Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 4A Further Pure 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎡ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	(table below)	M1 A1 A1	1.1b 1.1b 1.1b

$\times_{14}$	**1**	**3**	**5**	**9**	**11**	**13**
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11	**3**	**13**	**9**
9	9	13	**3**	11	**1**	**5**
11	11	5	**13**	**1**	9	**3**
13	13	11	**9**	**5**	**3**	1

Question	Scheme	Marks	AOs
1(a)		**(3)**	
(b)	$\{1, 13\}$	B1	2.2a
		(1)	
(c)	States the groups are cyclic or identifies a generator for each. E.g. $3^2 = 9, 3^3 = 13, 3^4 = 11, 3^5 = 5, 3^6 = 1$ so 3 generates G or $q^2 = t, q^3 = s, q^4 = r, q^5 = u, q^6 = p$ so q generates H	M1	3.1a
	E.g. $(3^2 = 9, 3^3 = 13, 3^4 = 11, 3^5 = 5, 3^6 = 1)$ 3 (or 5) generates G **and** $(q^2 = t, q^3 = s, q^4 = r, q^5 = u, q^6 = p)$ q (or u) generates H	A1	1.1b
	Both groups are cyclic of order 6 and so are isomorphic.	A1	2.1
		(3)	
Alt I	(table below)	M1 A1	3.1a 1.1b

G	1	3	5	9	11	13
H	p	q	u	t	r	s
Or ***H***	p	u	q	r	t	s

Question	Scheme	Marks	AOs
Alt I	Also $3^2 = 9$ and $\theta(3)^2 = q^2 = t = \theta(9) = \theta(3^2)$ so structure is preserved and therefore groups are **isomorphic**	A1	2.1
		(3)	
Alt II	(tables below)	M1 A1	3.1a 1.1b

G	1	3	5	9	11	13
Order	identity	6	6	3	3	2

H	p	q	r	s	t	u
Order	identity	6	3	2	3	6

Question	Scheme	Marks	AOs
Alt II	Since orders match and group is order 6 (so one of only two types) therefore the groups are isomorphic **isomorphic**	A1	2.1
		(3)	
		(7 marks)	

Notes:
(a) **M1:** At least 3 correct entries. **A1:** At least 6 correct entries. **A1:** All 12 entries correct. NB for the final A all entries must be given modulo 14. Do not accept e.g. 17 for 3.
(b) **B1:** Deduces the correct subgroup. This is the only subgroup of order 2. Accept with set brackets or round brackets as long as both elements are listed. Accept $\langle 13 \rangle$ as long as the angled brackets are clear.
(c) **M1:** State the groups are cyclic, or identifies a generator for each group. **A1:** Correctly generator identified for each group. **A1:** Correct work for showing a generator for each group and conclude both cyclic and hence isomorphic. Alt I **M1:** Attempts to identify an isomorphism between the elements – may be implied by • identifying at least 4 correct pairings • by attempting to rearrange group tables to have the same structure **A1:** For a correct pairing of all items for an isomorphism. Note that $3^2 = 9$ while $q^2 = t$ and this can be used to see if they have a fully correct pairing (check if $3 \leftrightarrow q$ then you need $9 \leftrightarrow t$, or if $3 \leftrightarrow u$ then you need $9 \leftrightarrow r$) **A1:** All correct pairings with demonstration of structure being preserved for at least one of the possible choices made, or via matching of group tables to show same structure, and concludes isomorphic. NB there are two choices for isomorphism, $1 \leftrightarrow p$ and $13 \leftrightarrow s$ are forced, 3 and 5 must map to q and u either way round, but then 9 and 11 need to be correctly selected for the order of 3 and 5 – and this must be shown for fully marks. They may likely choose the pairings by considering orders (as in Alt II) but still need a reason for the final choice. Alt II **M1:** Attempts to find the order for each element of a both groups – at least four correct for at least one of the groups. **A1:** All orders correct for all elements of both groups. **A1:** All orders correct, reason referring to small group order and therefore groups are **isomorphic**. Note if they go on to give an incorrect pairing it is wrong reasoning so A0. Note: In general matching orders of all elements is a necessary but not sufficient requirement for an isomorphism (e.g. for groups of order 16 there are distinct isomorphism classes which have the same number of elements of the same orders). However, for small groups (up to order 15) with few isomorphism classes this criterion is sufficient but for a fully correct answer some reference to the group order must be given.

Question	Scheme	Marks	AOs
2 (a)	$\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = 0$ leading to $(1-\lambda)(4-\lambda)-4=0$	M1	1.1b
	Solves $\lambda^2 - 5\lambda = 0 \Rightarrow \lambda = ...$	dM1	1.1b
	$\lambda = 5, 0$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \text{their '0'}\begin{pmatrix} x \\ y \end{pmatrix}$ leading to $x-2y=0 \Rightarrow x=2y$ or $\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \text{their '5'}\begin{pmatrix} x \\ y \end{pmatrix}$ leading to $x-2y=5x \Rightarrow 2y=-4x$ or $-2x+4y=5y \Rightarrow y=-2x$	M1	3.1a
	Eigenvectors $\lambda = 0: \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda = 5: \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ oe	A1 A1	2.2a 2.2a
	eg $\mathbf{P} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \end{pmatrix}$ **or** $\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$	B1ft	2.2a
	eg $\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix}$	A1ft	2.5
		(5)	
			(8 marks)

Notes:

(a)

M1: Attempts $\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = 0$

dM1: Dependent on the previous method mark. Solves their 3TQ to find a value for λ

A1: Correct values for λ

(b)

Note this appears on epen as M1A1A1A1A1 but is being marked as M1A1A1B1A1

M1: Uses $\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \text{'their } \lambda\text{'}\begin{pmatrix} x \\ y \end{pmatrix}$ to form an equation of the form $ax = by$ for at least one of their eigenvalues. May be implied by a correct eigenvector.

A1: Deduces one correct eigenvector for one of the (correct) eigenvalues. Accept any non-zero multiple.

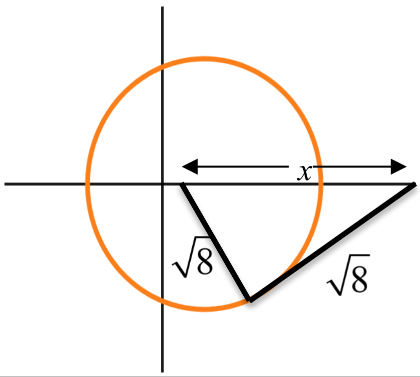
A1: Deduces both correct eigenvectors for the correct eigenvalues. Accept any non-zero multiples.

B1ft: Note: not dependent on the method mark. Deduces a correct matrix **P** or **D**, following through on their eigenvalues or **non-zero** eigenvectors. This may be scored for a correct **D** even if no attempt at the eigenvectors has been made, or a correct f.t. **P** even if the method for eigenvectors was incorrect.

A1ft: **Depends on the M having been scored.** Correct matrices **D** and **P** which are consistent. Follow through on their eigenvalues and **non-zero** eigenvectors.

Note if they assume the eigenvector for $\lambda = 0$ is **0** then do not allow this for the follow through mark(s) (though the first may be gained for correct **D**).

SC: If they mislabel **P** and **D** then allow A1ftA0ft for both correct but the wrong order, but A0A0 if only one is "correct" but wrong order.

Question	Scheme	Marks	AOs
3(a)	$(x-5)^2+(y+4)^2=4\left[(x-2)^2+(y+1)^2\right]$ or $\sqrt{(x-5)^2+(y+4)^2}=2\sqrt{(x-2)^2+(y+1)^2}$	**M1**	1.1b
	$(x-5)^2+(y+4)^2=4\left[(x-2)^2+(y+1)^2\right]$	**A1**	1.1b
	$3x^2-6x+3y^2-21=0$ $x^2-2x+y^2-7=0$ *	**A1***	2.1
		(3)	
(b)	$y=x-a$	**B1**	2.2a
	$x^2-2x+(x-a)^2-7=0$ leading to a 3TQ	**M1**	2.1
	$2x^2-(2+2a)x+a^2-7=0$	**A1**	1.1b
	$b^2-4ac=(2+2a)^2-4(2)(a^2-7)=0$	**M1**	3.1a
	Solve 3TQ $4a^2-8a-60=0\Rightarrow a=\ldots$	**dM1**	1.1b
	$a=5$ only $[a=-3$ must be clearly rejected$]$	**A1**	3.2a
		(6)	
	Alternative $x^2-2x+y^2-7=0\Rightarrow(x-1)^2+y^2=8$	**B1**	2.2a
	Centre (1, 0) & radius $=\sqrt{8}$	**M1** **A1**	2.1 1.1b
			
	E.g. $x^2='8'+'8'\Rightarrow x=\ldots\{4\}$	**M1**	3.1a
	$a='1'+'4'$	**dM1**	1.1b
	$a=5$	**A1**	3.2a
		(6)	
			(9 marks)

Notes
(a) **M1:** Obtains an equation in terms of x and y using the given information. Condone $(x-5)^2+(y+4)^2=2\left[(x-2)^2+(y+1)^2\right]$ for this mark and condone e.g $(y\pm 4)^2$ as s slip but **must** be + between the brackets. **A1:** Obtains any correct equation without the square root. **A1*:** Obtains printed answer, including the "=0", with no errors via an expanded intermediate step.
(b) **B1:** Deduces the equation of the tangent. Allow for $y = x +$ c used (ie correct gradient identified). Note an incorrect tangent $y=-x+a$ will give the same answers as below but must lose the A marks as from incorrect working. **M1:** Substitutes their equation of the tangent into the circle to form a 3TQ. This may be in terms of $y=\pm x+c$ for this mark. **A1:** Correct quadratic equation in x (or y) (may have c instead of $-a$), e.g. $2x^2-(2-2c)x+c^2-7=0$ **M1:** Uses the discriminant equal to 0 to produce a quadratic in a or c **M1:** Solves their 3TQ in a. If working with c they must return to find a value of a for this mark. **A1cso:** Deduces the correct value $a=5$, the solution $a=-3$ must clearly be rejected – e.g. accept underling 5 as the answer. Must be from correct work. **Alternative** **B1:** Correct completion of the square seen or implied. **M1:** Finds the centre and radius of the circle **A1:** Correct centre and radius **M1:** Uses a right-angled triangle with their radius and tangent to find the length x shown in the diagram. E.g. recognises the tangent length has the same length as the radius and uses Pythagoras to find the length x, or uses trigonometry in the right-angled triangle such as $\sin\frac{\pi}{4}=\frac{"\sqrt{8}"}{x}$ leading to x **M1:** Finds the value of a by adding their x-coordinate of the centre to their value of x **A1cso:** $a=5$ Must be from correct work.
(b) Alt II Use of Differentiation may also be seen. **B1:** $2x-2+2y\frac{dy}{dx}=0$ Correct derivative statement. **M1A1:** $\frac{dy}{dx}=\tan\left(-\frac{3}{4}\pi\right)=1\Rightarrow$ radial equation is $2x-2+2y=0$ Any correct equation. **M1:** $y=1-x\Rightarrow x^2-2x+(1-x)^2-7=0\Rightarrow x=...,y=...$ substitutions into the circle equation (either variable) and solves to find the points of intersection of tangent and circle $\left[(3,-2)\right.$ and/or $\left.(-1,2)\right]$ **dM1:** Tangent is $y-(-2)=x-3\Rightarrow y=x-5\Rightarrow a=...$ Correct process to find a including selecting the correct point (below the x-ais) to use. **A1cso:** Correct a Must be from correct work.

Question	Scheme	Marks	AOs
4(a)	$n=1: u_1 = 200(2)^1 - \frac{1}{10}(4)^1 = 399.6$ $n=2: u_2 = 200(2)^2 - \frac{1}{10}(4)^2 = 798.4$ (Hence true for n = 1 and n = 2)	**B1**	2.2a
	Assume true for some $n = k$ and $n = k + 1$, so $u_k = 200(2)^k - \frac{1}{10}(4)^k$ and $u_{k+1} = 200(2)^{k+1} - \frac{1}{10}(4)^{k+1}$	**M1**	2.4
	Then $u_{k+2} = 6\left[200(2)^{k+1} - \frac{1}{10}(4)^{k+1}\right] - 8\left[200(2)^k - \frac{1}{10}(4)^k\right]$	**M1**	1.1b
	e.g. $u_{k+2} = \left[3\left(200(2)^{k+2}\right) - 2\left(200(2)^{k+2}\right)\right] + \left[-\frac{6}{10}(4)^{k+1} + \frac{2}{10}(4)^{k+1}\right]$ $u_{k+2} = 200(2)^{k+2} - \frac{4}{10}(4)^{k+1}$ or $u_{k+2} = \left[3\left(200(2)^{k+2}\right) - 2\left(200(2)^{k+2}\right)\right] + \left[-\frac{3}{20}(4)^{k+2} + \frac{1}{20}(4)^{k+2}\right]$	**M1**	1.1b
	$u_{k+2} = 200\left(2^{k+2}\right) - \frac{1}{10}\left(4^{k+2}\right)$	**A1**	2.1
	Hence <u>if true for $n = k$ and $n = k + 1$ then true for $n = k + 2$</u>. As also <u>true for $n = 1$ and $n = 2$</u>, then <u>true for all n $\in$</u> by mathematical induction.	**A1**	2.4
		(6)	
(b)	$u_n = 200\left(2^n\right) - \frac{1}{10}\left(4^n\right) = 200\left(2^n\right) - \frac{1}{10}\left(2^n\right)^2 = 0$ Leading to $2^n = \ldots\{2000\}$ Alt : reaches $4^n > 2000 \times 2^n \Rightarrow n\log 4 > \log 2000 + n\log 2$	**M1**	3.1a
	Solves $2^n = 2000 \Rightarrow n = \log_2 2000 = \ldots\{10.96\}$ $2^n = 2000 \Rightarrow n = \frac{\log 2000}{\log 2} = \ldots\{10.96\}$ Alt: Solve the linear equation to find n	**dM1**	1.1b
	$u_{11} = -9830.4$ or $-\frac{49152}{5}$	**A1**	2.2a
		(3)	
			(9 marks)

Notes

(a)

B1: Checks the closed form works for n = 1 and n = 2. Allow if they use the recurrence to find u_3 and check for n =2 and n = 3, but a consecutive pair must be checked.

M1: Makes the inductive assumption. If not explicitly made, accept just stating "$n = k$ and $n = k + 1$" as making the assumption these are true – or implied by use of the relevant formulae, as long as the assumption is made clear in the conclusion. May use e.g. $n = k - 2$ and $n = k - 1$ instead and show true for $n = k$. It must be clear it is the closed forms they are assuming, not a recurrence form.

M1: Substitutes expression for $n = k$ and $n = k + 1$ (or equivalents) into the recurrence formula.

M1: Uses algebra in an attempt to achieve the required result

e.g. Uses the coefficients of 6 and 8 to

- write as 2^n terms as 2^{k+2} and simplify.
- write as 4^n terms as 4^{k+1} or 4^{k+2} and simplify.
  Note this is a method mark so you may score for the attempt even if some of the working is incorrect as long as the intent to reach the correct form is clear and at least one bit of indec work is correct.

A1: Completes the process correctly to the required form

A1: Correct conclusion made at the end. Depends on all three M's and the A being gained and an attempt at both n = 1 and n =2 having been shown true. Must convey the underlined ideas of

- true for n = 1 and n = 2
- if true for two successive cases, it is also true for the next case
- a suitable conclusion that it is true for all positive n

though accept equivalent wordings for these.

Note Accept work with n instead of k throughout the inductive step.

(b)

M1: Sets closed form = 0 or < 0 and solves to set up an inequality, or to find a non-zero value, for 2^n. Alternatively if the 4^n is not written in terms of 2^n score for a correct process for taking logs to get a linear equation in n. Be tolerant with incorrect inequalities for the M marks.

dM1: Solves $2^n = a$ where $a > 0$ by any valid means. May be by inspection. In the Alt it is for proceeding to a value for n from the linear equation.

A1: Deduces the first negative term of the sequence

Question	Scheme	Marks	AOs
5(i)(a)	Any two out of 26 or $10\times9\times8$ or 4 seen multiplied together	**M1**	1.1b
	$26\times10\times9\times8\times4=74880$	**A1**	1.1b
		(2)	
(i) (b)	their $'74880'\times\frac{5!}{3!}=1497600$	**B1ft**	1.1b
		(1)	
(ii)	$4^{12}\equiv1(\text{mod}13)$ or $4^{13}\equiv4(\text{mod}13)$	**B1**	1.2
	$(4^{12})^4\times4^2\equiv1\times4^2\,(\text{mod}13)$ or $(4^{13})^3\times4^{11}\equiv4^3\times4^{11}\equiv4^{14}\equiv4^{13}\times4^1\equiv...(\text{mod}13)$	**M1**	2.1
	$4^{50}\equiv3(\text{mod}13)$	**A1**	2.2a
		(3)	
(iii)	The number of different 1-digit odd numbers that do not contain the digit 2 is 5	**B1**	2.2a
	The number of different 2-digit odd numbers that do not contain the digit 2 is $8\times5\{=40\}$ The number of different 3-digit odd numbers that do not contain the digit 2 is $8\times9\times5\{=360\}$	**M1**	1.1b
	There are 500 odd numbers therefore 500 – (5 + 40 + 360)	**dM1**	2.1
	= 95	**A1**	1.1b
		(4)	
Alt 1	Identifies that 2 can only appear in the first and/or second position	**B1**	2.2a
	First digit is 2, second digit 10 possible, third digit 5 possible $=10\times5=...\{50\}$ First digit not 2, 9 options (including 0), second digit is 2, third digit 5 possible $=9\times5=...\{45\}$	**M1**	1.1b
	Therefore total number of digits is “50” + “45” = …	**dM1**	2.1
	= 95	**A1**	1.1b
		(4)	
Alt 2	1 digit number = 0 ways	**B1**	2.2a
	2 digit number = 1 x 5 =5 ways 3 digit number starting with 2 =1 x 10 x 5 = 50 ways Other 3 digit number with middle number 2 = 8 x 1 x 5 = 40 ways	**M1**	1.1b
	Total number = 0 + 5 + 50 + 40	**dM1**	2.1
	= 95	**A1**	1.1b
		(4)	

(10 marks)
Notes
(i) (a) **M1:** Any two out of 26 or $10\times9\times8$ or 4 seen multiplied together **A1:** Correct answer **(b)** **B1ft:** For the correct answer or follow through their answer to (a) multiplied by 20. Note they may start again and use $26\times{}^{10}C_3\times4\times5!$
(ii) **B1:** Recalls Fermat's Little Theorem correctly. May be implicit in the working rather than stated separately. Accept if stated in terms of *a* and *p*. **M1:** A complete attempt to write 4^{50} as powers of 4^{12} or 4^{13} and completes the process to find the residue. **A1:** Deduces the correct residue.
(iii) **B1:** Deduces that there are 3 different 1-digit odd numbers that do not contain the digit 2. Variation: if they consider 000 to 999 allow B1 for correct option of 9 for first digit. **M1:** Finds the number of different 2-digit and 3-digit odd numbers that do not contain the digit 2. (In the variation they would do $9\times9\times5$ possibilities without 2, giving 405 directly.) **dM1:** For 500 – the sum of the number of 1, 2, 3 digits that don't contain the digit 2. Must have been full method to find all the digits without 2 first. **A1:** Correct answer **Alternative 1** **B1:** Deduces that there are only two positions for 2, first and second. (May be implied) **M1:** Considers first digit is 2 and the possible digits for second and third position and considers second digit is 2 and the possible digits for first and third position. Allow if they have double counted some digits for this mark. **dM1:** Full method considering the possibilities for 2 in various positions and adds the results. They must subtract any repeats for this mark. **A1:** Correct answer **Alternative 2** **B1:** Deduces that there are no single digit numbers **M1:** Considers the possible 2 and 3 digit numbers. Allow if they have double counted some digits for this mark or omit some cases as long as at least some cases are considered. **dM1:** Adds up all the possible digits following consideration of all possible combinations. They must subtract any repeats for this mark. For 3 digit numbers they may have $1\times10\times5$ starting 2, $9\times1\times5$ with 2 in middle, but need to note 5 of these are repeated (22x), and so get $5+50+45-5=\ldots$ **A1:** Correct answer. Other Alternatives are possible. E.g. **B1:** Considers 3 digit numbers from 000 to 999. (May be implicit) **M1:** Starting with 2 : 50 possibilities (must be odd), middle 2 but not starting 2 : $9\times1\times5$ possibilities. **dM1:** So total number of possibilities is 50 + 45 = … **A1:** Correct answer.

Question	Scheme	Marks	AOs
6(a)	$\int(4-x^2)^n\,dx=\int 1\times(4-x^2)^n\,dx$ $u=(4-x^2)^n\Rightarrow\frac{du}{dx}=\lambda x(4-x^2)^{n-1}$ and $\frac{dv}{dx}=1\Rightarrow v=x$ $=x(4-x^2)^n-\int\lambda x^2(4-x^2)^{n-1}$	**M1**	2.1
	$\int(4-x^2)^n\,dx=x(4-x^2)^n+\int 2nx^2(4-x^2)^{n-1}\,dx$	**A1**	1.1b
	$=\left[x(4-x^2)^n\right]_0^2+\int 2n\{4-(4-x^2)\}(4-x^2)^{n-1}\,dx$ $=[0]+8n\int(4-x^2)^{n-1}\,dx-2n\int(4-x^2)^n\,dx$	**M1**	2.1
	$I_n=8nI_{n-1}-2nI_n$ $I_n=\frac{8n}{2n+1}I_{n-1}$ *	**A1***	1.1b
		(4)	
(a) Alt	$\int(4-x^2)^n\,dx=\int(4-x^2)(4-x^2)^{n-1}\,dx=4\int(4-x^2)^{n-1}\,dx-\int x.x(4-x^2)^{n-1}\,dx$ $u=x\Rightarrow\frac{du}{dx}=1$ and $\frac{dv}{dx}=x(4-x^2)^{n-1}\Rightarrow v=-\frac{1}{2n}(4-x^2)^n$ $\int(4-x^2)^n\,dx=4I_{n-1}-\left[Kx(4-x^2)^n-\int M(4-x^2)^n\,dx\right]$	**M1**	2.1
	$\int(4-x^2)^n\,dx=4I_{n-1}-\left[-\frac{x}{2n}(4-x^2)^n-\int-\frac{1}{2n}(4-x^2)^n\,dx\right]$	**A1**	1.1b
	$=4I_{n-1}-\left[-\frac{x}{2n}(4-x^2)^n\right]_0^2-\frac{1}{2n}\int_0^2(4-x^2)^n\,dx\;=4I_{n-1}+[0]-\frac{1}{2n}I_n$	**M1**	2.1
	$2nI_n=8nI_{n-1}-I_n\;\Rightarrow I_n=\frac{8n}{2n+1}I_{n-1}$ *	**A1***	1.1b
		(4)	
(b)	$I_1=\int_0^2(4-x^2)\,dx=\left[4x-\frac{x^3}{3}\right]_0^2=\left(8-\frac{8}{3}\right)-(0)=\ldots\left\{\frac{16}{3}\right\}$ Or $I_0=\int_0^2 1\,dx=[x]_0^2=\ldots\qquad\left\{I_1=\frac{8}{3}\times I_0=\ldots\left\{\frac{8}{3}\times 2=\frac{16}{3}\right\}\right\}$	**M1**	2.1
	$I_2=\frac{16}{5}\times$ their $\frac{16}{3}=\ldots\left\{\frac{256}{15}\right\}$	**M1**	1.1b
	$I_3=\frac{24}{7}\times$ their $\frac{256}{15}=\ldots\left\{\frac{2048}{35}\right\},I_4=\frac{32}{9}\times$ their $\frac{2048}{35}=\ldots\left\{\frac{65536}{315}\right\}$	**M1**	1.1b
	$n=4$	**A1**	2.2a
		(4)	
			(8 marks)

Notes

(a)

M1: Writes the integral as $\int 1\times\left(4-x^2\right)^n \,\mathrm{d}x$ and applies integration by parts with $u=\left(4-x^2\right)^n$ and $\frac{\mathrm{d}v}{\mathrm{d}x}=1$ to achieve the correct form

A1: Correct integration

M1: Writes x^2 as $4-\left(4-x^2\right)$ to split the integral into the sum of I_n and I_{n-1}

A1*: Completes the proof by making I_n the subject with no error or omissions seen. Allow recovery of missing brackets.

Alt:

M1: Writes the integral as $\int\left(4-x^2\right)\times\left(4-x^2\right)^{n-1}\,\mathrm{d}x$, splits and applies integration by parts with $u=x$ and $\frac{\mathrm{d}v}{\mathrm{d}x}=x\left(4-x^2\right)^{n-1}$ to achieve the correct form.

A1: Correct integration.

M1: Evaluates the limits and identifies I_n and I_{n-1} within the integral.

A1*: Completes the proof by making I_n the subject with no error or omissions seen. Allow recovery of missing brackets.

NB: There is a possible method via using $4-x^2=\left(2-x\right)\left(2+x\right)$ and integration by parts on these terms. You may score:

M1A1: $\int_0^2\left(2-x\right)^n\left(2+x\right)^n\,\mathrm{d}x=\left[\left(2-x\right)^n\frac{\left(2+x\right)^{n+1}}{n+1}\right]_0^2-\int_0^2 -n\left(2-x\right)^{n-1}\frac{\left(2+x\right)^{n+1}}{n+1}\mathrm{d}x$ oe with *u, v*' reversed, then M1 for full method to reduce integral to terms of *I*'s and A1 if all correct. If unsure send to review. E.g. for second M

$$=-\frac{2^{2n+1}}{n+1}+\frac{n}{n+1}\int_0^2\left(4-x^2\right)^{n-1}\left(4+4x+x^2\right)\mathrm{d}x=-\frac{2^{2n+1}}{n+1}+\frac{n}{n+1}\left(4I_{n-1}+4\int_0^2 x\left(4-x^2\right)^{n-1}\mathrm{d}x+\int_0^2 x^2\left(4-x^2\right)^{n-1}\mathrm{d}x\right)$$

$$=-\frac{2^{2n+1}}{n+1}+\frac{n}{n+1}\left(4I_{n-1}+4\left[-\frac{\left(4-x^2\right)^n}{2n}\right]_0^2-\int_0^2\left(4-x^2-4\right)\left(4-x^2\right)^{n-1}\mathrm{d}x\right)=-\cancel{\frac{2^{2n+1}}{n+1}}+\frac{n}{n+1}\left(4I_{n-1}+\cancel{4\frac{4^n}{2n}}-I_n+4I_{n-1}\right)$$

$$\Rightarrow (n+1)I_{n-1}=8nI_{n-1}-nI_n\Rightarrow I_n=\frac{8n}{2n+1}I_{n-1}$$

(b)

M1: Integrates to find the value of $I_0=\int_0^2 1\,\mathrm{d}x$ or $I_1=\int_0^2\left(4-x^2\right)\mathrm{d}x$ Need not be fully simplified at this stage.

M1: Correct method to find I_2 Again, need not be simplified.

M1: Uses the reduction formula correctly to find the value of I_4 Again, need not be simplified.

A1: Deduces the value of *n*

Question	Scheme	Marks	AOs
7(i)	$\gcd(36, 21) = 3$	M1	1.1b
	3 does not divide into 11 therefore no solutions	A1	2.4
		(2)	
(ii)	$12\times 3y \equiv 12\times 16 \pmod{40}$	M1	1.1b
	$\gcd(12, 40) = 4$ and divides through by 4	M1	1.1b
	$3y \equiv 16 \pmod{10} \Rightarrow 3y \equiv 6 \pmod{10}$ $\gcd(3,10) = 1$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv$ 2,12,22,32 (mod 40)	A1	2.2a
		(4)	
	Alternative $36y \equiv 192 \equiv 32 \pmod{40}$	M1	1.1b
	$\text{hcf}(36,40) = 4 \Rightarrow 9y \equiv 8 \pmod{10}$	M1	1.1b
	E.g. $9y \equiv 18 \pmod{10} \Rightarrow y = 2 \pmod{10}$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv$ 2,12,22,32 (mod 40)	A1	2.2a
		(4)	
			(6 marks)

Notes:

(i)
M1: Finds the gcd of 36 and 21 or in some way considers the hcf of these two numbers.
A1: States that the gcd is 3 and does not divide into 11 therefore no solutions

(ii)
M1: Identifies $36y$ as kay and 192 as kb (may be implied by cancelling).
M1: Finds the gcd of k and 40. Then divides through by the gcd of k and 40, Modulus must be reduced to 10.
dM1: Completes the solution by appropriate means. There will be many variations on how they can do this. Accept just $y = 2$ as an attempt to find the solution. Note the multiplicative inverse of 9 modulo 10 is 9. If a multiplicative inverse is attempted, it must be from a correct method.
A1: Deduces the correct solution
Alternative I
M1: Reduces RHS modulo 40 to give $36y \equiv 32 \pmod{40}$
M1: Finds the gcd of 36 and 40 and divides through by 4. This must include the modulus being reduced to 10.
dM1: Depends on previous M. Finds a value for y by any appropriate means. There will be many variations on how they can do this. Accept just $y = 2$ as an attempt to find the solution.
A1: Deduces the correct solution

Note: Candidates to start by finding the hcf of 36 and 40 and divide through to get $9y \equiv 48 \pmod{10}$ will score M2 at the start of the solution.			
(b) Alt II	$36y \equiv 192 \pmod{40} \Rightarrow 36y = 192 + 40k$	M1	1.1b
	$\Rightarrow 9y = 48 + 10k$	M1	1.1b
	$\Rightarrow 10y - y = 50 - 2 + 10k \Rightarrow y = 10y - 50 - 10k + 2$ $\Rightarrow y = 2 + 10(y - 5 - k)$	dM1	2.1
	$y \equiv 2 \pmod{10}$ or $y \equiv$ 2,12,22,32 (mod 40)	A1	2.2a
		(4)	
(6 marks)			

Notes:

(b) Alt II
M1: Writes the congruence as a linear equation with $40k$
M1: Divides the equation through by 4
dM1: Completes the solution by appropriate means. One example shown in scheme.
A1: Deduces the correct solution.

Question	Scheme	Marks	AOs
8(a)	$30^2 = A(10) \Rightarrow A = ...$	M1	3.3
	$A = 90$	A1	1.1b
		(2)	
(b)	$2y\dfrac{dy}{dx} = \text{their } 90$ or $\dfrac{dy}{dx} = \dfrac{\sqrt{90}}{2}x^{-\frac{1}{2}}$ oe	B1ft	3.4
	$SA = 2\pi\int y\sqrt{1+\left(\dfrac{45}{y}\right)^2}\,dx$	M1	3.4
	$SA = 2\pi\int y\sqrt{\dfrac{y^2+2025}{y^2}}\,dx = 2\pi\int\sqrt{y^2+2025}\,dx = 2\pi\int\sqrt{90x+2025}\,dx$	M1	3.1a
	$\int(\alpha x+\beta)^{0.5}\,dx = K(\alpha x+\beta)^{1.5}$ (NB α may be 1 if they take the 90 out)	M1	1.1b
	$\int(90x+2025)^{0.5}\,dx = \dfrac{(90x+2025)^{1.5}}{90\times1.5}$ oe e.g. $\dfrac{3\sqrt{10}(x+22.5)^{1.5}}{1.5}$ FT is $\dfrac{\sqrt{A}(4x+A)^{1.5}}{12} = \dfrac{2\sqrt{A}\left(x+\frac{A}{4}\right)^{1.5}}{3}$ with their A	A1ft	1.1b
	Use of correct limits $x = 0$ and $x = 10$ $\{2\pi\}\left[\dfrac{(90(10)+2025)^{1.5}}{135} - \dfrac{(90(0)+2025)^{1.5}}{135}\right]$	M1	3.4
	SA = AWRT 3100 (cm^2)	A1	1.1b
		(7)	
(c)	Score for any reasonable comment about a limitation of the model. Eg the dish will not be smooth, the curve may not be accurate, the measurements might not have been exact.	B1	3.5b
		(1)	
			(10 marks)

Notes:

(a)
M1: Uses the model $x = 10$ and $y = 30$ to find a value for A
A1: Correct value for A

(b)
B1: Correct derivative.
M1: Uses the model, their $\dfrac{dy}{dx}$ (which must not be a constant) and the surface area of revolution formula $SA = 2\pi\int y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$ to form an expression of the inner surface area. The 2π must be

present either here or later, but may be missing in between. If never considers this mark and the final A will be lost but the in between marks may be gained. Must be integral with respect to x. Attempting to integrate wrt y will likely gain no further marks in the question as they will not obtain correct forms.

M1: Uses correct algebra to manipulate their integral into the form $\int(\alpha x+\beta)^{0.5}\,\mathrm{d}x \quad (\alpha,\beta\neq 0)$

M1: Integrates to the correct form $\int(\alpha x+\beta)^{0.5}\mathrm{d}x = K(\alpha x+\beta)^{1.5}$ (allow $\beta=0$ for this mark)

A1ft: Correct integration, follow through on their value of A – not on incorrect manipulation.

M1: Use of correct limits $x = 0$ and $x = 10$ on an attempt at an integral, need not be correct but must have come from an attempt at $\int y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$

A1: Correct answer **awrt** 3100 (units not required).

(c)

B1: Any appropriate comment referencing the model. E.g. their may be imperfections in the dish, it may not be smooth. The answer must be referring to the model of the curve of the dish for the inner surface, not other features of the satellite.

Do **not** accept answer such as "there is an antenna" which do not in any way refer to the curve unless the specifically refer to how it might affect the inner surface.

Do **not** accept answer about the thickness of the dish – the model is specifically referring to the curved inner surface of the dish, so thickness is not relevant to this aspect of the model.

Question	Scheme	Marks	AOs
9	$w(z+2\text{i}) = z \Rightarrow wz + 2w\text{i} = z \Rightarrow 2w\text{i} = z - wz$ $\Rightarrow 2w\text{i} = z(1-w) \Rightarrow z = \dfrac{2w\text{i}}{(1-w)}$	M1 A1	3.1a 1.1b
	$\lvert z\rvert = 4 \Rightarrow \left\lvert \dfrac{2w\text{i}}{(1-w)} \right\rvert = 4 \Rightarrow \lvert 2w\text{i}\rvert = 4\lvert 1-w\rvert$	dM1	1.1b
	$w = u + v\text{i}$ $\lvert 2u\text{i} - 2v\rvert = 4\lvert (1-u) - v\text{i}\rvert \Rightarrow (2u)^2 + (2v)^2 = 16\left[(1-u)^2 + v^2\right]$ oe	ddM1	2.1
	$\left\{4u^2 + 4v^2 = 16 - 32u + 16u^2 + 16v^2\right\}$ $12u^2 - 32u + 12v^2 + 16 = 0$ oe	A1	1.1b
	$u^2 - \dfrac{8}{3}u + v^2 + \dfrac{4}{3} = 0 \Rightarrow \left(u - \dfrac{4}{3}\right)^2 - \dfrac{16}{9} + v^2 + \dfrac{4}{3} = 0$	M1	1.1b
	Centre $\left(\dfrac{4}{3}, 0\right)$ **or** radius $= \dfrac{2}{3}$	A1	1.1b
	Centre $\left(\dfrac{4}{3}, 0\right)$ **and** radius $= \dfrac{2}{3}$	A1	2.2a
		(8)	
(8 marks)			
Notes:			
M1: A complete method of rearranging to make z the subject. Condone slips rearranging **A1:** Correct expression for z NB if the only error is incorrect sign, this will correct when taking modulus – allow recovering of later marks, losing just this first A mark. **dM1:** Dependent on the previous method mark. Sets $\lvert \text{their } z\rvert = 4$ and forms a linear equation **ddM1:** Dependent on previous method mark. Uses $w = u + v\text{i}$ and Pythagoras to form an equation in u and v only. Condone $(2u)^2 + (2v)^2 = 4\left[(1-u)^2 + v^2\right]$ or slips in signs on $1 - (u + v\text{i})$. They may factor out the 2 first, which is fine. **A1:** Correct equation **M1:** Having reached a circle equation, completes the square on u (and v if appropriate). **A1:** Correct centre or radius of C from correct work **A1:** Centre and radius both correct from correct work. Accept centre as coordinates or as $\dfrac{4}{3} + 0\text{i}$			
Alt I	$w(z+2\text{i}) = z \Rightarrow wz + 2w\text{i} = z \Rightarrow 2w\text{i} = z - wz$ $\Rightarrow 2w\text{i} = z(1-w) \Rightarrow z = \dfrac{2w\text{i}}{(1-w)}$	M1 A1	3.1a 1.1b
	$z = \dfrac{2u\text{i} - 2v}{1 - u - v\text{i}} \times \dfrac{1 - u + v\text{i}}{1 - u + v\text{i}} = \ldots$	dM1	1.1b

	$\lvert z \rvert = 4 \Rightarrow \left(\dfrac{2v}{(u-1)^2+v^2}\right)^2 + \left(\dfrac{2u^2+2v^2-2u}{(u-1)^2+v^2}\right)^2 = 16$ $\Rightarrow 12v^4+24u^2v^2-56uv^2+28v^2+12u^4-56u^3+92u^2-64u+16=0$ $\Rightarrow 4\left(v^2+u^2-2u+1\right)\left(3v^2+3u^2-8u+4\right)=0$	ddM1	2.1
	$\left\{4u^2+4v^2=16-32u+16u^2+16v^2\right\}$ $12u^2-32u+12v^2+16=0$ oe	A1	1.1b
	$u^2-\dfrac{8}{3}u+v^2+\dfrac{4}{3}=0 \Rightarrow \left(u-\dfrac{4}{3}\right)^2-\dfrac{16}{9}+v^2+\dfrac{4}{3}=0$	M1	1.1b
	Centre $\left(\dfrac{4}{3},0\right)$ **or** radius $=\dfrac{2}{3}$	A1	1.1b
	Centre $\left(\dfrac{4}{3},0\right)$ **and** radius $=\dfrac{2}{3}$	A1	1.1b
		(8)	
Notes:			
M1: A complete method of rearranging to make z the subject. Condone slips rearranging **A1:** Correct expression for z **dM1:** Dependent on the previous method mark. Applies $w = u + v\text{i}$ and rationalises the denominator with correct conjugate used. **ddM1:** Dependent on previous method mark. Extracts real and imaginary components, sets $\lvert \text{their } z \rvert = 4$ and applies $\text{Re}^2+\text{Im}^2=16$ (condone "=4") and proceeds to simplify to a quadratic expression in u and v. This mark is not likely to be successfully earned but is possible. The most likely approach is shown but identifying and cancelling a factor $(u-1)^2+v^2$ is also possible. **A1:** Correct equation **M1:** Having reached a circle equation, completes the square on u (and v if appropriate). **A1:** Correct centre or radius of C from correct work **A1:** Centre and radius both correct from correct work. Accept centre as coordinates or as $\dfrac{4}{3}+0\text{i}$			
9 Alt II	$w=\dfrac{x+\text{i}y}{x+\text{i}y+2\text{i}}\times\dfrac{x-(y+2)\text{i}}{x-(y+2)\text{i}}=\ldots$ $=\dfrac{x^2-x(y+2)\text{i}+\text{i}xy+y(y+2)}{x^2+(y+2)^2}$	M1 A1	3.1a 1.1b
	$\lvert z\rvert=4\Rightarrow x^2+y^2=16\Rightarrow w=\dfrac{x^2+y^2+2y-\text{i}(xy+2x-xy)}{x^2+y^2+4y+4}=\dfrac{16+2y-2x\text{i}}{4y+20}$	dM1	1.1b
	$u=\dfrac{8+y}{2y+10}=\dfrac{1}{2}+\dfrac{3}{2y+10},\ v=\dfrac{-x}{2y+10}$ $\Rightarrow u^2+v^2=\dfrac{(8+y)^2+x^2}{(2y+10)^2}=\dfrac{64+16y+y^2+x^2}{4(y+5)^2}=\dfrac{64+16y+16}{4(y+5)^2}$ $=\dfrac{16y+80}{4(y+5)^2}=\dfrac{16(y+5)}{4(y+5)^2}=\dfrac{4}{y+5}=4\times\dfrac{2}{3}\left(u-\dfrac{1}{2}\right)$	ddM1	2.1

	$\Rightarrow u^2+v^2=\frac{8}{3}\left(u-\frac{1}{2}\right)$	A1	1.1b
	$u^2-\frac{8}{3}u+v^2+\frac{4}{3}=0\Rightarrow\left(u-\frac{4}{3}\right)^2-\frac{16}{9}+v^2+\frac{4}{3}=0$	M1	1.1b
	Centre $\left(\frac{4}{3},0\right)$ **or** radius $=\frac{2}{3}$	A1	1.1b
	Centre $\left(\frac{4}{3},0\right)$ **and** radius $=\frac{2}{3}$	A1	2.2a
		(8)	
			(8 marks)

Notes:

M1: Replaces z by x + iy and rationalises the denominator.
A1: Correct expression.
dM1: Dependent on the previous method mark. Uses $|z|=4\Rightarrow x^2+y^2=16$ in the equation to simplify coefficients
ddM1: Dependent on previous method mark. Extracts the real and imaginary parts of the equation and attempts to find u^2+v^2 eliminates all x's and y's
A1: Correct equation.
M1: Having reached a circle equation, completes the square on u (and v if appropriate).
A1: Correct centre or radius of C from correct work
A1: Centre and radius both correct from correct work

9 Alt III	$\lvert z\rvert=4\Rightarrow\pm4,\pm4\text{i}$ on circle so e.g $\frac{\pm4}{\pm4+2\text{i}},\frac{\pm4\text{i}}{\pm4\text{i}+2\text{i}}$ on C $\Rightarrow\frac{2}{3},2,\frac{4}{5}\pm\frac{2}{5}\text{i}$ on C	M1 A1	3.1a 1.1b
	Perpendicular bisector of points $\frac{2}{3},2$ is $x=\frac{4}{3}$	dM1	1.1b
	Perpendicular bisector of points $\frac{4}{5}\pm\frac{2}{5}\text{i}$ is $y=0$	ddM1	2.1
	$x=\frac{4}{3}$ and $y=0$	A1	1.1b
	Centre is intersection of these lines = …	M1	1.1b
	Centre $\left(\frac{4}{3},0\right)$ **or** radius $=\frac{2}{3}$	A1	1.1b
	Centre $\left(\frac{4}{3},0\right)$ **and** radius $=\frac{2}{3}$	A1	2.2a
		(8)	
			(8 marks)

Notes:
M1: Identifies at least 2 points on the original circle and finds the images on *C* **A1:** Correct simplified two points on *C* **dM1:** Dependent on the previous method mark. Finds the perpendicular bisector of two of their points found on *C* **ddM1:** Dependent on previous method mark. Finds a third point on *C* (or two more points on *C*) and attempts the perpendicular bisector for another pair of points. **A1:** Correct equations for the bisectors. **M1:** Solves the equations of the bisectors to find the centre of the circle. **A1:** Correct centre or radius of *C* from correct work **A1:** Centre and radius both correct from correct work. There may be variations on approach (e.g. using diametrically opposite points to work out centre and radius). If solutions are not fully correct by such approaches, use the review system for guidance.

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