

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 21 Further Pure Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded.
 Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS General Instructions for Marking

- 1. The total number of marks for the paper is 40.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- ***** The answer is printed on the paper
- L The second mark is dependent on gaining the first mark
- 4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
 If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
- 6. Ignore wrong working or incorrect statements following a correct answer.
- 7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

1(a) $x = 0 \Rightarrow 12 - 7.5 = 4.5^{\circ}C$ B1 3.4 (b) $S = 12 - \frac{15}{2} \left(\frac{1-t^2}{1+t^2}\right) - \frac{27}{10} \left(\frac{2t}{1+t^2}\right)$ M1 1.1b $= \frac{120(1+t^2) - 75(1-t^2)3 - 54t}{10(1+t^2)}$ M1 2.1 (c) $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2$ M1 3.4 $95t^2 - 54t - 55 = 0$ A1 1.1b $t = 1.09, \Rightarrow \frac{x}{2} = \tan^{-1}1.09 = \{47.4\text{or } 0.828\}$ or $-0.528 \Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) = \{-27.83\text{or } -0.485\}$ Alternative: Leading to a value for x $t = 1.09 \Rightarrow \sin x = \frac{2(1.09)}{1 + (1.09)^2} \text{ or } \cos x = \frac{1 - (1.09)^2}{1 + (1.09)^2}$ $\frac{dM1}{1 + (1.09)^2} = \frac{3.4}{1 + (-0.528)^2}$ $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1 \Rightarrow x =$ $x = 360 + \sin^{-1}\left(\frac{2(-0.528)}{1 + (-0.528)^2}\right)$ ddM1 3.1b $x = 360 - \cos^{-1}\left(\frac{1 - (-0.528)^2}{1 + (-0.528)^2}\right)$ ddM1 3.2a	Question	Scheme	Marks	AOs
(b) $S = 12 - \frac{15}{2} \left(\frac{1-t^2}{1+t^2} \right) - \frac{27}{10} \left(\frac{2t}{1+t^2} \right) $ M1 1.1b $= \frac{120(1+t^2) - 75(1-t^2)3 - 54t}{10(1+t^2)}$ M1 1.1b $= \frac{195t^2 - 54t + 45}{10(1+t^2)}$ A1 2.1 (c) $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2$ M1 3.4 $95t^2 - 54t - 55 = 0$ A1 1.1b $t = 1.09, \Rightarrow \frac{x}{2} = \tan^{-1}(.09 =(47.4or\ 0.828)$ or $-0.528\Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) =(-27.83or\ -0.485)$ Alternative: Leading to a value for x $t = 1.09\Rightarrow \sin x = \frac{2(1.09)}{1 + (1.09)^2} \text{ or } \cos x = \frac{1 - (1.09)^2}{1 + (1.09)^2}$ Or $t = -0.528\Rightarrow \sin x = \frac{2(-0.528)}{1 + (-0.528)^2} \text{ or } \cos x = \frac{1 - (-0.528)^2}{1 + (-0.528)^2}$ $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1\Rightarrow x =$ $x = 360 + \sin^{-1}\left(\frac{2(-0.528)}{1 + (-0.528)^2}\right) \text{ ddM1} 3.1b$ $x = 360 - \cos^{-1}\left(\frac{1 - (-0.528)^2}{1 + (-0.528)^2}\right)$ 304 days A1 3.2a	1(a)	$x = 0 \Rightarrow 12 - 7.5 = 4.5$ °C	B1	3.4
$S = 12 - \frac{13}{2} \left(\frac{1+t^2}{1+t^2} \right) - \frac{21}{10} \left(\frac{2t}{1+t^2} \right) $ M1 1.1b $= \frac{120(1+t^2) - 75(1-t^2) 3 - 54t}{10(1+t^2)}$ M1 1.1b $= \frac{195t^2 - 54t + 45}{10(1+t^2)}$ A1 2.1 $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2$ M1 3.4 $95t^2 - 54t - 55 = 0$ A1 1.1b $t = 1.09, \Rightarrow \frac{x}{2} = \tan^{-1}1.09 = \{47.4\text{or } 0.828\}$ or $-0.528 \Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) = \{-27.83\text{or } -0.485\}$ Alternative: Leading to a value for x $t = 1.09 \Rightarrow \sin x = \frac{2(1.09)}{1 + (1.09)^2} \text{ or } \cos x = \frac{1 - (1.09)^2}{1 + (1.09)^2}$ Or $t = -0.528 \Rightarrow \sin x = \frac{2(-0.528)}{1 + (-0.528)^2} \text{ or } \cos x = \frac{1 - (-0.528)^2}{1 + (-0.528)^2}$ $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1 \Rightarrow x =$ $x = 360 + \sin^{-1}\left(\frac{2(-0.528)}{1 + (-0.528)^2}\right)$ ddM1 3.1b $x = 360 - \cos^{-1}\left(\frac{1 - (-0.528)^2}{1 + (-0.528)^2}\right)$ ddM1 3.1b			(1)	
$= \frac{195t^2 - 54t + 45}{10(1+t^2)} $ A1 2.1 $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2 $ M1 3.4 $\frac{95t^2 - 54t - 55 = 0}{10(1+t^2)} $ A1 1.1b $t = 1.09, \Rightarrow \frac{x}{2} = \tan^{-1}1.09 =\{47.4\text{or } 0.828\} $ or $-0.528\Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) =\{-27.83\text{or } -0.485\} $ Alternative: Leading to a value for x $t = 1.09\Rightarrow \sin x = \frac{2(1.09)}{1 + (1.09)^2} \text{ or } \cos x = \frac{1 - (1.09)^2}{1 + (1.09)^2} $ dM1 3.4 $t = -0.528\Rightarrow \sin x = \frac{2(-0.528)}{1 + (-0.528)^2} \text{ or } \cos x = \frac{1 - (-0.528)^2}{1 + (-0.528)^2} $ $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1\Rightarrow x =$ $x = 360 + \sin^{-1}\left(\frac{2(-0.528)}{1 + (-0.528)^2}\right) $ ddM1 3.1b $x = 360 - \cos^{-1}\left(\frac{1 - (-0.528)^2}{1 + (-0.528)^2}\right) $ A1 3.2a	(b)	$S = 12 - \frac{15}{2} \left(\frac{1 - t^2}{1 + t^2} \right) - \frac{27}{10} \left(\frac{2t}{1 + t^2} \right)$	M1	1.1b
(c) $\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2 $ M1 3.4 $95t^2 - 54t - 55 = 0$ A1 1.1b $t = 1.09, \Rightarrow \frac{x}{2} = \tan^{-1}1.09 =\{47.4\text{or } 0.828\}}$ or $-0.528 \Rightarrow \frac{x}{2} = \tan^{-1}(-0.528) =\{-27.83\text{or } -0.485\}}$ Alternative: Leading to a value for x $t = 1.09 \Rightarrow \sin x = \frac{2(1.09)}{1 + (1.09)^2} \text{ or } \cos x = \frac{1 - (1.09)^2}{1 + (1.09)^2}$ or $t = -0.528 \Rightarrow \sin x = \frac{2(-0.528)}{1 + (-0.528)^2} \text{ or } \cos x = \frac{1 - (-0.528)^2}{1 + (-0.528)^2}$ $\frac{x}{2} = \tan^{-1}(-0.528) + 180 = 152.1 \Rightarrow x =$ $x = 360 + \sin^{-1}\left(\frac{2(-0.528)}{1 + (-0.528)^2}\right) $ ddM1 3.1b $x = 360 - \cos^{-1}\left(\frac{1 - (-0.528)^2}{1 + (-0.528)^2}\right)$ 304 days A1 3.2a		$=\frac{120(1+t^2)-75(1-t^2)3-54t}{10(1+t^2)}$	M1	1.1b
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304 days A1 3.2a			ddM1	3.1b
-		$x = 360 - \cos^{-1}\left(\frac{1 - \left(-0.528\right)^2}{1 + \left(-0.528\right)^2}\right)$		
(5)		304 days	A1	3.2a
			(5)	

(9 marks)

Notes

(a)

B1: Obtains the correct temperature of 4.5°C (must include units)

(b)

M1: Uses the correct t-formulae to obtain S in terms of t

M1: Correct method to obtain a common denominator,

$$A + \frac{B(1-t^2)}{1+t^2} + \frac{Ct}{1+t^2} = \frac{A(1+t^2) + B(1-t^2) + Ct}{(1+t^2)}$$

A1: Correct answer in the correct form

(c)

M1: Uses S = 10 with the model and multiplies up to obtain a quadratic equation in t

A1: Correct 3TQ

dM1: Solves their 3TQ in t and proceeds to obtain at least one value of $\frac{x}{2}$ as suggested by the

model. Alternative uses t formulae for $\sin x$ or $\cos x$ to find a value for x, degrees or radians ddM1: A fully correct strategy to find the required value of x from the negative root of the quadratic equation in t. They must be using degrees for this mark. This mark may be award even if more than one value found e.g x = 95.2, 304.3

A1: Correct number of days, must be clear that this is their answer

Question	Scheme	Marks	AOs
2	Steps are 25 minutes so $h = \frac{25}{60}$	B1	3.3
	$\left(\frac{dH}{dt}\right)_0 = \frac{-4\sqrt{2}}{3(5\times2-2^2)} (=-0.314)$	M1	3.4
	$\Rightarrow \left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_0 = \frac{H_1 - H_0}{\frac{5}{12}} \Rightarrow H_1 = \frac{5}{12} \left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_0 + 2$	M1	1.1b
	$= 1.86905 \left(= \frac{108 - 5\sqrt{2}}{54} \right)$	A1	1.1b
	$\left(\frac{dH}{dt}\right)_1 = \frac{-4\sqrt{H_1}}{3\left(5\times H_1 - H_1^2\right)} (=-0.311)$	M1	3.4
	$\Rightarrow \left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_1 = \frac{H_2 - H_1}{\frac{5}{12}} \Rightarrow y_2 = \frac{5}{12} \left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_1 + H_1 \left(=1.73926\right)$	M1	1.1b
	Awrt 1.74 m	A1	3.2a
		(7)	

(7 marks)

Notes

B1: Uses the given information to set up the correct value of h for the model

M1: Uses H = 2 in the given differential equation to find $\left(\frac{dH}{dt}\right)_0$

M1: Uses the approximation formula with their h and their $\left(\frac{dH}{dt}\right)_0$ to find H after 35 minutes

A1: Correct value for H after 35 minutes (accept the exact value or awrt 1.87)

M1: Uses their H after 35 minutes in the given differential equation to find $\left(\frac{dH}{dt}\right)_1$

M1: Uses the approximation formula with their h and their $\left(\frac{dH}{dt}\right)_1$ to find H after 1 hour

A1: Correct answer. Allow awrt 1.74 m with units

Note if they use more than two iterations, they can score a maximum of B0M1M1A0M0M0A0

If uses
$$h = \frac{1}{6}$$
 then $H_1 = \frac{54 - \sqrt{2}}{27} = 1.9476$

It may be seen in a table

n	H_n	Т	$\left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_n$	H_{n+1}
0	2	$\frac{1}{6}$	$-\frac{2\sqrt{2}}{9} = -0.314$	1.869
1	1.869	$\frac{7}{12}$	- 0.311	1.739
2		1		

Question	Scheme	Marks	AOs
3(a)	$\overline{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \ \overline{AD} = \begin{pmatrix} 1 \\ -1 \\ t - 7 \end{pmatrix}$	M1	1.1b
	$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix} = \begin{pmatrix} 2t-14-4 \\ -(t-7+4) \\ -1-2 \end{pmatrix}$	dM1	1.1b
	$\begin{pmatrix} 2t-18\\ 3-t\\ -3 \end{pmatrix} \text{ or } (2t-18)\mathbf{i} - (t-3)\mathbf{j} - \mathbf{k}$	A1	2.2a
		(3)	
(b)	$\overrightarrow{AE} = \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix} \Rightarrow V = \begin{pmatrix} 2t-18 \\ 3-t \\ -3 \end{pmatrix} \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix} = 2t^2 - 6t - 18t + 54 - 9 + 3t - 9$ Or $\begin{vmatrix} t-3 & -3 & 3 \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix}$ $= (t-3) \left[2(t-7) - 4 \right] + 3[t-7+4] + 3[-1-2]$	M1 A1	1.1b 1.1b
	$2t^{2} - 21t + 36 = 9 \Rightarrow t = \dots$ or $2t^{2} - 21t + 36 = -9 \Rightarrow t = \dots$	dM1	1.1b
	$2t^{2} - 21t + 36 = 9 \Rightarrow t = \dots$ and $2t^{2} - 21t + 36 = -9 \Rightarrow t = \dots$	M1	1.1b
	$t = 9, \frac{3}{2}$ or $t = 3, \frac{15}{2}$	A1	1.1b
	$t = 9, \frac{3}{2}$ and $t = 3, \frac{15}{2}$	A1	2.2a
		(6)	marks)

(9 marks)

Notes

(a)

M1: Attempts to find both vectors by subtraction. Allow either direction. If no method shown then two correct values is sufficient.

dM1: Uses the correct process of the vector product. May be implied by 2 correct components if working not shown.

A1: Correct vector from correct work, isw

(b)

M1: Attempts $\pm \overrightarrow{AE}$ and the scalar triple product using their vector from part (a). Alternatively uses determinant approach

A1: Correct expression in any form, may be unsimplified

dM1: Sets their triple product = 9 or –9 and attempts to solve 3TQ using a correct method

M1: Sets their triple product = 9 and -9 and attempts to solve both 3TQ's using a correct method

A1: One correct pair of values or any 2 correct values

A1: All 4 values correct

Questi on	Scheme	Mark s	AOs
4	$\frac{x-8}{x} \le \frac{7}{x(x-2)}$		
	$\frac{(x-2)(x-8)-7}{x(x-2)} \le 0$ or $x(x-8)(x-2)^2 - 7x(x-2) \le 0$ or $x^3(x-8)(x-2)^2 - 7x^3(x-2) \le 0$	M1	2.1
	$\frac{(x-9)(x-1)}{x(x-2)} \le 0 \text{ or } x(x-1)(x-2)(x-9) \le 0$ $\text{Or } x^3(x-1)(x-2)(x-9) \le 0$	dM1	1.1b
	Critical values 0 and 2	B1	1.1b
	All 4 critical values 0, 1, 2, 9	A1	1.1b
	$\{x \in \Box : 0 < x \le 9\} \cup \{x \in \Box : 2 < x \le 9\}$	ddM1 A1	2.2a 2.5
		(6)	mawka)

(6 marks)

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by $x^2(x-2)^2$ or $x^4(x-2)^2$ and gathers terms on one side

dM1: Factorises numerator or factorises into 4 factors

B1: Identifies the critical values 0 and 2, may be seen in an inequality

A1: All 4 correct critical values, must have s cored dM1

ddM1: Deduces 2 "inside" inequalities are required with critical values in ascending order as shown

A1: Exactly 2 correct intervals using correct notation minimum $0 < x \le 9 \cup 2 < x \le 9$

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \frac{8}{8t} = \frac{1}{t}$ or $y = 4\sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{x}} = \frac{1}{t}$ or $2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{1}{t}$ Or $\frac{dy}{dx} = \frac{2a}{y} = \frac{1}{t}$	B1	1.1b
	$y - 8t = \frac{1}{t}(x - 4t^{2})$ Or $y = \frac{1}{t}x + c \Rightarrow 8t = \frac{1}{t} \times 4t^{2} \Rightarrow c =\{4t\}$	M1	1.1b
	$ty - 8t^{2} = x - 4t^{2} \Rightarrow yt - x = 4t^{2} *$ Or $y = \frac{1}{t}x + 4t \Rightarrow yt - x = 4t^{2} *$	A1*	2.1
<i>a</i> >		(3)	
(b)	$y = -tx$ $yt - x = 4t^{2}, \ y = -tx \Rightarrow -t^{2}x - x = 4t^{2} \Rightarrow x = \dots$ or $yt - x = 4t^{2}, \ y = -tx \Rightarrow yt + \frac{y}{t} = 4t^{2} \Rightarrow y = \dots$	M1	2.2a 2.1
	$\left(\frac{-4t^2}{t^2+1}, \frac{4t^3}{t^2+1}\right) \text{ or seen as } x = \dots y = \dots$	A1	1.1b
(c)	$\frac{y}{x} = -t \Rightarrow x = \frac{-4\frac{y^2}{x^2}}{\frac{y^2}{x^2} + 1} \text{ or } y = \frac{-4\frac{y^3}{x^3}}{\frac{y^2}{x^2} + 1}$	(3) M1	1.1b
	$\Rightarrow y^2 = \dots$	dM1	1.1b
	$\Rightarrow y^2 = \dots$ $y^2 = \frac{-x^3}{x+4}$	A1	1.1b
		(3)	

	Alternative 1		
	$y^{2} = \left(\frac{4t^{3}}{1+t^{2}}\right)^{2} = \frac{16t^{6}}{\left(1+t^{2}\right)^{2}} \qquad x^{3} = \left(\frac{-4t^{2}}{1+t^{2}}\right)^{3} = \frac{-64t^{6}}{\left(1+t^{2}\right)^{3}}$		
	Leading to $y^2 = -\frac{1}{4}x^3(1+t^2)$ or $\frac{y^2}{x^3} = -\frac{1}{4}(1+t^2)$	M1	1.1b
	$x = -\frac{4t^2}{\left(1 + t^2\right)} \Longrightarrow t^2 = -\frac{x}{x + 4} \Longrightarrow t^2 + 1 = \frac{4}{x + 4}$		
	$y^2 = -\frac{1}{4}x^3\left(\frac{4}{x+4}\right)$	JM1	1 11
	Or	dM1	1.1b
	$\frac{y^2}{x^3} = \frac{1+t^2}{-4} = \frac{A}{x+B} = \frac{A}{\left(\frac{-4t^2}{1+t^2}\right) + B} = \frac{A(1+t^2)}{-4t^2 + B(1+t^2)}$		
_	Leading to values for A and B		
	$y^2 = \frac{-x^3}{x+4}$	A1	1.1b
	Alternative 2	(3)	
	$y = -tx \Longrightarrow y^2 = t^2 x^2$	M1	1.1b
	$x = -\frac{4t^2}{t^2 + 1} \Longrightarrow xt^2 + x = -4t^2 \Longrightarrow t^2 = -\frac{x}{4 + x}$		
	$y^2 = \left(-\frac{x}{4+x}\right)x^2$	dM1	1.1b
	$y^2 = \frac{-x^3}{x+4}$	A1	1.1b
_	Alternative 3	(3)	
	$y^{2} = \left(\frac{4t^{3}}{1+t^{2}}\right)^{2} = \frac{16t^{6}}{\left(1+t^{2}\right)^{2}} = \frac{A\left(\frac{-4t^{2}}{1+t^{2}}\right)^{3}}{\left(\frac{-4t}{1+t^{2}}\right)+B}$	M1	1.1b
	$\frac{A(-64t^{6})}{(1+t^{2})^{3}} \div \frac{-4t^{2} + B(1+t^{2})}{(1+t^{2})} = \frac{A(-64t^{6})}{(1+t^{2})^{2}(-4t^{2} + B + Bt^{2})}$ Leading to $B = 4$		
	Detaining to D - T	<u> </u>	

$\frac{A(-64t^6)}{4(1+t^2)^2} = \frac{16t^6}{(1+t^2)^2} \Rightarrow A =\{-1\}$ Correct values for A and B and draws the conclusion that therefore	dM1	1.1b
Correct values for A and B and draws the conclusion that therefore $y^2 = \frac{-x^3}{x+4}$	A1	1.1b
	(3)	

(9 marks)

Notes

(a)

B1: Obtains the correct tangent gradient

M1: Correct strategy for the equation of the tangent

A1*: Correct proof with no errors

(b)

M1: Deduces the correct equation of l

M1: Solves simultaneously their equation of the normal (using a changed gradient) and the answer to (a) to find x or y

A1: Correct simplified coordinates

(b)

M1: Eliminates t to obtain a Cartesian equation

dM1: Rearranges to obtain the form $y^2 = f(x)$

A1: Correct equation

Alternative 1

M1: Finds their y^2 and their x^3 substitutes into the given equation and simplifies

dM1: Uses their x to find an expression in terms of x for $1+t^2$ or compares the simplified $\frac{y^2}{x^3}$

with $\frac{A}{x+B}$ uses correct algebra to find as a single fraction and find values for A and B

A1: Achieves the correct equation

Alternative 2

M1: Finds y^2 and uses the x coordinates to find t^2 in terms of x

dM1: Substitutes their t^2 into y^2 to find in terms of x only

Alternative 3

M1: Substitutes their y^2 and their x into given equation. Uses correct algebra to simply to a single fraction to deduce a value for B.

dM1: Uses their value of B and equates to find a value for A

A1: Correct value for A and B and draws the conclusion therefore $y^2 = \frac{-x^3}{x+4}$

Note: There may be other methods, please send to review if you are unsure