Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 4D Pure Mathematics

Summer 2025
Question Paper Log Number P75692A
Publications Code 9FM0_4D_2506_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\hat{\surd}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎡ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	EMV for A is $0.55(370)+0.25(250)+0.2(-75)=251$	M1	3.4
	EMV for B is $0.75(245)+0.25(195)=232.5$		
	EMV for C is $0.45(390)+0.4(325)+0.15(-280)=263.5$	A1	1.1b
	The optimal EMV is £263.50, which makes option C the best choice (using the EMV criterion)	A1	2.2a
		(3)	
			(3 marks)

Notes for Question 1

Note: Working (or values) may be seen on the diagram as well as on the lined page. If both seen, the answers on the lined page take precedent. Completion of the diagram is not necessary.

M1: Correct method for calculation EMV for either A, B or C

A1: Correct values of EMV for A, B and C

A1: Correct deduction of optimal EMV (dependent on all three correct EMVs) C must be clearly identified in some way, either stated or indicated with e.g. an arrow. If all working is shown on the diagram with 263.50 written in the decision node, they must still indicate C in some way.

Question	Scheme	Marks	AOs
2(a)	Add an additional dummy column with equal values (e.g. 0) to create a square array	B1	3.5c
	Input a suitable large number (e.g. 100 but > 43) in cell BR	B1	1.1b
		(2)	
(b)	e.g. $\left(\begin{array}{cccccc} & P & Q & R & S & X \\ A & 25 & 32 & 43 & 28 & 0 \\ B & 41 & 37 & 100 & 38 & 0 \\ C & 43 & 35 & 37 & 39 & 0 \\ D & 40 & 33 & 37 & 41 & 0 \\ E & 37 & 38 & 43 & 40 & 0 \end{array}\right)$	B1	1.1b
	Reducing (rows and) columns gives		
	$\left(\begin{array}{cccccc} & P & Q & R & S & X \\ A & 0 & 0 & 6 & 0 & 0 \\ B & 16 & 5 & 63 & 10 & 0 \\ C & 18 & 3 & 0 & 11 & 0 \\ D & 15 & 1 & 0 & 13 & 0 \\ E & 12 & 6 & 6 & 12 & 0 \end{array}\right)$	M1	1.1b
	Three lines required to cover the zeros hence solution is not optimal (augment by 1)		
	$\left(\begin{array}{cccccc} & P & Q & R & S & X \\ A & 0 & 0 & 7 & 0 & 1 \\ B & 15 & 4 & 63 & 9 & 0 \\ C & 17 & 2 & 0 & 10 & 0 \\ D & 14 & 0 & 0 & 12 & 0 \\ E & 11 & 5 & 6 & 11 & 0 \end{array}\right)$	M1	1.1b
	Four lines (e.g. A, Q, R and X) required to cover the zeros hence solution is not optimal (augment by 9)		
	e.g. $\left(\begin{array}{cccccc} & P & Q & R & S & X \\ A & 0 & 9 & 16 & 0 & 10 \\ B & 6 & 4 & 63 & 0 & 0 \\ C & 8 & 2 & 0 & 1 & 0 \\ D & 5 & 0 & 0 & 3 & 0 \\ E & 2 & 5 & 6 & 2 & 0 \end{array}\right)$	M1 A1	1.1b 1.1b
	Five lines required to cover the zeros hence solution is optimal		
	Allocation: A to P, B to S, C to R, and D to Q (E does no task)	A1ft	2.2a
		(6)	

(c)	133 (mins)	B1	1.1b
		(1)	
(9 marks)			

Notes for Question 2

(a)

B1: Explain the need to add a dummy column to create a square array

B1: Input a large value in cell BR

(b) See Example for large number in dummy

B1: Mark awarded when both steps complete (addition of extra column with 0s in every cell and large value in cell BR) (Note may be implied by sight of table with (rows and) columns reduced)

M1: Simplifying the initial matrix by reducing columns

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 4 lines needed to 5 lines needed

A1: CSO on final table (so must have scored all previous marks in this part with no errors)

A1ft: Deduction of the correct allocations following through their 5 line final table (must have scored all previous M marks)

(c)

B1: CAO (‘mins’ not required)

Alternatives for **(b)** – after 4 lines may possibly see another 4 lines before 5 lines or two further 4 lines before 5 lines

The 3rd M mark in this case is only awarded when 5 lines are needed

	P	Q	R	S	X
A	0	0	7	0	1
B	15	4	63	9	0
C	17	2	0	10	0
D	14	0	0	12	0
E	11	5	6	11	0

	P	Q	R	S	X
A	0	0	7	0	5
B	11	0	59	5	0
C	17	2	0	10	4
D	14	0	0	12	4
E	7	1	2	7	0

	P	Q	R	S	X
A	0	5	12	0	10
B	6	0	59	0	0
C	12	2	0	5	4
D	9	0	0	7	4
E	2	1	2	2	0

	P	Q	R	S	X
A	0	0	7	0	1
B	15	4	63	9	0
C	17	2	0	10	0
D	14	0	0	12	0
E	11	5	6	11	0

	P	Q	R	S	X
A	0	0	9	0	3
B	13	2	63	7	0
C	15	0	0	8	0
D	14	0	2	12	2
E	9	3	6	9	0

	P	Q	R	S	X
A	0	0	9	0	5
B	11	0	61	5	0
C	15	0	0	8	2
D	14	0	2	12	4
E	7	1	4	7	0

	P	Q	R	S	X
A	0	5	14	0	10
B	6	0	61	0	0
C	10	0	0	3	2
D	9	0	2	7	4
E	2	1	4	2	0

Example – Large Number in Dummy – (b) B1 M1 M1 M1 A1 A1

	P	Q	R	S	X
A	25	32	43	28	100
B	41	37	100	38	100
C	43	35	37	39	100
D	40	33	37	41	100
E	37	38	43	40	100

	P	Q	R	S	X
A	0	7	18	3	75
B	4	0	63	1	63
C	8	0	2	4	65
D	7	0	4	8	67
E	0	1	6	3	63

	P	Q	R	S	X
A	0	7	16	2	12
B	4	0	61	0	0
C	8	0	0	3	2
D	7	0	2	7	4
E	0	1	4	2	0

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3 (a)</td><td>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th></tr>
<tr><td>A</td><td>35</td><td></td><td></td></tr>
<tr><td>B</td><td>22</td><td>19</td><td></td></tr>
<tr><td>C</td><td></td><td>12</td><td>17</td></tr>
<tr><td>D</td><td></td><td></td><td>45</td></tr>
</table>
</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td></td><td></td><td>27</td><td>28</td><td>21</td></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>– 5</td><td>4</td></tr>
<tr><td>2</td><td>B</td><td>X</td><td>X</td><td>5</td></tr>
<tr><td>5</td><td>C</td><td>– 3</td><td>X</td><td>X</td></tr>
<tr><td>15</td><td>D</td><td>– 10</td><td>– 9</td><td>X</td></tr>
</table>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th></tr>
<tr><td>A</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>$22-\theta$</td><td>$19+\theta$</td><td></td></tr>
<tr><td>C</td><td></td><td>$12-\theta$</td><td>$17+\theta$</td></tr>
<tr><td>D</td><td>θ</td><td></td><td>$45-\theta$</td></tr>
</table>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th></tr>
<tr><td>A</td><td>35</td><td></td><td></td></tr>
<tr><td>B</td><td>10</td><td>31</td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>29</td></tr>
<tr><td>D</td><td>12</td><td></td><td>33</td></tr>
</table>
Entering cell is DP and exiting cell is CQ
</td><td>M1
A1
M1
A1</td><td>2.1
1.1b
1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>Let x_{ij} be the number of units (of stock) transported from (supply point) i to (demand point) j</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R\}\left(x_{ij} \geq 0\right)$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>Minimise $27x_{AP} + 23x_{AQ} + 25x_{AR} + 29x_{BP} + 30x_{BQ} + 28x_{BR} + 29x_{CP} + 33x_{CQ} + 26x_{CR} + 32x_{DP} + 34x_{DQ} + 36x_{DR}$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>$\sum x_{Aj} \leqslant 35, \sum x_{Bj} \leqslant 41, \sum x_{Cj} \leqslant 29, \sum x_{Dj} \leqslant 45$ accept =</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>$\sum x_{iP} \geqslant 57, \sum x_{iQ} \geqslant 31, \sum x_{iR} \geqslant 62$ accept =</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Notes for Question 3

(a)

B1: CAO for north-west corner method

(b)

M1: Finding 7 shadow costs and 6 improvement indices

A1: Shadow costs and II correct (Alternative SC rows 27, 29, 32, 42 columns 0, 1, -6)

M1: A valid route, their most negative II chosen, only one empty square used, θ's balance

A1: CSO (for (b)) so all previous marks in this part must have been awarded – including exiting and entering cells stated correctly. Improved solution must be 6 numbers only with no additional 0 in CQ

(c) Check all suffixes carefully for accuracy and consistency

B1: Correct definition of x_{ij} must clearly state that this is the **number** of units transported

B1: Correctly defining the set of values that i and j can take

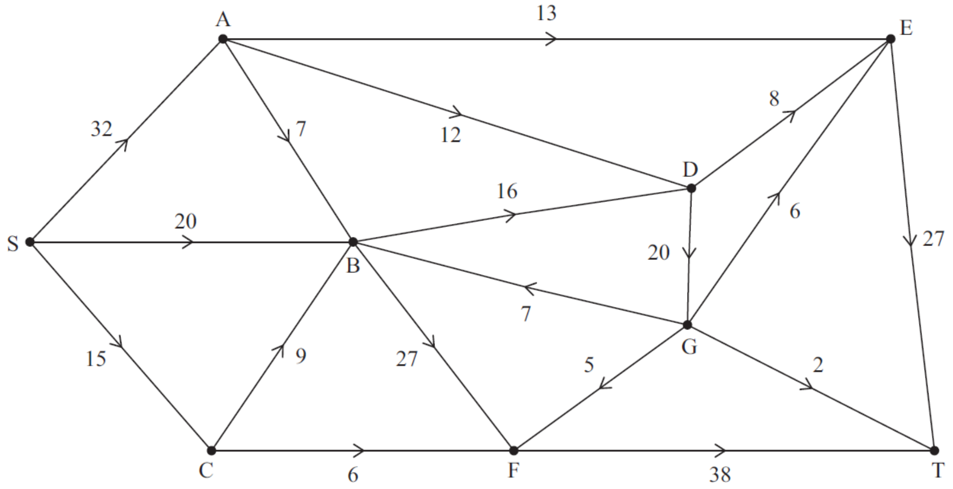
B1: 'Minimise' + correct objective function

B1: Correct supply constraints with unit coefficients

(allow 'equals' or written out in full e.g. $x_{\text{AP}}+x_{\text{AQ}}+x_{\text{AR}} \leqslant 35$)

B1: Correct demand constraints with unit coefficients

(allow 'equals' or written out in full e.g. $x_{AP} + x_{BP} + x_{CP} + x_{DP} \geqslant 57$)

Question	Scheme	Marks	AOs
4(a)	$C_1(=40+35+19+6)=100$	B1	1.1b
	$C_2(=13+12+16-2-3+43)=79$	B1	1.1b
		(2)	
(b)	Deduces the maximum possible flow is $\leqslant 79$ litres per second	B1ft	2.2a
		(1)	
(c)	Initial flow = 58	B1	1.1b
		(1)	
(d)	e.g. SAET – 4, SADET – 2, SBGFT – 2, SCFT – 1	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(e)	e.g.  	B1	2.2a
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through AE, AD, BD, DG, GE, GT, GF, BF and CF Value of flow = 67 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(11 marks)

Notes for Question 4

(a)

B1: CAO

B1: CAO

(b)

B1ft: deduced from their least value given in (a) - must include ‘less than or equal to’ (oe)

(c)

B1: CAO

(d)

M1: One flow augmenting route found from S to T

A1: Two correct routes + flow values

A1: CSO – increasing the flow by 9

Note possible flow augmenting routes are only _ADET + 2 _AET + 4 _BGFT + 2 _CFT + 1 where _ represents either S, SB, SC or SCB

(SADGFT + 2 is also valid but this prevents DET being increased)

(e)

B1: CAO – one number only per arc (only SA, SB, SC, AB and CB can vary depending on their flow augmenting routes)

(f)

M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs) (A valid cut through saturated arcs must either be listed or drawn)

May be drawn on either diagram or listed as arcs or set notation {S, A, B, C, G} {D, E, F, T}

A1: Use appropriate process of finding a minimum cut: cut + value correct and value of flow stated

A1: Correct deduction that the flow is maximal – must see all 4 words max flow min cut and conclusion **-** dependent on previous A mark

Question	Stage	State	Action	Dest	Value	Marks	AOs
5(a)	March	3	0	0	210 = 210*		
	(3)	2	1	0	140 + 60 = 200*		
		1	2	0	70 + 120 = 190*		
		0	3	0	180 = 180*		
	Feb	3	1	0	210 + 60 +180 = 450*		
	(4)		2	1	210 + 120 + 190 = 520		
			3	2	210 + 180 + 200 = 590	M1	3.1a
			4	3	210 + 240 + 400 + 210 = 1060	A1	1.1b
		2	2	0	140 + 120 + 180 = 440*	A1	1.1b
			3	1	140 + 180 + 190 = 510	A1	1.1b
			4	2	140 + 240 + 400 + 200 = 980		
			5	3	140 + 300 + 400 + 210 = 1050		
		1	3	0	70 + 180 + 180 = 430*		
			4	1	70 + 240 + 400 + 190 = 900		
			5	2	70 + 300 + 400 + 200 = 970		
		0	4	0	240 + 400 + 180 = 820*		
			5	1	300 + 400 + 190 = 890		
	Jan	3	3	0	210 + 180 + 820 = 1210*		
	(6)		4	1	210 + 240 + 400 + 430 = 1280		
			5	2	210 + 300 + 400 + 440 = 1350	M1	3.1a
		2	4	0	140 + 240 + 400 + 820 = 1600	A1ft	1.1b
			5	1	140 + 300 + 400 + 430 = 1270*	A1	1.1b
		1	5	0	70 + 300 + 400 + 820 = 1590*		
	Dec	3	3	1	210 + 180 + 1590 = 1980*		
	(5)		4	2	210 + 240 + 400 + 1270 = 2120		
			5	3	210 + 300 + 400 + 1210 = 2120		
		2	4	1	140 + 240 + 400 + 1590 = 2370	M1	1.1b
			5	2	140 + 300 + 400 + 1270 = 2110*	A1ft	1.1b
		1	5	1	70 + 300 + 400 + 1590 = 2360*		
	Nov	0	3	1	180 + 2360 = 2540*		
	(2)		4	2	240 + 400 + 2110 = 2750	A1	1.1b
			5	3	300 + 400 + 1980 = 2680		

Month	November	December	January	February	March
Number made	3	5	5	4	3

Question	Scheme	Marks	AOs
	(month table above)	B1	2.2a
	Minimum cost: (£) 2540	B1	2.2a
		(12)	
(b)	Additional cost is (£) 140	B1	2.2a
		(1)	

(13 marks)

Notes for Question 5

All M marks – must bring earlier optimal results into calculations. Ignore extra rows. Must have right 'ingredients' (storage costs, overhead costs and workers) at least once per stage. Penalise lack of * only once per question. Additional rows which are not rejected are penalised by the loss of the final A mark in each stage

(a)

M1: Second stage completed. At least 9 rows, something in each cell. Condone extra rows
A1: Any one state correct
A1: All four states complete (13 rows) with any two states correct
A1: CAO for second stage no extra rows

M1: Third stage completed. 6 rows, something in each cell. Condone extra rows
A1ft: Any one state correct – ft their optimal values
A1: CAO for third stage no extra rows

M1: Fourth stage completed. 6 rows, something in each cell. Condone extra rows
A1ft: Correct fourth stage – ft their optimal values
A1: CSO all stages

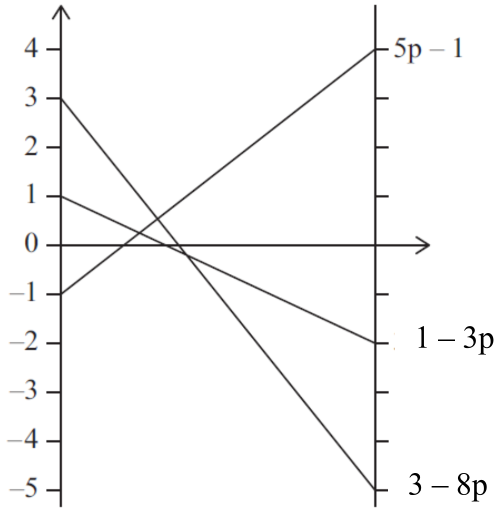
B1: Correct allocation (dependent on all previous M marks)
B1: Correct minimum cost (dependent on all previous M marks)

(b)
B1: CAO (dependent on all M marks in (a))

Special Cases Maximises see below Max 7/12 in (a) If Minimax or Maximin seen send to review

SC Maximises Max M1 A1 A1 A0 M1 A1 A0 M1 A1 A0 B0 B0 (Max 7/12 in (a))

Stage	State	Action	Dest	Value
March	3	0	0	210 = 210*
(3)	2	1	0	140 + 60 = 200*
	1	2	0	70 + 120 = 190*
	0	3	0	180 = 180*
Feb	3	1	0	210 + 60 +180 = 450
(4)		2	1	210 + 120 + 190 = 520
		3	2	210 + 180 + 200 = 590
		4	3	210 + 240 + 400 + 210 = 1060*
	2	2	0	140 + 120 + 180 = 440
		3	1	140 + 180 + 190 = 510
		4	2	140 + 240 + 400 + 200 = 980
		5	3	140 + 300 + 400 + 210 = 1050*
	1	3	0	70 + 180 + 180 = 430
		4	1	70 + 240 + 400 + 190 = 900
		5	2	70 + 300 + 400 + 200 = 970*
	0	4	0	240 + 400 + 180 = 820
		5	1	300 + 400 + 190 = 890*
Jan	3	3	0	210 + 180 + 890 = 1280
(6)		4	1	210 + 240 + 400 + 970 = 1820
		5	2	210 + 300 + 400 + 1050 = 1960*
	2	4	0	140 + 240 + 400 + 890 = 1670
		5	1	140 + 300 + 400 + 970 = 1810*
	1	5	0	70 + 300 + 400 + 890 = 1660*
Dec	3	3	1	210 + 180 + 1660 = 2050
(5)		4	2	210 + 240 + 400 + 1810 = 2660
		5	3	210 + 300 + 400 + 1960 = 2870*
	2	4	1	140 + 240 + 400 + 1660 = 2440
		5	2	140 + 300 + 400 + 1810 = 2650*
	1	5	1	70 + 300 + 400 + 1660 = 2430*
Nov	0	3	1	180 + 2430 = 2610
(2)		4	2	240 + 400 + 2650 = 3290
		5	3	300 + 400 + 2870 = 3570*

Question	Scheme	Marks	AOs
6(a)	Row minima: – 5, – 1 max is – 1 Column maxima: 4, 1, 3 min is 1	M1	1.1b
	Row maximin (– 1) $\neq$ Column minimax (1) so not stable	A1	2.4
		(2)	
(b)	If B plays option P, A's gains are $4p + (-1)(1-p) = -1 + 5p$ If B plays option Q, A's gains are $-2p + 1(1-p) = 1 - 3p$ If B plays option R, A's gains are $-5p + 3(1-p) = 3 - 8p$	M1 A1	1.1b 1.1b
		M1 A1	1.1b 1.1b
	$1 - 3p = -1 + 5p \Rightarrow p = 1/4$	A1	1.1b
	A should play option X with probability 1/4 and option Y with probability 3/4	A1ft	3.2a
		(6)	

<table>
<tr><td>(c)</td><td>e.g. $\begin{pmatrix} 4 & -2 & -5 \\ -1 & 1 & 3 \\ 4 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 9 & 3 & 0 \\ 4 & 6 & 8 \\ 9 & 4 & 6 \end{pmatrix}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>e.g.
$V - 9x - 4y - 9z + r = 0$
$V - 3x - 6y - 4z + s = 0$
$V - 8y - 6z + t = 0$
$x + y + z + u = 1$
$P - V = 0$</td><td></td><td></td></tr>
<tr><td></td><td>e.g.
<table>
<tr><td>b.v.</td><td>V</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>u</td><td>Value</td></tr>
<tr><td>r</td><td>1</td><td>-9</td><td>-4</td><td>-9</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>1</td><td>-3</td><td>-6</td><td>-4</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>t</td><td>1</td><td>0</td><td>-8</td><td>-6</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td>B1
B1
B1</td><td>3.3
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(d)</td><td>$x = 0, y = \frac{5}{7} \Rightarrow z = \frac{2}{7}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>e.g. $V \leqslant \frac{38}{7}, \frac{38}{7}, \frac{52}{7} \Rightarrow V = \frac{38}{7}$ so the value of the 3×3 game is $\frac{3}{7}$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>The value of the 3×3 game is $\frac{3}{7}$ which is $\frac{5}{28}$ better than the value of the 2×3 game which was $\frac{1}{4}$</td><td>A1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(15 marks)</td></tr>
</table>

Notes for Question 6

(a)

M1: Attempt to calculate row minima and column maxima (all 5 values) – condone one error

A1: Correct reasoning that the game is not stable (accept $-1 \neq 1$ + statement) – dependent on correct row maximin and column minimax which must be clearly identified either around the table or in their statement (must have all 5 max and min values correct)

(b)

M1: setting up three expressions in terms of p

A1: all three expressions correct

M1: axes correct, at least two lines correctly drawn for their expressions, horizontal scale from 0 to 1

A1: correct graph

A1: using the graph to obtain the correct probability expressions leading to the correct value of p

A1ft: interpret their value of p in the context of the question – must refer to play and name options

(c) Condone use of p_1 p_2 p_3 instead of x y z

B1: Correct augmentation (by at least 5) – possibly implied by later working in tableau

B1: Any one (numerical in nature) constraint row (r, s, t or u) correct (ignore labelling of b.v. column) or one correct constraint equation stated (so must be using columns)

B1: Any two (numerical in nature) constraint row (r, s, t or u) correct (ignore labelling of b.v. column) or two correct constraint equations stated

B1: CAO (including b.v. column)

(d)

B1: correct value for z (may be implied by subsequent working)

M1: Attempts to calculate the value of the 3×3 game – must substitute in to at least two of their equations which may either be augmented or unaugmented

(if using the original game then expect to see (if correct) $V \leqslant \frac{3}{7}, \frac{3}{7}, \frac{17}{7} \Rightarrow V = \frac{3}{7}$ so the value of the 3×3 game is $\frac{3}{7}$)

A1: Correct values for both games seen together with some indication of how much better the new game is (could be given as a % increase, for example '71.4% better', or equivalent or as a multiplier e.g, $\frac{12}{7}$ or 1.714)

Question	Scheme	Marks	AOs
7(a)	aux. equation is $m-\alpha=0\Rightarrow$complementary function is $A\alpha^{n}$	B1	2.1
	Particular solution try $u_{n}=Bk^{n}$ and substitute into recurrence relation	M1	1.1b
	$Bk^{n+1}-\alpha Bk^{n}=-\beta k^{n}\Rightarrow Bk-\alpha B=-\beta$ and solve for B	dM1	1.1b
	$u_{n}=A\alpha^{n}+\dfrac{\beta k^{n}}{\alpha-k}$	A1	2.2a
		(4)	
(b)	From given conditions: $u_{0}=L\Rightarrow A+\dfrac{\beta}{\alpha-k}=L$ and $u_{T}=0\Rightarrow A\alpha^{T}+\dfrac{\beta k^{T}}{\alpha-k}=0$	M1	3.4
	Eliminating β which if correct is $A\alpha^{T}+(L-A)k^{T}=0$	dM1	3.1a
	Re-arrange to get an expression for A in terms of L, k, T and α (which for reference if correct is $A=\dfrac{Lk^{T}}{k^{T}-\alpha^{T}}$)	ddM1	3.1a
	Eliminate A and β from their general solution to obtain an expression for u_{n} in terms of L, k, T, n and αonly $u_{n}=\left(\dfrac{Lk^{T}}{k^{T}-\alpha^{T}}\right)\alpha^{n}+\left(L-\dfrac{Lk^{T}}{k^{T}-\alpha^{T}}\right)k^{n}$	dddM1	1.1b
	$u_{n}=\dfrac{Lk^{n}}{\alpha^{T}-k^{T}}\left(\alpha^{T}-k^{T-n}\alpha^{n}\right)=\dfrac{Lk^{n}}{\frac{\alpha^{T}}{\alpha^{T}}-\frac{k^{T}}{\alpha^{T}}}\left(\dfrac{\alpha^{T}}{\alpha^{T}}-\dfrac{k^{T-n}\alpha^{n}}{\alpha^{T}}\right)$ $u_{n}=\dfrac{Lk^{n}}{1-\left(\frac{k}{\alpha}\right)^{T}}\left(1-\left(\dfrac{k}{\alpha}\right)^{T-n}\right)^{*}$	A1	2.2a
		(5)	
(c)	(i) $\alpha=1.06$ (ii) $k=1.04$ (iii) β is the amount that Taylor repaid on the first anniversary of the loan being taken out	B1 B1 B1	2.2a 2.2a 3.1b
		(3)	
(d)	$u_{30}=\dfrac{L(1.04)^{30}}{1-\left(\frac{1.04}{1.06}\right)^{40}}\left[1-\left(\dfrac{1.04}{1.06}\right)^{40-30}\right]$	M1	3.4
	$u_{30}=(1.05\ldots)L>L$ so, after 30 years the outstanding debt (which is (1.05…)L) is larger than the original loan (L)	A1	3.2a
		(2)	
			(14 marks)

Notes for Question 7

(a)

B1: CAO for complementary function (condone if stated as $u_n = A\alpha^n$)

M1: correct form for particular solution and substituted into recurrence relation

dM1: simplifying (by eliminating *n*) and solving for their constant (*B*)

A1: correct general solution (must be $u_{\text{n}} =$) (must be fully simplified)

(b)

M1: Form two equations using their general solution, and both given conditions

dM1: Eliminate either *A* or β from their two equations (formed from the given conditions)

ddM1: Re-arrange to obtain an expression for either β or *A* in terms of *L*, *k*, *T*, *n* and α

They may eliminate *A* and then obtain an expression for $\dfrac{\beta}{\alpha - k}$ and then substitute for this

dddM1: Eliminate both *A* and β from their general solution to obtain an expression for u_n in terms of *L*, *k*, *T*, *n* and αonly

A1: CSO – correctly derive given result (so sufficient working must be shown)

(c)(i)

B1: CAO for α

(ii)

B1: CAO for *k*

(iii)

B1:CAO for the relevance of β in (*) must be clear that this is the amount repaid one year after the loan is taken out. Accept e.g original amount repaid or initial repayment and condone reference to 0th year if clear that this is the first repayment (do not accept annual repayment)

(d)

M1: Substitute $T = 40$, $n = 30$ and their values of α, k into correct particular solution (accept sight of 1.05L for this mark even if full calculation not shown)

A1: CAO that the outstanding debt after 30 years is larger than the original debt (so must see some reference to 1.05*L* (or better) and *L*) (accept 1.05*L* > *L*) (must follow from a correct calculation)

In part (c) for those that at attempt to verify (rather than show the given result) then the first 3 M marks are as in the main scheme (M1 for using the given conditions to form two equations, dM1 for eliminating *A*, ddM1 for obtaining an expression for β (which for reference if correct is $\beta = \dfrac{L\alpha^T(\alpha - k)}{\alpha^T - k^T}$), then the fourth M mark is for obtaining an expression for $u_{n+1} - \alpha u_n$ in terms of *L*, *k*, *T*, *n* and α only – which if correct should be equivalent to $u_{n+1} - \alpha u_n = \frac{Lk^n}{1 - \left(\frac{k}{\alpha}\right)^T}\left\{k - k\left(\frac{k}{\alpha}\right)^{T-n}\left(\frac{\alpha}{k}\right) - \alpha + \alpha\left(\frac{k}{\alpha}\right)^{T-n}\right\} = -k^n\left\{\frac{L(\alpha - k)}{1 - \left(\frac{k}{\alpha}\right)^T}\right\}$. The final A mark is for correctly showing that $\dfrac{L(\alpha - k)}{1 - \left(\frac{k}{\alpha}\right)^T}$ is equivalent to $\dfrac{L\alpha^T(\alpha - k)}{\alpha^T - k^T}$ (so explicitly showing that $u_{n+1} - \alpha u_n$ is equal to $-\beta(k^n)$ with a correct expression for β explicitly seen (in terms of *L*, *k*, *T* and α)).

Alternative for (b)

If they eliminate *A* first

$$\beta = \frac{L\alpha^T(\alpha - k)}{(\alpha^T - k^T)} \text{ and } A = L - \frac{L\alpha^T}{(\alpha^T - k^T)} = \frac{-Lk^T}{\alpha^T - k^T}$$

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