Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 3D Further Decision 1

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Summer 2025
Question Paper Log Number P74082
Publications Code 9FM0_3A_2506_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Bin 1: $\boxed{4.7}$ $\boxed{2.9}$ $\boxed{1.4}$ **2.8** Bin 2: $\boxed{5.5}$ **5.8** Bin 3: **3.8** **6.5** Bin 4: 5.1 6.3 Bin 5: 4.1	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	Bin 1: $\underline{6.5}$ $\underline{6.3}$ Bin 2: $\underline{5.8}$ $\underline{5.5}$ 1.4 Bin 3: $\underline{5.1}$ $\underline{4.7}$ 2.9 Bin 4: $\underline{4.1}$ $\underline{3.8}$ 2.8	M1 A1	1.1b 1.1b
		(2)	
			(5 marks)

Notes:

a1M1: First four items placed correctly (the boxed values) and at least eight values placed in bins – condone cumulative totals for M1 only

a1A1: First eight items placed correctly (the boxed **and** bold values)

a2A1: CSO (no additional/repeated values)

b1M1: First eight items placed correctly (the underlined values) and at least ten values placed in bins (no additional/repeated values) - condone cumulative totals for M1 only

b1A1: CSO

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)</td><td>e.g. Middle right
<table>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Pivots</td></tr>
<tr><td>271</td><td>828</td><td>182</td><td>845</td><td>904</td><td>523</td><td>536</td><td>028</td><td>747</td><td>135</td><td>523</td></tr>
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</table></td><td>M1
A1
A1 ft
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td>Middle left
<table>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Pivots</td></tr>
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</table></td><td></td><td></td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>In the worst case, the first pass requires $(n-1)$ comparisons, the second $(n-2)$, and so on

$\sum_{r=1}^{n-1} r = \frac{1}{2}n(n-1)$</td><td>M1

A1</td><td>3.1a

2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
</table>

(c)

a	b	c	d	decimal	3 sig fig	
1	1	1	1	1	1	Row 1
		1	2	2	2	Row 2
	2	2	$\frac{5}{2}$	2.5	2.5	Row 3
	3	6	$\frac{8}{3}$	2.666…	2.67	Row 4
	4	24	$\frac{65}{24}$	2.708333…	2.71	Row 5
	5	120	$\frac{163}{60}$	2.71666…	2.72	Row 6
	6	720	$\frac{1957}{720}$	2.7180555…	2.72	Row 7

Output is $\frac{1957}{720}$ (accept 2.7180555… or 2.72 if working to 3 s.f.)

M1 1.1b
A1 1.1b
A1 2.2a

SC Accept

a	b	c	d	decimal	3 sig fig
1	1	1	1	1	1
	2	1	2	2	2
	3	2	$\frac{5}{2}$	2.5	2.5
	4	6	$\frac{8}{3}$	2.666…	2.67
	5	24	$\frac{65}{24}$	2.708333…	2.71
	6	120	$\frac{163}{60}$	2.71666…	2.72
		720	$\frac{1957}{720}$	2.7180555…	2.72

(3)

(d)

$\frac{\left|\frac{1957}{720} - e\right|}{e} \times 100 = 0.00832(\%)$ If using 2.72 error = 0.0632(%)

B1 ft 3.2b

(1)

(10 marks)

Notes:

a1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing any other item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). Condone one error or omission in the list.

If only choosing one pivot per iteration then max of M1 only

Bubble sort is not a MR and scores M0.

If the list is sorted into ascending order, they can score M1 A1 A0 A0 for a fully correct sort even if reversed (if any error seen this can score M1 only).

If 028 is written as 28 at any stage this will lose the final CSO mark

The choice of pivots may only be seen once they have been used

a1A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct)

a2A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

If they alternate MR and ML pivots for the first and second passes, the pivots for the third pass must be consistent with their second pass

a3A1: CSO – so must have chosen and used 747 (MR) or 828 (ML) as a pivot for the fifth pass

b1M1: Considering the total number of comparisons in the worst case (possibly will see e.g. $(n-1)+(n-2)+\ldots[+2+1]$ which scores this mark (brackets not needed and sufficient terms to make sum clear. Accept a clear list without + signs for this mark)

b1A1: Using the standard series result that $\sum r = \frac{1}{2}n(n+1)$ with $n = (n-1)$ to get the correct quadratic expression for the total number of comparisons when sorting n values using bubble sort (allow any letter) Accept $(n-1) + (n-2) + \ldots + 2 + 1 = \frac{1}{2}n(n-1)$ for both marks (sight of $\frac{1}{2}n(n-1)$ with no working scores both marks) (ISW if correct expression incorrectly expanded)

Note: Rows are shown above, and they may be spread across multiple rows in the table. Each row ends when the appropriate value in column d is seen

c1M1: At least four rows of cells completed with a correct first and second row – condone repeated values in all columns or a single value in each row (Note SC accept Column B with 1, 2, 3, 4, 5, 6 with no empty cell between 1 and 2)

c1A1: CAO – the values in the third, fourth and fifth rows correct (accept either full recurring decimals or decimals consistently rounded to 3 significant figures)

c2A1: CAO - correct output following a correct seventh row (their output must be clearly indicated e.g. on the Output line or identified in or near the table) If a value is written on the output line, this takes precedence over the table.

d1B1ft: Follow through their answer to (c) provided that % error is **< 0.1%** (must be given to at least 3 significant figures) (condone missing %) (accept - 0.00832%) (if a numerical approximation to e is used, it must be given to at least 3 d.p. – so 2.718 or better) (accept 8.32×10^{-3}). If incorrect working is seen, even if followed by a correct answer, this is B0.

Note $\dfrac{\left|\frac{1957}{720} - e\right|}{\frac{1957}{720}} \times 100$ = 0.00832% scores B0

Sort Ascending Max M1 A1 A0 A0 e.g Middle right

										Pivots
271	828	182	845	904	**523**	536	028	747	135	523
271	182	**028**	135	523	828	845	**904**	536	747	028, 904
028	271	**182**	135	523	828	845	**536**	747	904	182, 536
028	135	182	271	523	536	828	**845**	747	904	845
028	135	182	271	523	536	828	**747**	845	904	747
028	135	182	271	523	536	747	828	845	904	

Middle left

										Pivots
271	828	182	845	**904**	523	536	028	747	135	904
271	828	182	845	**523**	536	028	747	135	904	523
271	**182**	028	135	523	828	**845**	536	747	904	182, 845
028	135	182	271	523	828	**536**	747	845	904	028, 536
028	135	182	271	523	536	**828**	747	845	904	828
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<tr><td>3(a)</td><td>
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td></tr>
<tr><td>A</td><td>-</td><td>9</td><td>∞</td><td>25</td><td>35</td><td>∞</td><td>11</td></tr>
<tr><td>B</td><td>∞</td><td>-</td><td>40</td><td>8</td><td>∞</td><td>35</td><td>∞</td></tr>
<tr><td>C</td><td>∞</td><td>40</td><td>-</td><td>17</td><td>∞</td><td>∞</td><td>47</td></tr>
<tr><td>D</td><td>∞</td><td>8</td><td>17</td><td>-</td><td>11</td><td>31</td><td>∞</td></tr>
<tr><td>E</td><td>35</td><td>∞</td><td>∞</td><td>11</td><td>-</td><td>14</td><td>∞</td></tr>
<tr><td>F</td><td>∞</td><td>35</td><td>∞</td><td>31</td><td>14</td><td>-</td><td>12</td></tr>
<tr><td>G</td><td>11</td><td>∞</td><td>47</td><td>∞</td><td>24</td><td>12</td><td>-</td></tr>
</table>
</td><td>B2, 1, 0</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>Fourth iteration:
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td></tr>
<tr><td>A</td><td>-</td><td>9</td><td>34</td><td>17</td><td>28</td><td>44</td><td>11</td></tr>
<tr><td>B</td><td>∞</td><td>-</td><td>25</td><td>8</td><td>19</td><td>35</td><td>72</td></tr>
<tr><td>C</td><td>∞</td><td>25</td><td>-</td><td>17</td><td>28</td><td>48</td><td>47</td></tr>
<tr><td>D</td><td>∞</td><td>8</td><td>17</td><td>-</td><td>11</td><td>31</td><td>64</td></tr>
<tr><td>E</td><td>35</td><td>19</td><td>28</td><td>11</td><td>-</td><td>14</td><td>46</td></tr>
<tr><td>F</td><td>∞</td><td>35</td><td>48</td><td>31</td><td>14</td><td>-</td><td>12</td></tr>
<tr><td>G</td><td>11</td><td>20</td><td>45</td><td>28</td><td>24</td><td>12</td><td>-</td></tr>
</table>
Fifth iteration:
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td></tr>
<tr><td>A</td><td>-</td><td>9</td><td>34</td><td>17</td><td>28</td><td>42</td><td>11</td></tr>
<tr><td>B</td><td>54</td><td>-</td><td>25</td><td>8</td><td>19</td><td>33</td><td>65</td></tr>
<tr><td>C</td><td>63</td><td>25</td><td>-</td><td>17</td><td>28</td><td>42</td><td>47</td></tr>
<tr><td>D</td><td>46</td><td>8</td><td>17</td><td>-</td><td>11</td><td>25</td><td>57</td></tr>
<tr><td>E</td><td>35</td><td>19</td><td>28</td><td>11</td><td>-</td><td>14</td><td>46</td></tr>
<tr><td>F</td><td>49</td><td>33</td><td>42</td><td>25</td><td>14</td><td>-</td><td>12</td></tr>
<tr><td>G</td><td>11</td><td>20</td><td>45</td><td>28</td><td>24</td><td>12</td><td>-</td></tr>
</table>
</td><td>M1

A1

M1

A1

A1</td><td>2.1

1.1b

2.1

1.1b

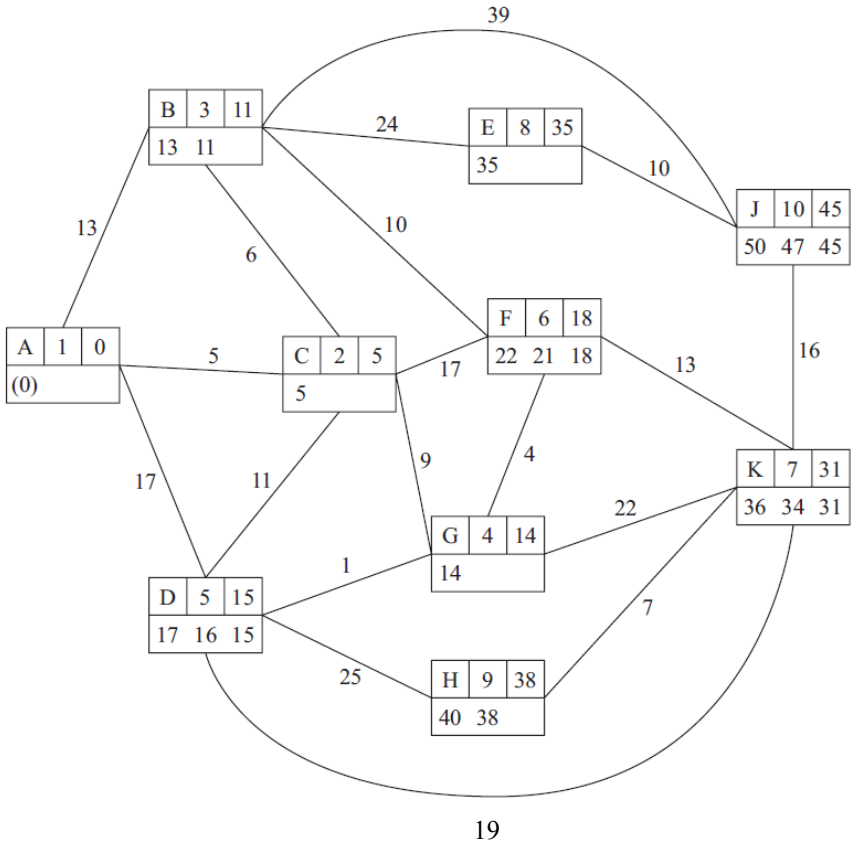
2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(c)(i)</td><td>NNA starting at B: B – D – E – F – G – A – C – B</td><td>B1</td><td>2.2a</td></tr>
<tr><td>(ii)</td><td>Time taken: 8 + 11 + 14 + 12 + 11 + 34 + 25 = 115 (minutes)</td><td>B1</td><td>1.1b</td></tr>
<tr><td>(iii)</td><td>Actual villages visited: BDEFGABDCDB</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Notes:

a1B1: At least three rows and three columns correct (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)
a2B1: CAO

b1M1: No change in the fourth row and fourth column with at least two values reduced correctly (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)
b1A1: CAO for the fourth iteration (changes in bold and blue shading)
b2M1: No change in the fifth row and fifth column with at least two values reduced correctly (follow through from their previous iteration) (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)
b2A1: At least eight values reduced correctly (from the values shown in bold and orange shading)
b3A1: CSO for the fourth and fifth iterations
(if correct changes for the fifth iteration are shown in bold and orange shading)

ci1B1: CAO (Nearest neighbour cycle starting at B – must include return to B) (condone the use of arcs BD DE EF FG GA AC CB – must be written in this manner and not AG or BC)
cii2B1: Correct time
ciii3B1: Correct route – must be stated as nodes

Question	Scheme	Marks	AOs
4(a)	 **(i)** Route from J to K via A: JEBCACGFK **(ii)** Length of shortest route from J to K via A is 76 (m)	**M1** **A1** (ACBGD) **A1** (FKE) **A1ft** (HJ) **A1** **A1ft**	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)(i)	BC + D(GF)K = 6 + 18 = 24 B(FG)D + C(GF)K = 15 + 26 = 41 B(F)K + C(G)D = 23 + 10 = 33 Edges to traverse twice are BC, DG, FG, FK	**M1** **A1** **A1** **A1**	3.1b 1.1b 1.1b 2.2a
(ii)	Length of route = 268 + 24 = 292 (m)	**A1ft**	2.2a
		(5)	
(c)	E.g. [24 –] (“45” – 13 – 11 – 16) Or [292 –] (268 + “45” – (13 + 11 + 16)) = 19 (m)	**M1** **A1**	3.1b 2.2a
		(2)	
			(13 marks)

Notes:

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 22 21 18 in that order (so 22 18 21 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,… will be penalised once (see notes below) but 1, 2, 3, 5, 6,… is fine. Errors in the final values and working values are penalised before errors in the order of labelling

If starting at J or K instead of A M0

a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either B or D or F or H or J or K

a1A1: All values at A, C, B, G and D correct and working values in the correct order. Condone lack of 0 in As working value

a2A1: All values at F, K and E correct and the working values in the correct order. Penalise order of labelling only once per question (F, K and E must be labelled in that order and F must be labelled after A, C, B, G and D)

a3A1ft: All values at H and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working values at H follow from the candidate's final values from the feeds into H (which will come from nodes D and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from B, E and K with the order of labelling determined by the candidate's order of labelled at B, E and K)

a4A1: CAO (correct path from J to K via A – JEBCACGFK) (accept in terms of arcs)

a5A1ft: ft their final value at J + their final value at K only (if 76 stated and 76 is not the sum of these final values then A0)

If route and length are written on the wrong lines this can still score both marks

bi1M1: The correct three pairings of the correct four nodes (B, C, D and K)

bi1A1: Two rows correct including pairings and totals

bi2A1: All three rows correct including pairings and totals

bi3A1: Correct repeats (BC, DG, FG, FK)

bii4A1ft: Length of route correct on the follow through (268 + smallest repeat) but must be from a choice of 3 totals

c1M1: For using their final value at J and the blocked lengths in attempt to work out the length or change in length of the new route (sight of "45" 13 11 and 16) (follow through their final value at J for this mark) (condone use of 40 for 13 + 11 + 16)

c1A1: CAO (19)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>The Simplex algorithm cannot be used as the problem has no obvious basic feasible solution since the origin is not in the feasible region

Alternatively

Simplex can only be used with non-negative values of variables</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$2(X+4)+2(Y+1)+2(-Z-3)\leqslant 21 \Rightarrow 2X+2Y-2Z\leqslant 17$

$2(X+4)-(Y+1)-(-Z-3)\leqslant 18 \Rightarrow 2X-Y+Z\leqslant 8$

$-3(X+4)+(Y+1)-2(-Z-3)\leqslant 1 \Rightarrow -3X+Y+2Z\leqslant 6$

$Q+10=3(X+4)+4(Y+1)+2(-Z-3) \Rightarrow Q=3X+4Y-2Z$

$(X\geqslant 0, Y\geqslant 0, Z\geqslant 0)$</td><td>M1

A1

A1</td><td>3.3

1.1b

1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td>$Q-\frac{5}{4}Z+\frac{15}{8}s_1+\frac{1}{4}s_3=\frac{267}{8}$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>$Q=\frac{5}{4}Z-\frac{15}{8}s_1-\frac{1}{4}s_3+\frac{267}{8}$ so therefore, we can increase the profit by increasing Z</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)(i)</td><td>
<table>
<tr><td>b.v.</td><td>X</td><td>Y</td><td>Z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>Value</td><td>Row Ops</td></tr>
<tr><td>X</td><td>1</td><td>0</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>0</td><td>$\frac{11}{2}$</td><td>r1 + $\frac{3}{4}\times R2$</td></tr>
<tr><td>Z</td><td>0</td><td>0</td><td>1</td><td>$\frac{1}{18}$</td><td>$\frac{4}{9}$</td><td>$\frac{1}{3}$</td><td>$\frac{13}{2}$</td><td>$\frac{4}{9}\times$ r2</td></tr>
<tr><td>Y</td><td>0</td><td>1</td><td>0</td><td>$\frac{7}{18}$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{3}$</td><td>$\frac{19}{2}$</td><td>r3 + $\frac{1}{4}\times R2$</td></tr>
<tr><td>Q</td><td>0</td><td>0</td><td>0</td><td>$\frac{35}{18}$</td><td>$\frac{5}{9}$</td><td>$\frac{2}{3}$</td><td>$\frac{83}{2}$</td><td>r4 + $\frac{5}{4}\times R2$</td></tr>
</table>
</td><td>B1

M1

A1

A1</td><td>1.1b

2.1

1.1b

1.1b</td></tr>
<tr><td>(ii)</td><td>$X=\frac{11}{2}, Y=\frac{19}{2}, Z=\frac{13}{2}, Q=\frac{83}{2}$ $(X=5.5\ Y=9.5\ Z=6.5\ Q=41.5)$
$x=\frac{19}{2}, y=\frac{21}{2}, z=-\frac{19}{2}, P=\frac{103}{2}$ $(x=9.5\ y=10.5\ z=-9.5\ P=51.5)$</td><td>M1
A1ft</td><td>3.4
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes:

a1B1: Correct reason why Simplex cannot be used to solve the LP problem. Please mark positively and award if a correct statement is seen.
SC accept Because $z \leqslant -3$ (and variables must be $\geqslant 0$)

b1M1: Substituting given equations into all three given inequalities to form inequalities or equations with slack variables in terms of *X*, *Y* and *Z* (condone at most two slips but see Special Case below)
b1A1: At least two of the four expressions simplified correctly (three inequalities and the new objective) (accept any equivalent rearrangement but the objective must be in terms of *Q* not *P*) (accept equations with slack variables)
b2A1: CAO (must be inequalities)

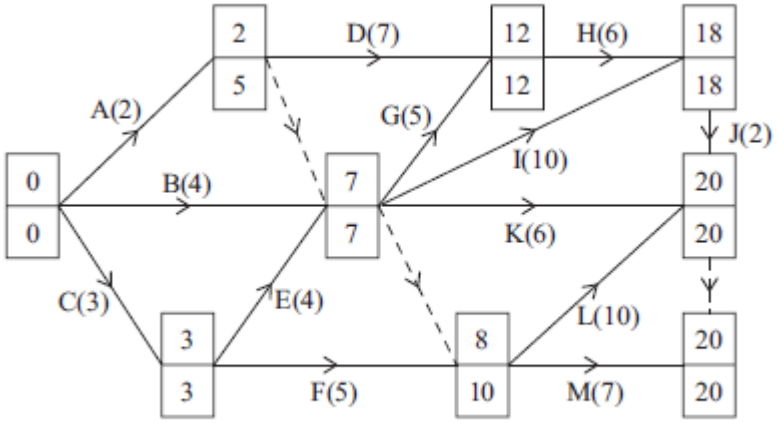
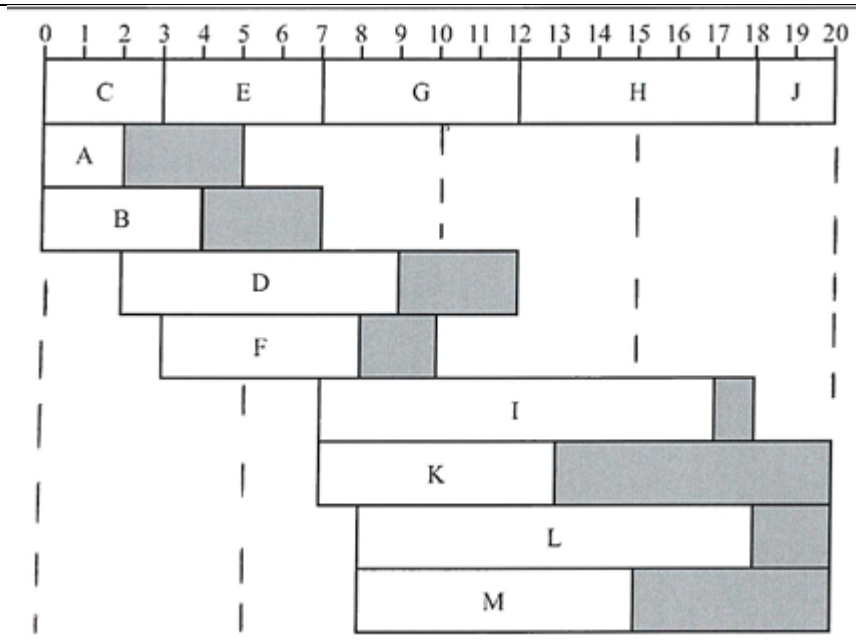
c1B1: CAO (must be in terms of Q)

d1B1: CAO (Must include mention that *Z* can be increased but do not award if any incorrect statement seen)
Do not accept just a statement that the objective row contains a negative value and therefore it is not optimal

Note – accept correct recurring decimals in place of fractions or any equivalent fractions
ei1B1: Pivot row (*Z* row) correct including change of b.v. (ignore row ops)
ei1M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (s_1, s_2, s_3or value) correct
ei1A1: Row operations used correctly at least twice, i.e. **two** of the non-pivot rows or **two** of the non zero and one columns (s_1, s_2, s_3or value) – ignore row operations for this mark
ei2A1: CAO all values and row operations correctly stated including b.v column (allow alternative numbering of rows as long as this is clear. Condone use of r_2 throughout. Do not accept in terms of b.v.) (Accept in terms of original r2 so r1 + 1/3r2, 4/9r2, r3 + 1/9r2, r4 + 5/9r2) (Row ops for pivot row may be written as r2 ÷ 9/4)

eii2M1: Stating (or implying) optimal values of *X*, *Y*, *Z* and *Q* (see special case)
eii3A1ft: CAO for *x*, *y*, *z* and *P* (the correct 4 values implies both marks) (follow through their values for *X*, *Y*, *Z* and *Q* from the tableau)

Special Case
(b) If they make a consistent error when substituting by using an incorrect expression for *X*, *Y* or *Z* (e.g. substituting $x = X + 1$ instead of $x = X + 4$ in all terms) they may score M1 A1 (for at least two of their four expressions correct) A0
(e) If they use the same incorrect substitution and do not state the values of *X*, *Y*, *Z* and *Q* they may score M1 A0 for the implied values of *X*, *Y*, *Z* and *Q* from their *x*, *y*, *z* and *P*

Question	Scheme	Marks	AOs
6(a)	<table><tr><th>Activity</th><th>IPA</th></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>-</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>C</td></tr><tr><td>F</td><td>C</td></tr></table> <table><tr><th>Activity</th><th>IPA</th></tr><tr><td>G</td><td>A, B, E</td></tr><tr><td>H</td><td>D, G</td></tr><tr><td>I</td><td>A, B, E</td></tr><tr><td>J</td><td>H, I</td></tr><tr><td>K</td><td>A, B, E</td></tr><tr><td>L</td><td>A, B, E, F</td></tr><tr><td>M</td><td>A, B, E, F</td></tr></table>	**B1** **B1**	1.1b 1.1b
		(2)	
(b)		**M1** **A1** **M1** **A1**	1.1b 1.1b 1.1b 1.1b
		(4)	
(c)		**M1** **A1** **A1** **A1**	2.1 1.1b 1.1b 1.1b
		(4)	

(d)	**Activity**	**Workers**		**Activity**	**Workers**	**B3, 2, 1**	3.4 1.1b 1.1b
	A	1		G	2		
	B	2		H	1		
	C	1		I	2		
	D	2		J	2		
	E	3		K	1		
	F	1		L	1		
				M	1		

(13 marks)

Notes:

a1B1: Five correct rows (not including A, B and C)
a2B1: All rows correct (accept blanks for A, B and C)

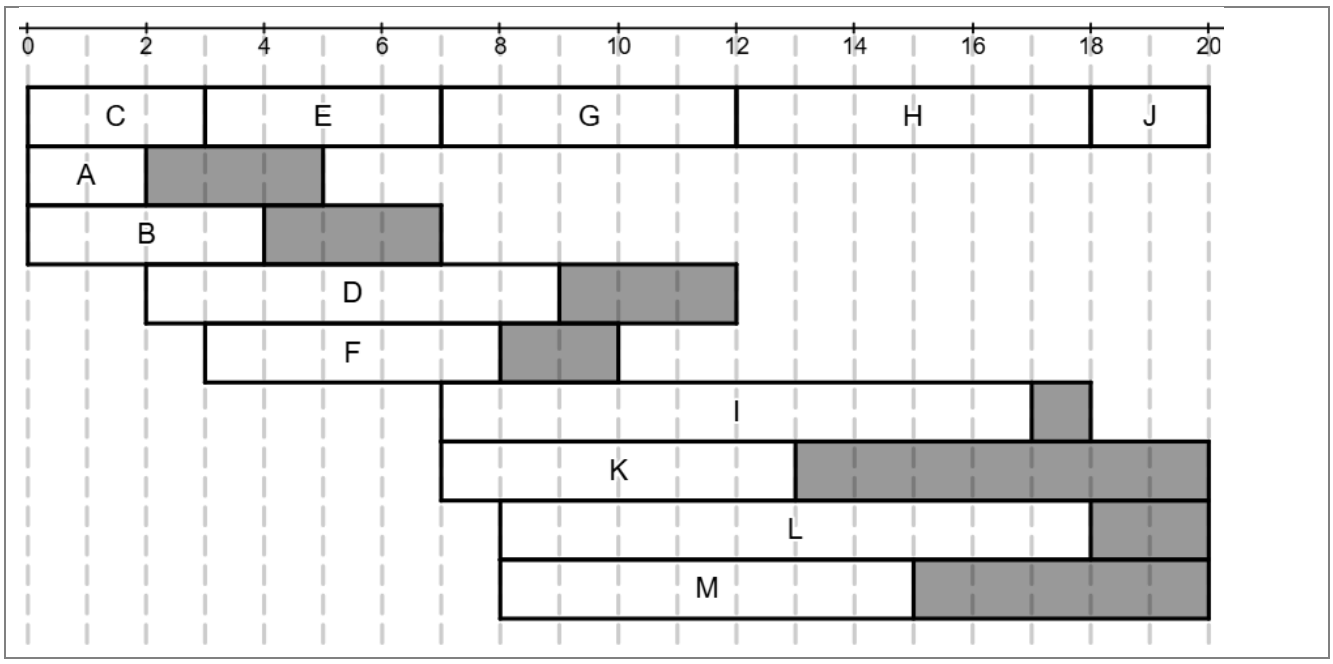
b1M1: All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue value which is a number in a top box greater than the subsequent value
b1A1: CAO – top boxes
b2M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue value which is a number in a bottom box greater than the previous value. Condone missing 0 and/or their 20 (at the end event) for the M mark only
b2A1: CAO – bottom boxes

c1M1: Gantt (cascade) chart with at least 9 activities labelled and at least four activities having non-zero float. A scheduling diagram scores M0
c1A1: Critical activities (C, E, G, H and J) correct (note this may be seen as separate activities listed alphabetically instead of at the top of the chart)
c2A1: At least 4 non-critical activities correct
c3A1: CAO All 13 activities present (just once). No errors**.**

d1B1: At least four correct
d2B1: At least eight correct
d3B1: CAO

For reference these are the early start, late end and length of float for the non-critical activities

Activity	Start	Late End	Float	Critical	Start	End
A	0	5	3	C	0	3
B	0	7	3	E	3	7
D	2	12	3	G	7	12
F	3	10	2	H	12	18
I	7	18	1	J	18	20
K	7	20	7			
L	8	20	2			
M	8	20	5			


0
2
4
6
8
10
12
14
16
18
20
C
E
G
H
J
A
B
D
F
I
K
L
M

Qu	Scheme	Marks	AOs
7(a)	$3x+4y \leqslant 20 \Rightarrow 3x+4y+s_1=20$ $3x+y \leqslant 8 \Rightarrow 3x+y+s_4=8$	**B1**	2.5
	$x \geqslant 1 \Rightarrow x-s_2+a_1=1$	**B1**	2.5
	$2x+3y \geqslant 6 \Rightarrow 2x+3y-s_3+a_2=6$	**B1**	1.1b
	$P=11x+ky-M\left(a_1+a_2\right)$ $a_1+a_2=7-3x-3y+s_2+s_3$ $P=11x+ky-M\left(7-3x-3y+s_2+s_3\right)$	**M1**	2.1
	$P-(11+3M)x-(k+3M)y+Ms_2+Ms_3=-7M$	**A1**	2.2a
	e.g. b.v. | x | y | s_1 | s_2 | s_3 | s_4 | a_1 | a_2 | Value s_1 | 3 | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 20 a_1 | 1 | 0 | 0 | -1 | 0 | 0 | 1 | 0 | 1 a_2 | 2 | 3 | 0 | 0 | -1 | 0 | 0 | 1 | 6 s_4 | 3 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 8 P | $-(11+3M)$ | $-(k+3M)$ | 0 | M | M | 0 | 0 | 0 | $-7M$	**M1** **A1**	3.3 2.2a
		(7)	
(b)	Using $x=\frac{18}{7}, y=\frac{2}{7}$ or stating $P=\frac{198}{7}+\frac{2}{7}k$	**B1**	3.4
	If optimal after the third iteration, then $\frac{11}{7}-\frac{3}{7}k \geqslant 0$ and $\frac{33}{7}-\frac{2}{7}k \geqslant 0$	**M1**	3.1a
	$(0<)\ k \leqslant \frac{11}{3}$	**A1**	2.2a
	Maximum P when $k=\frac{11}{3}$ and $P=11\left(\frac{18}{7}\right)+\frac{11}{3}\left(\frac{2}{7}\right)$	**dM1**	3.4
	Optimal value of P is $\frac{88}{3}$ accept answers from a stated value of k from $(0<)\ k \leqslant \frac{11}{3}$ so $\frac{198}{7} < P \leqslant \frac{88}{3}$	**A1**	2.2a
		(5)	
			(12 marks)

Notes:

If correct they must use two slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable
Please check suffices on *s* and *a* terms carefully – they may be in a different order – check that these are consistent

a1B1: Correctly re-writing the two ⩽ inequalities as equations with slack variables (can be implied by the corresponding two correct rows in Simplex tableau – our row 1 and 4)
a2B1: Correctly re-writing one of the ⩾ inequalities as an equation with surplus and artificial variables (can be implied by a correct corresponding row in Simplex tableau – our row 2 or 3)
a3B1: Correctly re-writing both ⩾ inequalities
a1M1: Forming an objective of the form $P = 11x + ky - M(a_1 + a_2)$ and substituting for a_1 and a_2 (we must see the substitution but this does not need to be a correct expression for this mark)
a1A1: CAO for new objective (accept equivalent equation with terms in *x* and *y* collected) (M1 A1 may be implied by a correct objective row in the tableau)
a2M1: Any two rows correct on the ft from the candidate's stated equations (ignore b.v. for this mark)
a2A1: CAO (including consistent b.v. column) – note that the candidate's order in which the rows appear in the tableau (and choice of slack variable) may be different

A fully correct tableau implies all marks in (a) provided that there are no errors seen in the formation of the objective function

b1B1: Either using the correct values of *x* and *y* in the objective function or stating $P = \frac{198}{7} + \frac{2}{7}k$
b1M1: Considering at least one of the expressions (s_3 or s_4 columns) in the *P* row that involve *k* and compare with 0 (accept any correct inequality or equals)
b1A1: Correct range of values for *k* (condone missing 0 <) but must have considered both possibilities and chosen 11/3 (may be implied by subsequent working) (allow *k* = 11/3 stated as the maximum value)
b2dM1: Dependent on previous M mark – using their *k* in given *P* which must come from a correct inequality or equation (they may choose a value of *k* from the correct range e.g. *k* = 3)
b2A1: CAO **-** Correct value of *P* **for their choice of *k***
(accept answers in the range $\frac{198}{7} < P \leqslant \frac{88}{3}$ if *k* = 3 *P* = 204/7)

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