Notes:			

(a)

M1: Uses the fact that the vectors are parallel and attempts a scale factor.

May be implied by e.g. $\begin{pmatrix} 4 \\ -6 \end{pmatrix} = \lambda \begin{pmatrix} 10 \\ p \end{pmatrix} \Rightarrow \lambda = \dots$ or $4 = 10\lambda \Rightarrow \lambda = \dots$

A1: $p = -15$ Sight of this with no incorrect working scores both marks including e.g. $\begin{pmatrix} 10 \\ -15 \end{pmatrix}$

Note $p = -15\mathbf{j}$ scores M1A0.

(b)

M1: Attempts a correct method of finding the modulus or modulus² for either vector $\mathbf{a} + \mathbf{c}$ or $\mathbf{d}$.
If using $\mathbf{a} + \mathbf{c}$ then they must clearly be adding the vectors.

Condone incorrect labelling, e.g., $|\mathbf{a} + \mathbf{c}| = (4+q)^2 + 9$ as it is not a “show that”.

dM1: Uses the information to set up a quadratic equation in q with square roots removed.
Dependent on the previous method mark.

When finding $|\mathbf{a} + \mathbf{c}|$ they must clearly be adding the vectors.

Correct algebraic work is required but condone poor squaring e.g. $(4+q)^2 = 16 + q^2$ and

numerical slips such as failure to square the 2 i.e. $2((4+q)^2 + 9) = 30^2 + 6^2$ or omission of the

2 i.e. $(4+q)^2 + 9 = 30^2 + 6^2$

Similarly, condone failure to multiply both components of $\mathbf{a} + \mathbf{c}$ by 2 leading to e.g.

$$\sqrt{(8+2q)^2 + 3^2} = \sqrt{30^2 + 6^2}$$

Use of $|\mathbf{a} + \mathbf{c}| \rightarrow |\mathbf{a}| + |\mathbf{c}|$ is dM0, as is poor square rooting e.g. $\sqrt{q^2 + 8q + 25} = \sqrt{q^2 + 8q} + 5$

ddM1: Full and complete attempt to find at least one value for q , condoning failure to square the 2 or similar.

Dependent on both previous method marks.

If they expand the brackets, then for this mark they must achieve a 3TQ and go on to solve it using the usual rules. They may also use a calculator, in which case values will need checking.

Note that the correct 3TQ is $4q^2 + 32q - 836 = 0$ or $q^2 + 8q - 209 = 0$

Note also that if they fail to square the 2 then they are likely to achieve $2q^2 + 16q - 886 = 0$ or $q^2 + 8q - 443 = 0$ which leads to $q = -4 \pm 3\sqrt{51}$

A1: cao Both $q = 11, -19$ and no others. Must be simplified. A0 if either value is rejected.

Note: You may see correct answers coming from incorrect work,

e.g. $4((4+q)^2 - 9) = 30^2 - 6^2$ which will score M0dM0ddM0A0

Alt(b): You may see perfectly valid responses such as

$$2 \begin{vmatrix} 4+q \\ -3 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix} \Rightarrow \begin{vmatrix} 8+2q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix} \Rightarrow \pm(8+2q) = 30 \Rightarrow q = 11, -19$$

M1: Sets up the modulus equation $\begin{vmatrix} 8+2q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix}$ which may be implied by either $|8+2q| = 30$

or **both** $8+2q = 30$ **and** $-8-2q = 30$

Condone a slip on the **i** component e.g. $\begin{vmatrix} 8+q \\ -6 \end{vmatrix} = \begin{vmatrix} 30 \\ 6 \end{vmatrix}$

dM1: Deduces **either** linear equation in q following the award of the previous mark.

ddM1: For solving **both** linear equations in q

A1: cao Both $q = 11, -19$ and no others. Must be simplified. A0 if either value is rejected.

Question Number	Scheme	Marks	AOs
7 (a)	$f(x) = \frac{2(x-5)^2}{\sqrt{x}} = \frac{2x^2 - 20x + 50}{\sqrt{x}} = \text{e.g. "2"x}^{\frac{3}{2}} - "20"x^{\frac{1}{2}} + "50"x^{-\frac{1}{2}}$	M1	1.1b
	$= 2x^{\frac{3}{2}} - 20x^{\frac{1}{2}} + 50x^{-\frac{1}{2}}$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left\{ \int f(x) dx = \right\} \frac{4}{5}x^{\frac{5}{2}} - \frac{40}{3}x^{\frac{3}{2}} + 100x^{\frac{1}{2}} + c$	M1 A1ft A1	1.1b 1.1b 1.1b
		(3)	
(c) (i)	$\left\{ f'(x) = \right\} 3x^{\frac{1}{2}} - 10x^{-\frac{1}{2}} - 25x^{-\frac{3}{2}}$	M1 A1ft	1.1b 1.1b
	Sets $3x^{\frac{1}{2}} - 10x^{-\frac{1}{2}} - 25x^{-\frac{3}{2}} = 0$ and $\Rightarrow 3x^2 - 10x - 25 = 0$ *	A1*	2.1
	(ii) $x = 5$ only	B1 (A1 on EPEN)	2.3
		(4)	
(10 marks)			
Notes:			

(a)

M1: Multiplies out the numerator to create a 3TQ and divides at least two of the terms by $\sqrt{x}$ to form at least two correct indices. The coefficients should be used to determine whether the correct indices are on the correct terms.

A1: At least 2 correct terms from $2x^{\frac{3}{2}} - 20x^{\frac{1}{2}} + 50x^{-\frac{1}{2}}$ with simplified coefficients and indices processed. Do not accept e.g. $\frac{50}{\sqrt{x}}$ for either A mark.

A1: Fully correct $2x^{\frac{3}{2}} - 20x^{\frac{1}{2}} + 50x^{-\frac{1}{2}}$ with simplified coefficients and indices processed.

(b) **Note that marks in (b) and (c) cannot be scored the wrong way round. If responses are unlabelled then the first attempt is marked as part (b) and the second is marked as (c)**

M1: Increases the index by 1 for at least one of their **fractional** indices from part (a). It must be clear that the index has increased by 1. The order might clearly imply this, or the coefficient may be used to imply this.

A1ft: At least 2 correct terms of $\frac{4}{5}x^{\frac{5}{2}} - \frac{40}{3}x^{\frac{3}{2}} + 100x^{\frac{1}{2}}$ o.e. which may have unsimplified coefficients (the indices must be processed) and follow through on correct integration of two of their terms from part (a) with **fractional** indices.
If they have the $+ c$ this is not considered to be one of their correct terms.

A1: $\frac{4}{5}x^{\frac{5}{2}} - \frac{40}{3}x^{\frac{3}{2}} + 100x^{\frac{1}{2}} + c$ o.e. including the $+ c$ and ISW after a correct integral is seen.

Condone inclusion of a spurious integral sign. Accept $-13.\dot{3}$ in place of $-\frac{40}{3}$ but not $-13.3...$

(c)(i) Mark parts (c)(i) and (c)(ii) together.

M1: Decreases the index by 1 for at least one of their **fractional** indices from part (a).

It must be clear that the index has decreased by 1. The order might clearly imply this, or the coefficient may be used to imply this.

A1ft: At least 2 correct terms of $3x^{\frac{1}{2}} - 10x^{-\frac{1}{2}} - 25x^{-\frac{3}{2}}$ o.e. which may have unsimplified coefficients (the indices must be processed) and follow through on correct differentiation of two of their terms from part (a) with **fractional** indices.

A1*: Achieves $3x^{\frac{1}{2}} - 10x^{-\frac{1}{2}} - 25x^{-\frac{3}{2}} = 0$ o.e. with processed indices **and** proceeds to $3x^2 - 10x - 25 = 0$ with no errors.

Must have come from a correct simplified $f(x)$ from part (a) (or a clear restart).

Must include the $= 0$ in the given answer and in at least one of the previous lines.

$3x^{\frac{1}{2}} - 10x^{-\frac{1}{2}} - 25x^{-\frac{3}{2}} = x^{-\frac{3}{2}}(3x^2 - 10x - 25) = 0 \Rightarrow 3x^2 - 10x - 25 = 0$ is also acceptable.

(c)(ii)

B1: $x = 5$ only. Any other answers must be discarded or $x = 5$ must be clearly selected.

Ignore any reference to y , so condone coordinates e.g. $(5, k)$ where k is any number.

Question Number	Scheme	Marks	AOs
8 (i)	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	B1	1.1a
	e.g. $2 \frac{\sin \theta}{\cos \theta} - 5 \sin \theta \cos \theta = 0 \Rightarrow \sin \theta (2 - 5 \cos^2 \theta) = 0$ or $2 \frac{\sin \theta}{\cos \theta} - 5 \sin \theta \cos \theta = 0 \Rightarrow \sin \theta (2 - 5 + 5 \sin^2 \theta) = 0$	M1	3.1a
	$\cos \theta = (\pm) \sqrt{\frac{2}{5}} \Rightarrow \theta = \dots$ or $\sin \theta = (\pm) \sqrt{\frac{3}{5}} \Rightarrow \theta = \dots$	dM1	1.1b
	Two of $\theta = 50.8^\circ, 129.2^\circ, 180^\circ, 230.8^\circ$ All $\theta = 50.8^\circ, 129.2^\circ, 180^\circ, 230.8^\circ$ with no extras	A1 A1	1.1b 2.1
		(5)	
(ii) (a) (b)	$H = 40 \sin(0.3t + 2)^\circ$		
	$t = 0 \Rightarrow H = 40 \sin 2^\circ$	M1	3.4
	awrt 1.40 metres	A1	3.2a
		(2)	
	$25 = 40 \sin(0.3T + 2)^\circ \Rightarrow \sin(0.3T + 2)^\circ = \frac{25}{40}$	M1	3.1b
	$\Rightarrow 0.3T + 2 = \text{either } 38.7 \text{ or } 141.3$	A1	1.1b
	$\Rightarrow 0.3T + 2 = "141.3" \Rightarrow T = \dots$	dM1	3.2a
	$\Rightarrow T = \text{awrt } 464$	A1	1.1b
		(4)	
(11 marks)			
Notes:			

(i) Do not be concerned about their argument throughout part (i), so they may be using e.g. x . With the exception of the B1, condone their argument going “missing”.

B1: States or uses a **correct** $\tan \theta = \frac{\sin \theta}{\cos \theta}$ which may be seen in the body of their work or as side working. There must be an argument for this mark, e.g. $\tan x = \frac{\sin x}{\cos x}$ but not just $\tan = \frac{\sin}{\cos}$

M1: Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$, multiplies by $\cos \theta$, takes out a factor of $\sin \theta$ **or** cancels $\sin \theta$ to form an equation in $\cos \theta$ only. Condone $2 \tan \theta \rightarrow \frac{2 \sin \theta}{2 \cos \theta}$ for the method marks only.

Alternatively, takes out a factor of $\sin \theta$ **or** cancels $\sin \theta$ **and** replaces $\pm \cos^2 \theta$ with $\pm 1 \pm \sin^2 \theta$ (in either order) to form an equation in $\sin \theta$ only (not just $5 \sin^3 \theta - 3 \sin \theta = 0$).

Note that this mark may be implied by e.g. $5 \sin^3 \theta - 3 \sin \theta = 0 \Rightarrow \sin \theta = \{\pm\} \frac{\sqrt{15}}{5}$

If they expand $5 \sin \theta (\pm 1 \pm \sin^2 \theta)$ incorrectly to $\pm 5 \sin \theta \pm 5 \sin^2 \theta$ i.e. not $\sin^3 \theta$ then they do not score this mark as they have materially altered the question.

Condone squaring in the wrong place e.g. $\cos \theta^2$ only if it is recovered by later work.

If they replace $\sin \theta$ with e.g. X in $5 \sin^3 \theta - 3 \sin \theta = 0$ to get $5X^3 - 3X = 0$ then score for any of e.g. $X(5X^2 - 3) = 0$, $5X^2 - 3 = 0$ or $X = \{\pm\} \frac{\sqrt{15}}{5}$ even if they do not replace X with $\sin \theta$

dM1: Dependent on the previous method mark.

Solves an equation of the form $\cos^2 \theta = k \Rightarrow \theta = \dots$ or $\sin^2 \theta = k \Rightarrow \theta = \dots$ with $0 < k < 1$ using the correct order of operations. Allow similarly e.g. $p \sin^2 \theta - q = 0 \Rightarrow \theta = \dots$ with $0 < \frac{q}{p} < 1$

You may need to check that their θ follows from their $\sin \theta = \{\pm\} \sqrt{k}$

Proceeding directly from a cubic such as $5 \sin^3 \theta - 3 \sin \theta = 0$ to $\sin \theta = \{\pm\} \frac{\sqrt{15}}{5} \Rightarrow \theta = \dots$ is also acceptable.

Note the correct equations are $\cos \theta = \pm \frac{\sqrt{10}}{5} \left\{ = \pm \frac{\sqrt{2}}{\sqrt{5}} = \text{awrt } \pm 0.63 \right\}$ and

$\sin \theta = \pm \frac{\sqrt{15}}{5} \left\{ = \pm \frac{\sqrt{3}}{\sqrt{5}} = \text{awrt } \pm 0.77 \right\}$

Condone squaring in the wrong place e.g. $\cos \theta^2$ only if it is recovered by later work.

A1: **Two** of $\theta = 50.8^\circ, 129.2^\circ, 180^\circ, 230.8^\circ$ with or without the degrees symbol.

For both A marks allow awrt for any of $\theta = 50.8^\circ, 129.2^\circ, 230.8^\circ$

A1: Full and rigorous solution to find all **four** answers $\theta = 50.8^\circ, 129.2^\circ, 180^\circ, 230.8^\circ$ with or without the degrees symbol. Note that the 180° may appear separate from their final line. Ignore any answers outside the domain $0 < \theta \leq 270^\circ$

(ii)(a)

M1: Uses the model by substituting $t = 0 \Rightarrow H = 40 \sin(0.3(0) + 2)$ o.e.

May be implied by awrt 1.4 (using degrees) or awrt 36.4 (using radians).

A1: cao including units. Sight of awrt 1.40m or 140cm only scores both marks but condone 1.4m

(ii)(b) Note: Use of t in place of T is acceptable throughout this part of the question.

M1: Uses the model by substituting $H = 25$ and proceeds to $\sin(0.3T + 2) = k$

A1: A "correct" value for $0.3T + 2$, i.e., either awrt 38.7 or awrt 141 (or truncated 38.6) but do not be concerned with the LHS. Cannot be implied by just awrt 122 or awrt 464 unless there is sight of

e.g. $0.3T + 2 = \sin^{-1}\left(\frac{25}{40}\right)$ or e.g. $T = \frac{\sin^{-1}\left(\frac{25}{40}\right) - 2}{0.3}$ as both awrt 122 and awrt 464 can be found directly from their calculator.

dM1: Full method to find the **second** smallest time $0.3T + 2 = 180 - \sin^{-1}\left(\frac{25}{40}\right) \Rightarrow T = \dots$

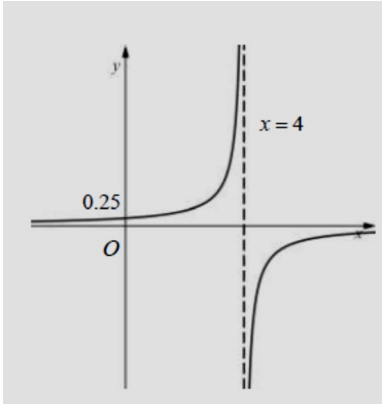
Dependent on the previous method mark.

Condone slips in the order of operations for solving $0.3T + 2 = 180 - \sin^{-1}\left(\frac{25}{40}\right)$

If using radians, they must convert back to degrees before solving $0.3T + 2 = \dots$

If they find the second smallest time but then reject it, they still score the dM1.

A1: $\{t / T = \}$ awrt 464 following award of the 3 previous marks. If there is more than one answer given, then awrt 464 must be clearly selected or the others rejected.

Question Number	Scheme	Marks	AOs
9 (a)		Shape and position Vertical asymptote Intercept	1.1b 1.1b 1.1b
		(3)	
(b) (i)	Sets $2x + k = \frac{1}{4-x} \Rightarrow 8x + 4k - 2x^2 - kx = 1$	M1	3.1a
	$2x^2 + (k-8)x + 1 - 4k = 0$	A1	1.1b
	Attempts $b^2 - 4ac = (k-8)^2 - 4 \times 2 \times (1-4k)$	dM1	1.1b
	$(k-8)^2 - 4 \times 2 \times (1-4k) \geq 0$ $\Rightarrow k^2 - 16k + 64 - 8 + 32k \geq 0 \Rightarrow k^2 + 16k + 56 \geq 0$	A1*	2.1
		(4)	
	At least one of $k = -8 + 2\sqrt{2}$ or $k = -8 - 2\sqrt{2}$	M1	1.1b
	$\{k : k \leq -8 - 2\sqrt{2}\} \cup \{k : k \geq -8 + 2\sqrt{2}\}$	A1	2.5
		(2)	
(9 marks)			
Notes:			

(a) **Requires a sketch to score any marks.**

B1: Correct shape and position for both branches of the curve with no significant overlap. Look for a negative reciprocal graph in quadrants 1, 2 and 4 only that does not cross or intersect the x -axis. It should look asymptotic to the x -axis but be generous here. If there is clearly a different horizontal asymptote to the x -axis then score B0.

B1: Correct equation of the asymptote $x = 4$ which can be scored if the curve $y = \frac{1}{x-4}$ is drawn. The curve must be asymptotic at both sides of the asymptote but be tolerant on distance. If there is more than one vertical asymptote shown, then score B0. If in doubt, the sketch takes precedent.

B1: Correct y intercept, which may be seen as $\frac{1}{4}$ or 0.25 labelled on the axes in the correct place. If there is more than one y intercept shown, then score B0.

Special Case: Score B1B0B0 for translating $y = -\frac{1}{x}$ left, with correct shape, asymptotic to the x -axis and the asymptote $x = -4$ labelled (i.e. a sketch of $y = -\frac{1}{x+4}$), ignoring any y intercept.

(b)(i)

- M1:** For the key step of equating $\frac{1}{4-x}$ with $2x+k$, cross multiplying, expanding brackets and proceeding to a quadratic equation. Condone slips in their expansion of brackets but expect there to be at least 3 terms of the correct form.
- A1:** Correct simplified quadratic equation with terms **collected** on one side.
Not scored for e.g. $2x^2 + kx - 8x + 1 - 4k = 0$ unless implied by use of correct values of a , b and c in their attempt at the discriminant but note that $2x^2 - (8-k)x + 1 - 4k = 0$ is correct.
No brackets are needed around $1 - 4k$. Condone bracketing error $2x^2 + (k-8)x (+1-4k) = 0$
The $= 0$ may be implied by later work and condone e.g. $2x^2 + (k-8)x + 1 - 4k \geq 0$ (which will be penalised in the final A1*).
- dM1:** Attempts $b^2 - 4ac$ or $b^2 \dots 4ac$ (where ... is any equality or inequality) with their values of a , b and c which must not include any x terms. Dependent on the previous method mark.
- A1*:** For constructing a rigorous argument and proceeding to the given answer showing all key steps.
There must be a correct intermediate line between e.g. $(k-8)^2 - 4 \times 2 \times (1-4k) \geq 0$ and $k^2 + 16k + 56 \geq 0$
Allow invisible brackets to be recovered.
The inequality cannot just appear at the end of the proof, but might only be seen previously in e.g. $b^2 - 4ac \geq 0$ or $b^2 \geq 4ac$
Penalise incorrect inequality work such as $2x^2 + (k-8)x + 1 - 4k \geq 0$ in this mark.
Condone an earlier bracketing error $2x^2 + (k-8)x (+1-4k) = 0$

(b)(ii)

- M1:** Solves the given equation to find at least one critical value (usual rules for solving a 3TQ apply but they may use a calculator in which case allow awrt -5.2 or awrt -10.8).
Do not be concerned what their critical values are labelled as for this mark (e.g. x or y).
- A1:** Uses correct notation to state the **exact** range of values for k
e.g. $\{k : k \leq -8 - 2\sqrt{2}\} \cup \{k : k \geq -8 + 2\sqrt{2}\}$ or $(-\infty, -8 - 2\sqrt{2}] \cup [-8 + 2\sqrt{2}, \infty)$
Condone the absence of " $k :$ " and $k \in \mathbb{R}$, but must be using k and not e.g. x or y .
Note that $\{k \mid k \leq -8 - 2\sqrt{2}\} \cup \{k \mid k \geq -8 + 2\sqrt{2}\}$ is also acceptable.
Condone $\{k \leq -8 - 2\sqrt{2} \cup k \geq -8 + 2\sqrt{2}\}$
Do not condone $\{k : k \leq -8 - 2\sqrt{2}, k \geq -8 + 2\sqrt{2}\}$
Allow $\sqrt{8}$ in place of $2\sqrt{2}$ throughout.
Use of the intersection symbol $\cap$ is incorrect and scores A0.
If using interval notation then $\pm\infty$ should not be included in their range.

Question Number	Scheme	Marks	AOs
10 (a)	$x^2 + y^2 - 8x + 5y - 20 = 0$		
	$(x-4)^2 + \left(y + \frac{5}{2}\right)^2 = \left\{16 + \frac{25}{4} + 20\right\}$	M1	1.1b
	Centre $\left(4, -\frac{5}{2}\right)$	A1	1.1b
	Radius = $\frac{13}{2}$	A1	1.1b
		(3)	
(b)	$x^2 - 8x - 20 = 0 \Rightarrow x = 10$	B1	2.2a
	Attempts the gradient of the radius: $\{m_r = \frac{0 - \left(-\frac{5}{2}\right)}{"10" - "4"}\}$	M1	1.1b
	$= \frac{5}{12}$	A1	1.1b
	Forms the equation of the tangent at P: $y - 0 = -\frac{12}{5}(x - 10)$	dM1	2.1
	$12x + 5y - 120 = 0$	A1	1.1b
		(5)	
(8 marks)			
Notes			

(a)

M1: Attempts to complete the square on both x and y terms.

Look for $(x \pm 4)^2 \pm \left(y \pm \frac{5}{2}\right)^2 \pm \dots$ where $\dots$ may be 0. May be implied by a centre of $\left(\pm 4, \pm \frac{5}{2}\right)$

A1: Correct centre $\left(4, -\frac{5}{2}\right)$ which may be given $x = \dots y = \dots$

A1: Correct exact radius. Allow $\frac{13}{2}$ or 6.5 but not e.g. $\sqrt{\frac{169}{4}}$

(b)

Watch out: Using $(-2, 0)$ for P gives a gradient of the radius of $-\frac{5}{12}$ and leads to

$12x - 5y - 24 = 0$ but will score maximum B0M1A0dM1A0

B1: Deduces that the x coordinate of P is 10. May be implied by use of $x = 10$ in their later work. If both $x = -2$ and $x = 10$ are found, then they must clearly indicate that P is when $x = 10$ or use the $x = 10$ in their later work.

Condone $P(0, 10)$ if $x = 10$ seen or used.

M1: Attempts the gradient of the radius using their centre and their P which should be $(\lambda, 0)$, where λ comes from a valid method e.g. by solving $x^2 - 8x - 20 = 0$ or $(x - "4")^2 + \left(\frac{5}{2}\right)^2 = \frac{169}{4}$

using the usual rules (if using a calculator then λ should be 10 or -2).

Note that they may use Pythagoras to find $P(\lambda, 0)$: $k^2 + \left(\frac{5}{2}\right)^2 = \left(\frac{13}{2}\right)^2 \Rightarrow k = \dots \{\pm 6\}$

followed by $\lambda = "4" \pm "6" = \dots \{10 \text{ or } -2\}$

Alternatively, they may differentiate implicitly to reach the form $\pm\alpha x \pm \beta y \frac{dy}{dx} \pm \gamma \pm \delta \frac{dy}{dx} = 0$

(where $\alpha, \beta, \gamma, \delta$ are non-zero) and substitute their $(\lambda, 0)$ (λ as above) to find a value for $\frac{dy}{dx}$

FYI: $2x + 2y \frac{dy}{dx} - 8 + 5 \frac{dy}{dx} = 0 \rightarrow 2("10") - 8 + 5 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left\{ -\frac{12}{5} \right\}$

A1: Correct gradient of the radius $= \frac{5}{12}$ which may be implied by later work and may be

unsimplified. Note that they might go directly to the gradient of the tangent $\left\{ -\frac{12}{5} \right\}$ and find

$$m_T = -\frac{"10" - "4"}{0 - \left(-\frac{5}{2} \right)} \text{ o.e. such as through implicit differentiation, which would imply this mark.}$$

dM1: Full method to find the equation of the tangent at P . Dependent on the previous method mark. Must be using the negative reciprocal of their gradient of the radius (note that they may have already found the gradient of the tangent through e.g. use of implicit differentiation).

If using $y = mx + c$ they must proceed as far as $c = \dots$

A1: $12x + 5y - 120 = 0$ but allow any integer multiple of this. The $= 0$ must be present.

ISW after a correct equation in the required form seen.

Not scored for just $12x + 5y = 120$.

Question Number	Scheme	Marks	AOs
11 (a)	$2\log_2(x-4) + \log_2\left(\frac{4}{3}x\right) = 2 + \log_2(x+2)$		
	One correct log law e.g. $2\log_2(x-4) = \log_2(x-4)^2$ or $2 = \log_2 4$	B1	1.1b
	Correct attempt to combine 2 log terms e.g. $2\log_2(x-4) + \log_2\left(\frac{4}{3}x\right) = \log_2\left(\frac{4}{3}x(x-4)^2\right)$	M1	1.1b
	e.g. $\frac{4}{3}x(x-4)^2 = 4(x+2)$ or e.g. $\frac{4x(x-4)^2}{3(x+2)} = 4$	A1	1.1b
	$4x^3 - 32x^2 + 64x - 12x - 24 = 0 \Rightarrow x^3 - 8x^2 + 13x - 6 = 0$ *	A1*	2.1
		(4)	
(b)	$\{f(1)=\} 1^3 - 8 \times 1^2 + 13 \times 1 - 6 = 0 \Rightarrow (x-1)$ is a factor	B1	2.4
		(1)	
(c)	$x^3 - 8x^2 + 13x - 6 = (x-1)(x^2 - 7x + 6)$	M1	1.1b
	$x^2 - 7x + 6 = (x-1)(x-6) = 0 \Rightarrow x = \dots$	dM1	1.1b
	$x = 1, 6$	A1	1.1b
		(3)	
(d)	$x = 6$ only	B1ft	2.2b
		(1)	
(9 marks)			
Notes:			

(a) Condone the absence of a base on their log terms throughout.

B1: States or uses one correct log law on either $2\log_2(x-4) = \log_2(x-4)^2$ or $2 = \log_2 4$

(which may be implied by e.g. $\log_2(\dots) = 2 \rightarrow (\dots) = 2^2$).

It is scored for use of either of the above, and not for addition or subtraction laws.

M1: Correct attempt to combine 2 log terms from the equation using the addition or subtraction laws.

Not scored for combining 2 log terms that have come from incorrect log work and not scored

for just e.g. $\log_2\left(\frac{4}{3}x\right) \rightarrow \log_2(4x) - \log_2 3$

A1: Correct unsimplified equation with logs removed e.g. $\frac{4}{3}x(x-4)^2 = 4(x+2)$ (that is not the given answer). Note that some candidates are proceeding directly to e.g. $\frac{4}{3}x(x-4)^2 = 4(x+2)$ and will score B1M1A1 if there are no errors seen prior to this but will score A0* even if they then achieve the given answer as they have not shown all necessary steps involving the log work.

A1*: Proceeds to the given answer showing all necessary steps and no errors.

There must be at least one correct intermediate line between their $4x(x-4)^2 = 12(x+2)$ with brackets expanded and the given answer of $x^3 - 8x^2 + 13x - 6 = 0$

(b)

B1: Correct attempt showing $\{f(1) =\} 1^3 - 8 \times 1^2 + 13 \times 1 - 6 = 0$ with a conclusion.

Requires the $= 0$ (which may be separate) and a conclusion e.g. “ $(x - 1)$ is a factor”.

The conclusion should refer to $(x - 1)$ or “it” being a factor in some way. Either e.g. ✓ or “shown” would be insufficient conclusions.

There is no need to evaluate any of the individual terms.

If they use a preamble, e.g., “if $(x - 1)$ (or “it”) is a factor then $x = 1$ is a root” and then show $\{f(1) =\} 1^3 - 8 \times 1^2 + 13 \times 1 - 6 = 0$ they must have a minimal conclusion so e.g. ✓ and “shown” are acceptable as they have already referred to $(x - 1)$ being a factor in some way.

Underlining $= 0$ is not a sufficient conclusion for either approach.

(c)

M1: Divides by $(x - 1)$ to produce a quadratic factor. Look for $x^2 \pm 7x \pm \dots$ via algebraic division or $x^2 \pm \dots x \pm 6$ by inspection, where $\dots$ may be 0. May be seen in part (b).

Note that they cannot just proceed directly to e.g. $(x - 1)^2 (x - 6)$

dM1: Solves their 3TQ using the usual rules, but there must be a method.

Calculator use is not acceptable here. Dependent on the previous method mark.

A1: Both $x = 1$ and $x = 6$ from correct work, and no other solutions.

Note that some candidates are labelling their answers, sometimes including e.g. factorisation, as part (d). Allow these candidates to score the dM1A1 for the two correct answers (following correct work) seen in (d).

(d)

B1ft: Deduces that the correct solution is 6 only. Ignore any justification for rejecting other roots. If awarding for $x = 6$ only, then they must have $x = 6$ in part (c) and have no other solutions larger than 4 in part (c).

Follow through on their solutions to part (c) if they select all solutions that are larger than 4.

So, if they have e.g. $x = 1, 5, 7$ in part (c) then they require $x = 5, 7$ in part (d).

and if they have e.g. $x = 1, 2, 3$ in part (c) then they should conclude “no solutions” o.e.

Question Number	Scheme	Marks	AOs
12 (a)	$\left\{ \frac{dy}{dx} = \right\} 3x^{-\frac{1}{2}} - x$	M1 A1	1.1b 1.1b
	Substitutes $x = 4$ into $\frac{dy}{dx} = \frac{3}{2} - 4 = \dots \left\{ -\frac{5}{2} \right\}$	dM1	1.1b
	$y - \frac{53}{2} = -\frac{5}{2}(x - 4)$	ddM1	2.1
	$5x + 2y = 73$ *	A1*	1.1b
		(5)	
(b)	$x = 9 \Rightarrow y = 6\sqrt{9} - \frac{1}{2} \times 9^2 + \frac{45}{2} = 0 \quad \checkmark$	B1	2.4
		(1)	
(c)	$\left\{ \int \left(6\sqrt{x} - \frac{1}{2}x^2 + \frac{45}{2} \right) dx = \right\} 4x^{\frac{3}{2}} - \frac{1}{6}x^3 + \frac{45}{2}x \{+c\}$	M1 A1	1.1b 1.1b
	Correct method for area under line e.g. $5x + 2(0) = 73 \Rightarrow x = \dots \left(\frac{73}{5} \right)$ $\Rightarrow \text{Area} = \frac{1}{2} \times \left(\frac{73}{5} - 4 \right) \times \frac{53}{2} \left\{ = \frac{2809}{20} \right\}$	M1 (B1 on EPEN)	1.1b
	Shaded Area = $\frac{1}{2} \times \left(\frac{73}{5} - 4 \right) \times \frac{53}{2} - \left[4x^{\frac{3}{2}} - \frac{1}{6}x^3 + \frac{45}{2}x \{+c\} \right]_4^9$ $= \frac{2809}{20} - \left(189 - \frac{334}{3} \right)$	ddM1	3.1a
	$= \frac{3767}{60}$ (allow awrt 62.8)	A1	1.1b
		(5)	
(11 marks)			
Notes:			

(a) Throughout the question, be generous on the labelling of parts.

M1: Attempts to differentiate the curve achieving at least one correct index which may be unprocessed for this mark or allow $\frac{45}{2} \rightarrow 0$

A1: Correct differentiation of the curve which may be unsimplified but indices must be processed.

dM1: Substitutes $x = 4$ into their $\frac{dy}{dx}$ and proceeds as far as a value.

Dependent on the previous method mark.

ddM1: Full and complete method to find an equation for l .

Dependent on both previous method marks. If using $y = mx + c$ they must proceed as far as $c = \dots$

A1*: Proceeds to $5x + 2y = 73$ from correct working and no errors seen.

ISW after the given answer is achieved. Not scored for just e.g. $5x + 2y - 73 = 0$

If they set e.g. $\frac{dy}{dx} = 0$ or 4 but then ignore this and complete the question correctly then do not withhold the A1*. Condone e.g. $l =$ on the LHS provided they have $5x + 2y = 73$

(b)

B1: Substitutes $x = 9$ into the equation for y leading to an answer of 0 with some minimal conclusion (a $\checkmark$ or “shown” is sufficient). Underlining $= 0$ is not a sufficient conclusion. Note that they may rearrange the equation first, but they must substitute $x = 9$ in and show that it satisfies the equation. Solving the equation on a calculator is B0.

(c) **Note: There are many ways to find the area of region R. The two main ways are listed below, but the general approach is:**

M1: Attempt to integrate either the curve or (line – curve).

If there is no algebraic integration seen, then the maximum score is M0A0M1ddM0A0.

A1: Correct algebraic integration of either the curve or (line – curve).

M1: Correct method to find the area of the remaining section of region R using any appropriate combination of triangles and/or trapezia.

If there **is** an attempt to integrate the curve or (line – curve), then to score this mark the area found **must** be able to be combined with their integration to find the area of region R .

If there is **no** attempt to integrate the curve or (line – curve), then this can be scored provided the area found **could** be combined with some integration to find the area of region R .

ddM1: Full and complete method that without errors would find the correct area.

A1: For a correct answer.

Main scheme: Triangle – area under curve

M1: Attempts to integrate the curve achieving at least one correct index for one term which may be unprocessed. Condone multiplying by 2 before integrating.

A1: Correct integration of the curve (may be unsimplified but the indices must be processed now).

M1: Correct method to find the area under the line between 4 and their attempt at $x = \frac{73}{5} \{=14.6\}$

which may be implied by sight of $\frac{2809}{20} \{=140.45\}$

Requires:

- setting $y = 0$ in the given equation for l (or their previous equation) and proceeding to $x = \dots$

- either a **correct** attempt at the triangle using $\left(4, \frac{53}{2}\right)$ and their x **or** an attempt to integrate the equation of the line (with y the subject) between 4 and their x . Look for

$$\frac{1}{2} \times \left(\frac{73}{5} - 4 \right) \times \frac{53}{2} = \dots \quad \text{or} \quad \int_4^{\frac{73}{5}} \left(\frac{73}{2} - \frac{5}{2}x \right) dx = \left[\frac{73}{2}x - \frac{5}{4}x^2 \{+c\} \right]_4^{\frac{73}{5}} = \dots$$

condoning slips in rearranging or in coefficients when integrating.

Condone attempts that find the area under the triangle using calculator integration, so if they have $\frac{2809}{20}$ they score this M1.

ddM1: Full and complete method of finding the area of region R (with correct limits) using triangle – area under curve.

If they substitute their limits in the wrong way round at any stage, then allow recovery if they make the area positive.

Dependent on both previous method marks.

A1: $\frac{3767}{60}$ but allow awrt 62.8 following award of all previous marks.

Alt: **Triangle + area under line – curve**

M1: Attempts to integrate $\pm(\text{line} - \text{curve})$ achieving a correct index for one term which may be unprocessed. Condone multiplying by 2 before integrating.

A1: Correct integration of $\pm(\text{line} - \text{curve})$ (may be unsimplified but the indices must be processed).

$$\left\{ \int \left(\frac{73}{2} - \frac{5}{2}x - \left(6\sqrt{x} - \frac{1}{2}x^2 + \frac{45}{2} \right) \right) dx = \left\{ \frac{73}{2}x - \frac{5}{4}x^2 - 4x^{\frac{3}{2}} + \frac{1}{6}x^3 - \frac{45}{2}x \right\} + c \right\}$$

M1: Correct method to find the area under the line between 9 and their attempt at $x = \frac{73}{5}$ which may be implied by sight of $\frac{196}{5} \{= 39.2\}$

Requires:

- setting $y = 0$ in the given equation for l (or their previous equation) and proceeding to $x = \dots$
- either a **correct** attempt at the triangle using their x and (9, “14”) (coming from substituting $x = 9$ into l) **or** an attempt to integrate the equation of the line (with y the subject) between 9 and their x .

$$\text{Look for } \frac{1}{2} \times \left(\frac{73}{5} - 9 \right) \times \text{“14”} = \dots$$

$$\text{or } \int_9^{\frac{73}{5}} \left(\frac{73}{2} - \frac{5}{2}x \right) dx = \left[\frac{73}{2}x - \frac{5}{4}x^2 \right]_9^{\frac{73}{5}} = \dots$$

condoning slips in rearranging or in coefficients when integrating.

Condone attempts that find the area under the triangle using calculator integration, so if they have $\frac{196}{5}$ they score this M1.

ddM1: Full and complete method of finding the area of region R (with correct limits) using triangle + area between line and curve.

If they substitute their limits in the wrong way round at any stage, then allow recovery if they make the area positive.

Dependent on both previous method marks.

A1: $\frac{3767}{60}$ but allow awrt 62.8 following award of all previous marks.

Question Number	Scheme	Marks	AOs
13 (a)	$23 = A + 12e^0 + 9e^0 \Rightarrow A = \dots$	M1	3.1b
	$V = 2 + 12e^{-0.2t} + 9e^{-0.1t}$	A1	3.3
		(2)	
(b)	$5 = 2 + 12e^{-0.2T} + 9e^{-0.1T} \Rightarrow 12e^{-0.2T} + 9e^{-0.1T} - 3 = 0$	M1	3.4
	e.g. $\Rightarrow 3\left(4e^{-0.1T} - 1\right)\left(e^{-0.1T} + 1\right) = 0 \Rightarrow e^{-0.1T} = \dots$ or implied by $e^{-0.1T} = \frac{1}{4}$	dM1	3.1a
	$\Rightarrow e^{-0.1T} = \frac{1}{4} \Rightarrow -0.1T = \ln \frac{1}{4}$	ddM1	2.1
	$\{T = \}$ awrt 13.86	A1	1.1b
		(4)	
(6 marks)			
Notes:			

(a)

M1: Attempts to find the value of A using $t = 0$ and $V = 23$ (**not** $V = 23000$).

A1: Correct complete equation for the model $V = 2 + 12e^{-0.2t} + 9e^{-0.1t}$ including $V =$
If just $A = 2$ seen in part (a) this mark may be awarded for a complete model seen in part (b).

(b) **Note: Use of t or e.g. x in place of T is acceptable throughout this part of the question.**

M1: Uses their model with $V = 5$ and their numerical A and proceeds to a 3TQ which may not all be on one side. The constant terms must be combined.

Note that they may not realise that they have a 3TQ but will score the mark for reaching an appropriate form.

Allow this mark to be scored for $V = 5000$ if they have used $V = 23000$ in (a) but they are not eligible to score any more marks in this question and condone slips e.g. 50000 or 2300.

dM1: For the key step in recognising the equation as a quadratic equation in $e^{\pm 0.1T}$ and attempting to solve. The usual methods apply for solving their 3TQ and roots for $e^{\pm 0.1T}$ may come from a calculator provided they are correct to 2sf. This is **not** implied by a correct value for T **only**.

Must be clearly solving for $e^{\pm 0.1T}$, so if they replace with X then they must revert back to $e^{\pm 0.1T}$

Note that the equation in $e^{0.1T}$ is $3e^{0.2T} - 9e^{0.1T} - 12 = 0$ which leads to $e^{0.1T} = 4$

Dependent on the previous method mark.

ddM1: Full and complete method to find T with correct use of lns but condone slips on dividing by ± 0.1

Dependent upon both previous method marks.

A1: $\{t / T = \}$ awrt 13.86 following award of all previous marks. Ignore any reference to units.

Must be a decimal and not just $\frac{1}{-0.1} \ln \frac{1}{4}$ or $10 \ln 4$

Question Number	Scheme	Marks	AOs
14	Attempts $n^2 + 2$ with $n = 3k + 1$ or $n = 3k + 2$ o.e. Look for e.g. $(3k + 1)^2 + 2 = \dots$	M1	3.1a
	e.g. $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ which is a multiple of 3.	A1	2.4
	Attempts $n^2 + 2$ with $n = 3k + 1$ and $n = 3k + 2$ o.e	dM1	2.1
	e.g. both $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ which is a multiple of 3 and $(3k + 2)^2 + 2 = 9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$ which is a multiple of 3 and e.g. Hence proven	A1	2.4
		(4)	
(4 marks)			
Notes:			

Note: If using $n = 3n + 1$ etc. then allow maximum 1110.

Ignore any attempts at $n = 3k$ throughout.

Ignore any spurious setting = 0 throughout.

M1: Starts to solve the problem by setting $n = 3k + 1$ or $n = 3k + 2$ o.e such as $n = 3k - 1$ and attempts $n^2 + 2$
Do not be concerned about their attempt to expand the brackets.
If using $n = 6k + 1$ etc. then score for one valid attempt.

A1: Achieves $(3k + 1)^2 + 2 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ **or** $(3k + 2)^2 + 2 = 9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$
or $(3k - 1)^2 + 2 = 9k^2 - 6k + 3 = 3(3k^2 - 2k + 1)$ **or** $(3k - 2)^2 + 2 = 9k^2 - 12k + 6 = 3(3k^2 - 4k + 2)$

and concludes that it is a multiple of 3 (or e.g. divisible by 3).

Condone e.g. " $n^2 + 2$ can be divided by 3" for this mark only.

To show it is a multiple of 3 they can either factorise or comment on each term individually.

You may also see e.g. $\frac{9k^2 + 6k + 3}{3} = 3k^2 + 2k + 1$ which is acceptable.

Note that the conclusion may be seen in an overall conclusion at the end.

dM1: Attempts two cases that cover all $n \in \mathbb{N}$ that are not multiples of 3.

So, sets $n = 3k + 1$ **and** $n = 3k + 2$ o.e. such as $n = 3k - 1$ and attempts $n^2 + 2$

Do not be concerned about their attempt to expand the brackets.

If using $n = 6k + 1$ then they require 4 valid attempts that cover all $n \in \mathbb{N}$ that are not multiples of 3:

e.g. $n = 6k + 1, n = 6k + 2, n = 6k + 4, n = 6k + 5$

A1: Requires

- **both** $(3k+1)^2 + 2 = 9k^2 + 6k + 3$ **and** $(3k+2)^2 + 2 = 9k^2 + 12k + 6$ o.e. correct
- sufficient reasons to conclude that **both** expressions are multiples of 3
 - e.g. $9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ or explains that each term is a multiple of 3
 - **and** e.g. $9k^2 + 12k + 6 = 3(3k^2 + 4k + 2)$ or explains that each term is a multiple of 3
- a minimal conclusion e.g. “hence proven” or “{when n is **not** a multiple of 3,} $n^2 + 2$ is always a multiple of 3” Condone the absence of “when n is **not** a multiple of 3”. Do not condone e.g. “ $n^2 + 2$ can be divided by 3” for this mark.

Alt:

M1: Starts to solve the problem by stating that if n is not a multiple of 3 then one of $(n+1)$ or $(n-1)$ must be a multiple of 3

A1: Deduces that $(n+1)(n-1)$ must be a multiple of 3

dM1: Connects the two expressions $(n+1)(n-1) + 3 \equiv n^2 + 2$

A1: Explains that as $(n+1)(n-1)$ must be a multiple of 3, $(n+1)(n-1) + 3$ must also be a multiple of 3, followed by a minimal conclusion, e.g. “hence $n^2 + 2$ is always a multiple of 3”.