A Level Physics CIE

25. Astronomy & Cosmology

CONTENTS

25.1 Astronomy

25.1.1 Luminosity & Radiant Flux

Defining Luminosity

* Luminosity L is defined as:

   The total power output of radiation emitted by a star

* It is measured in units of **Watts (W)**
* Radiant flux intensity F is defined as:

   The observed amount of intensity, or the radiant power transmitted normally through a surface per unit of area, of radiation measured on Earth

* The best way to picture this is:
   ○ The luminosity is the total radiation that **leaves** the star
   ○ The radiant flux intensity is the amount of radiation **measured on Earth**
   ○ By the time the radiation reaches the Earth, it will have spread out a great deal, therefore, it will only be a fraction of the value of the luminosity
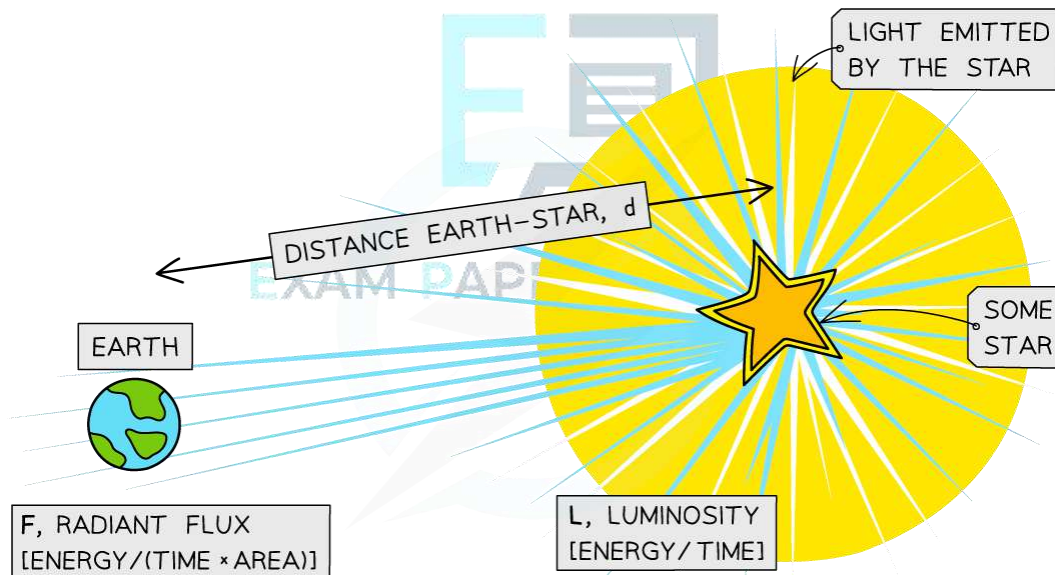

The luminosity is the total power output of the star, whereas the radiant flux is what is measured on Earth

Inverse Square Law of Flux

* Light sources which are further away appear fainter because the light it emits is spread out over a greater area
* The moment the light leaves the surface of the star, it begins to spread out uniformly through a spherical shell
    ○ The surface area of a sphere is equal to **4πr²**

* The radius r of this sphere is equal to the distance d between the star and the Earth
* By the time the radiation reaches the Earth, it has been spread over an area of $4\pi d^2$
* The inverse square law of flux can therefore be calculated using:

$$F = \frac{L}{4\pi d^2}$$

* Where:
    ○ F = radiant flux intensity, or observed intensity on Earth (W m^{-2})
    ○ L = luminosity of the source (W)
    ○ d = distance between the star and the Earth (m)

* This equation assumes:
    ○ The power from the star radiates uniformly through space
    ○ No radiation is absorbed between the star and the Earth

* This equation tells us:
    ○ For a given star, the luminosity is constant
    ○ The radiant flux follows an inverse square law
    ○ The greater the radiant flux (larger F) measured, the closer the star is to the Earth (smaller d)
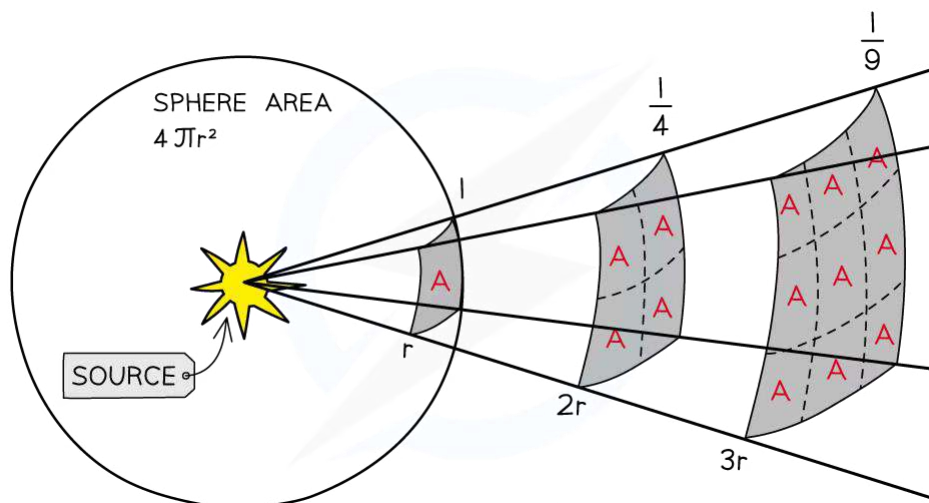
Inverse square law; when the light is twice as far away, it has spread over four times the area, hence the intensity is four times smaller

? Worked Example

A star has a luminosity that is known to be 4.8×10^{29} W. A scientist observing this star finds that the radiant flux intensity of light received on Earth from the star is 2.6 nW m^{-2}Determine the distance of the star from Earth.

Step 1: Write down the known quantities

$$\text{Luminosity, } L = 4.8 \times 10^{29} \text{ W}$$

$$\text{Radiant flux intensity, } F = 2.6 \text{ nW m}^{-2} = 2.6 \times 10^{-9} \text{ W m}^{-2}$$

Step 2: Write down the inverse square law of flux

$$F = \frac{L}{4\pi d^2}$$

Step 3: Rearrange for distance d, and calculate

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{4.8 \times 10^{29}}{4\pi \times (2.6 \times 10^{-9})}} = 3.8 \times 10^{18} \text{ m}$$

Standard Candles

- A standard candle is defined as:

   An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object

- Examples of standard candles are:
- Cepheid variable stars
   - A type of pulsating star which increases and decreases in brightness over a set time period
   - This variation has a well defined relationship to the luminosity

- Type 1a supernovae
   - A supernova explosion involving a white dwarf
   - The luminosity at the time of the explosion is always the same
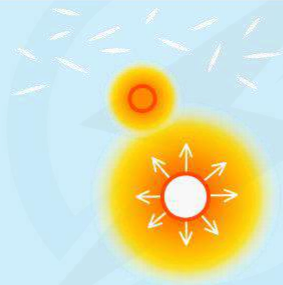
Using Standard Candles as a Distance Indicator

- Measuring astronomical distances accurately is an extremely difficult task
- A direct distance measurement is only possible if the object is close enough to the Earth
- For more distant objects, indirect methods must be used – this is where standard candles come in useful
- If the luminosity of a source is known, then the distance can be estimated based on how bright it appears from Earth
    - Astronomers measure the radiant flux intensity, of the electromagnetic radiation arriving at the Earth
    - Since the luminosity is known (as the object is a standard candle), the distance can be calculated using the inverse square law of flux

- Each standard candle method can measure distances within a certain range
- Collating the data and measurements from each method allows astronomers to build up a larger picture of the scale of the universe
    - This is known as the **cosmic distance ladder**

HUBBLE'S LAW: $d = \dfrac{V}{H_0}$

GALAXY CLUSTERS
$(10^{10}$ ly$)$

TYPE 1a
SUPERNOVAE

TULLY–FISHER
RELATION

NEARBY GALAXIES
$(10^7$ ly$)$

LUMINOSITY

PERIOD

CEPHEIDS

MILKY WAY
$(10^5$ ly$)$

RELATIVE APPARENT BRIGHTNESS

HYADES

PLEIADES

SURFACE TEMPERATURE (KELVIN)
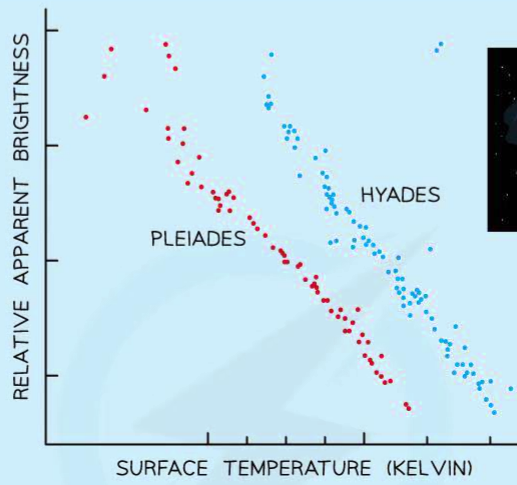
MAIN–SEQUENCE
FITTING

NEARBY STARS
(10^2 ly)

PARALLAX

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SUN

RADAR RANGING

VENUS
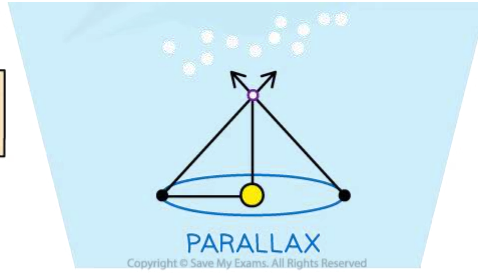
SOLAR SYSTEM
(10^{-4} ly)

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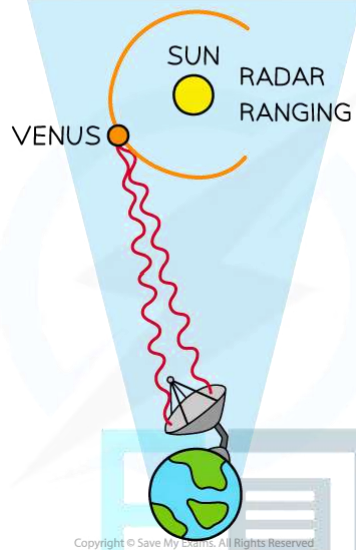
A combination of methods involving standard candles allows astronomers to build up a cosmic distance ladder from nearby stars to distant galaxies

Wien's Displacement Law

* Wien's displacement law relates the observed wavelength of light from a star to its surface temperature, it states:

   The black body radiation curve for different temperatures peaks at a wavelength which is inversely proportional to the temperature

* This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

* λ_{max} is the maximum wavelength emitted by the star at the peak intensity
* A black-body is an object which:
    ○ Absorbs all the radiation that falls on it, and is also a good emitter
    ○ Does not reflect or transmit any radiation

* A black-body is a theoretical object, however, stars are the best approximation there is
* The radiation emitted from a black-body has a characteristic spectrum that is determined by the temperature alone
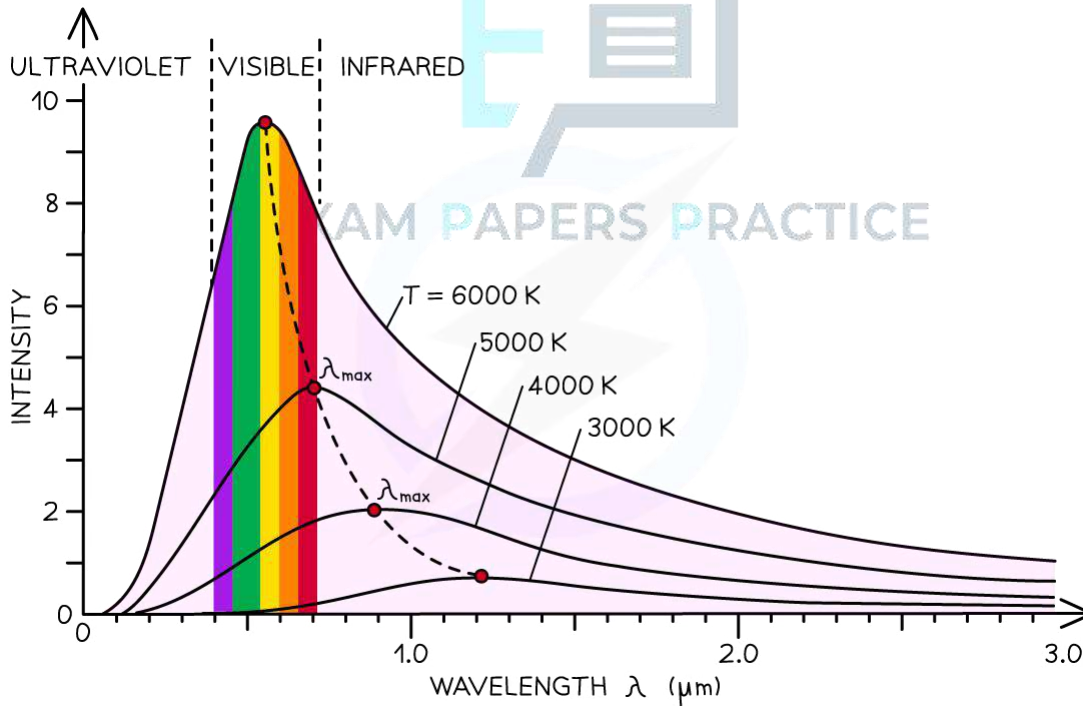
The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different stars

* The full equation for Wien's Law is given by

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

* Where:
  - λ_{max} = peak wavelength of the star (m)
  - T = thermodynamic temperature at the surface of the star (K)

* This equation tells us the higher the temperature of a body:
  - The shorter the wavelength at the peak intensity, so **hotter** stars tend to be **white or blue** and **cooler** stars tend to be **red or yellow**
  - The greater the intensity of the radiation at each wavelength

Table to compare surface temperature and star colour

Colour of star	Temperature / K
blue	> 33 000
blue−white	10 000 − 30 000
white	7 500 − 10 000
yellow−white	6 000 − 7 500
yellow	5 000 − 6 000
orange	3 500 − 5 000
red	< 3 500

? Worked Example

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm.Which of these two stars is cooler, Betelgeuse or Rigel?

Step 1: Write down Wien's displacement law

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

Step 2: Rearrange for temperature T

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}}$$

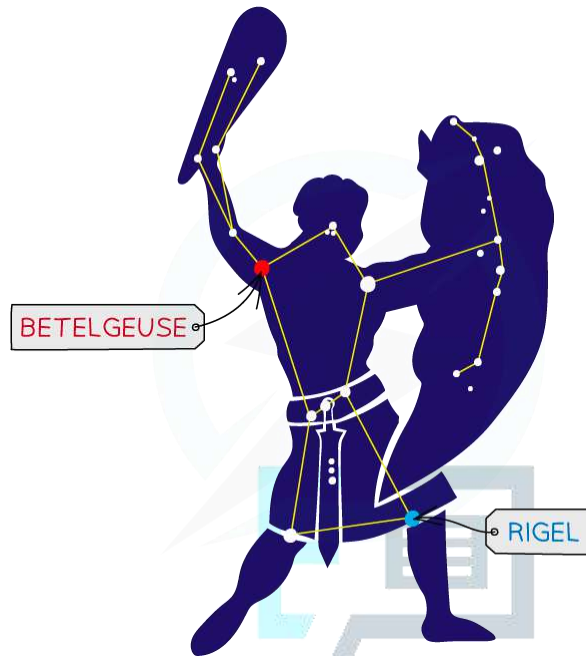**Step 3:** Calculate the surface temperature of each star

$$\text{Rigel:} \quad T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11026 = \textbf{11 000 K}$$

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Betelguese: $T = \dfrac{2.9 \times 10^{-3}}{\lambda_{max}} = \dfrac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = \mathbf{3\ 500\ K}$

Step 4: Write a concluding sentence

Betelgeuse has a surface temperature of 3500 K, therefore, it is much cooler than Rigel

The Orion Constellation; cooler stars, such as Betelguese, appear red or yellow, while hotter stars, such as Rigel, appear white or blue

> ### ♀ Exam Tip
>
> Note that the temperature used in Wien's Law is in **Kelvin** (K). Remember to convert from °C if the temperature is given in degrees in the question before using the Wien's Law equation.

Stefan-Boltzmann Law

* A star's luminosity depends on two factors:
    ◦ Its surface temperature
    ◦ Its radius

* The relationship between these is known as the **Stefan–Boltzmann Law**, which states:

   The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body

* It is equal to:

$$L = 4\pi r^2 \sigma T^4$$

* Where:
    ◦ L = luminosity of the star (W)
    ◦ r = radius of the star (m)
    ◦ σ = the Stefan-Boltzmann constant
    ◦ T = surface temperature of the star (K)

Estimating the Radius of Stars

- The radius of a star can be estimated by combining Wien's displacement law and the Stefan–Boltzmann law
- The procedure for this is as follows:
  ○ Using Wien's displacement law to find the surface temperature of the star
  ○ Using the inverse square law of flux equation to find the luminosity of the star (if given the radiant flux and stellar distance)
  ○ Then, using the Stefan–Boltzmann law, the stellar radius can be obtained

SUMMARY OF EQUATIONS

RADIANT FLUX INTENSITY

INVERSE SQUARE LAW OF FLUX

$$F = \frac{L}{4\pi d^2}$$

DISTANCE BETWEEN THE STAR AND EARTH

PEAK WAVELENGTH

WIEN'S DISPLACEMENT LAW

$$\lambda_{max} \propto \frac{1}{T}$$

SURFACE TEMPERATURE

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

STELLAR RADIUS

STEFAN–BOLTZMANN LAW

$$\boxed{}\, \sigma T^4$$
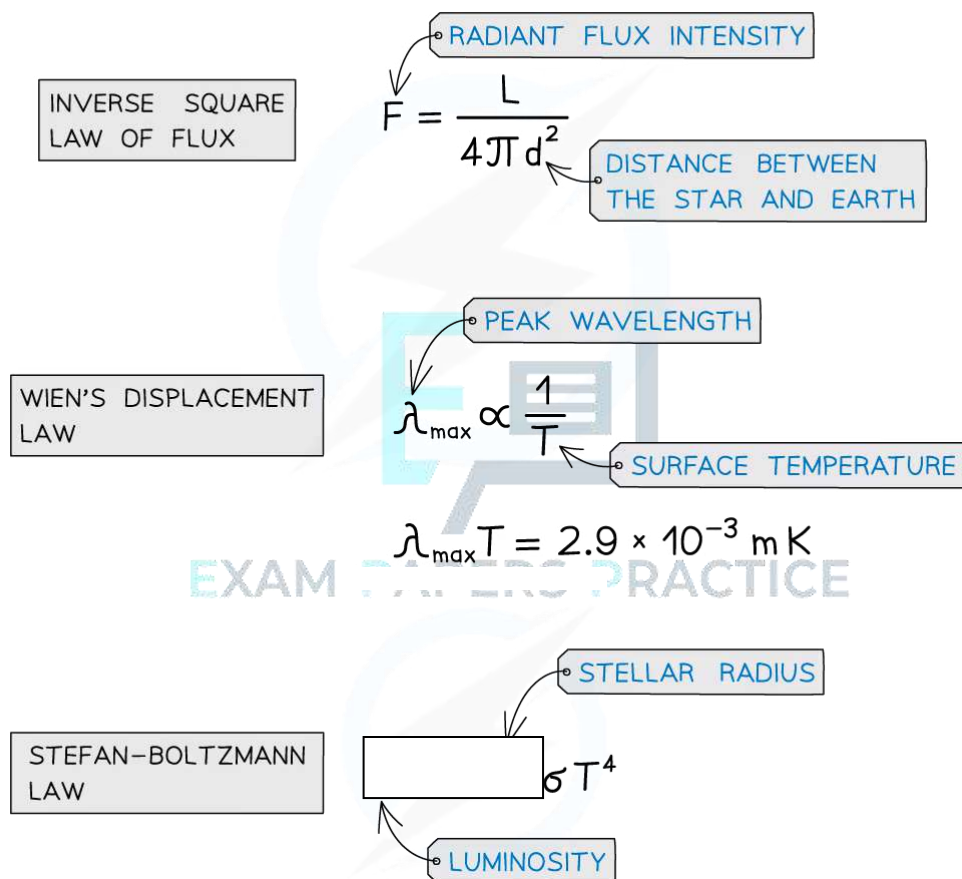
LUMINOSITY

? Worked Example

Betelguese is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm. Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_s. Radius of the sun r_s = 6.95×10^8 m.

Step 1: Write down Wien's displacement law

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

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Step 2: Rearrange Wien's displacement law to find the surface temperature of Betelguese

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{850 \times 10^{-9}} = 3410 \text{ K}$$

Step 3: Write down the Stefan–Boltzmann law

$$L = 4\pi r^2 \sigma T^4$$

Step 4: Rearrange for r and calculate the stellar radius of Betelguese

$$r_B = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{(4.49 \times 10^{31})}{4\pi \times (5.67 \times 10^{-8}) \times (3410)^4}} = 6.83 \times 10^{11} \text{ m}$$

Step 5: Calculate the ratio r_B / r_s

$$\frac{r_B}{r_s} = \frac{6.83 \times 10^{11}}{6.95 \times 10^8} = 983$$

**Therefore, the radius of Betelguese is about 1000 times larger than the Sun's radius**

Emission Spectra

* Astronomers are very limited in how they can investigate objects in the space
* All of the techniques used involve analysing the light emitted from the star, or galaxy
* One of these techniques involves analysing the **emission** and **absorption spectra** of stars
    ○ More details on this can be found in the revision notes "Line Spectra" in the Quantisation of Energy topic

* Elements in the star, predominantly hydrogen and helium, absorb some of the emitted wavelengths
* Therefore, characteristic lines are present when the spectrum is analysed
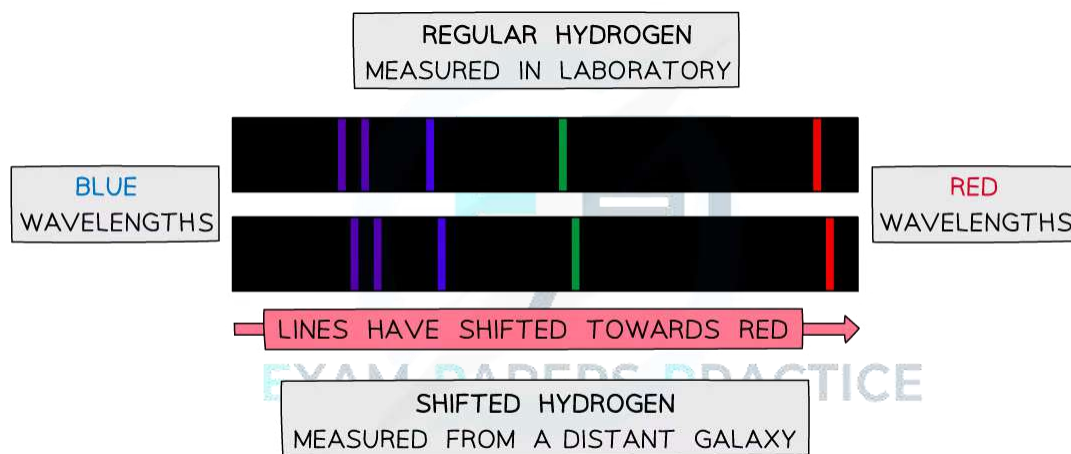
The top emission spectra shows spectral lines of hydrogen measured on Earth

The bottom emission spectra shows the shifted spectral lines of hydrogen measured from a distant galaxy

* When astronomers observe light from distant galaxies, they observe differences in the spectral lines to the light from the Sun
* The lines have the same characteristic pattern, meaning the element can still be easily identified, they just appear to be shifted sightly
    ○ The lines in the spectra from distant galaxies show an **increase in wavelength**
    ○ The lines are moved, or shifted, towards the **red end of the spectrum**

Redshift of EM Radiation

* Recall the Doppler effect is defined as:

  The apparent change in wavelength or frequency of the radiation from a source due to its relative motion away from or toward the observer

* On Earth, the Doppler effect of sound can be easily observed when sound waves moves past an observer at a notable speed
* In space, the Doppler effect of light can observed when spectra of distant stars and galaxies are observed, this is known as:
  * **Redshift** if the object is moving away from the Earth, or
  * **Blueshift** if the object is moving towards the Earth

* Redshift is defined as:

  The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other

* For non-relativistic galaxies, Doppler redshift can be calculated using:

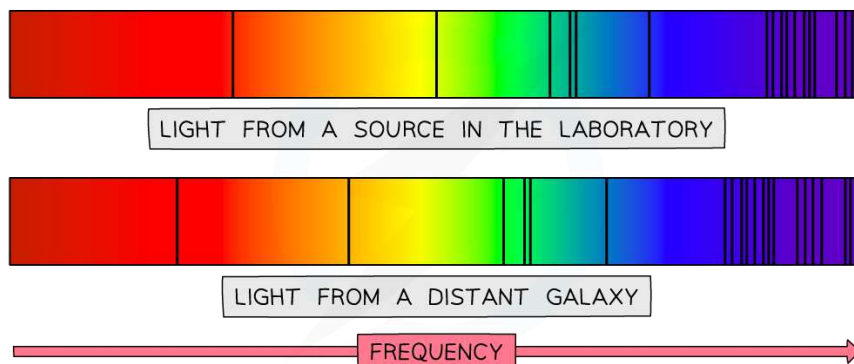$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$$

* Where:
  * $\Delta\lambda$ = shift in wavelength (m)
  * λ = wavelength emitted from the source (m)
  * Δf = shift in frequency (Hz)
  * f = frequency emitted from the source (Hz)
  * v = speed of recession (m s^{-1})
  * c = speed of light in a vacuum (m s^{-1})

? Worked Example

The spectra below show dark absorption lines against a continuous visible spectrum.

LIGHT FROM A SOURCE IN THE LABORATORY

LIGHT FROM A DISTANT GALAXY

FREQUENCY

A particle line in the spectrum of light from a source in the laboratory has a frequency of 4.570×10^{14} Hz. The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz. What speed is the distance galaxy moving in relation to the Earth? Is it moving towards or away from the Earth?

Step 1: Write down the known quantities

Emitted frequency, f = **4.570×10^{14} Hz**

Shift in frequency, Δf = **$(4.547 - 4.570) \times 10^{14} = -2.3 \times 10^{12}$ Hz**

Speed of light, c = **3.0×10^8 m s^{-1}**

Step 2: Write down the Doppler redshift equation

$$\frac{\Delta f}{f} = \frac{v}{c}$$

Step 3: Rearrange for speed v, and calculate

$$v = \frac{c\Delta f}{f} = \frac{(3.0 \times 10^8) \times (2.3 \times 10^{12})}{4.570 \times 10^{14}} = \textbf{1.5} \times \textbf{10}^\textbf{6} \textbf{ m s}^{-1}$$

Step 4: Write a concluding sentence

The observed frequency is **less** than the emitted frequency (the light from a laboratory source), therefore, the source is **receding**, or moving away, from the Earth at **1.5×10^6 m s^{-1}**

An Expanding Universe

* After the discovery of Doppler redshift, astronomers began to realise that almost all the galaxies in the universe are receding
* This lead to the idea that the space between the Earth and the galaxies must be **expanding**
* This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum
* The more red-shifted the light from a galaxy is, the **faster** the galaxy is moving away from Earth
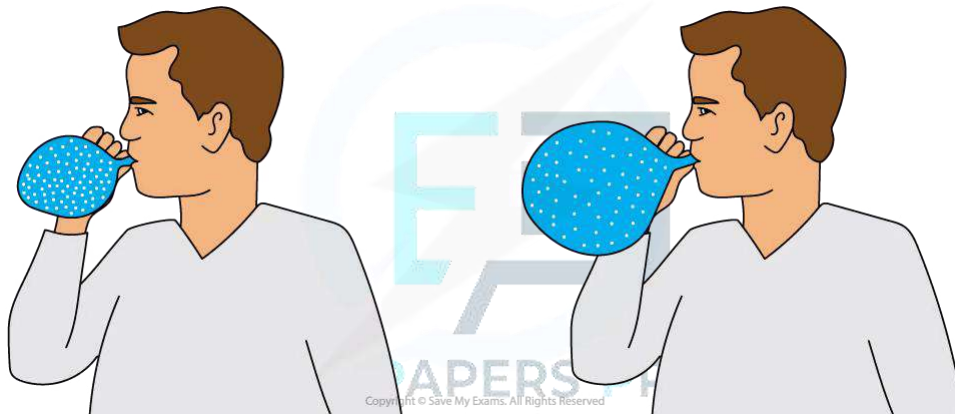
A balloon inflating is similar to the stretching of the space between galaxies

* The expansion of the universe can be compared to dots on an inflating balloon
    ◦ As the balloon is inflated, the dots all move away from each other
    ◦ In the same way as the rubber stretches when the balloon is inflated, space itself is **stretching out between galaxies**
    ◦ Just like the dots, the galaxies move away from each other, however, they themselves do not move

Hubble's Law & the Big Bang Theory

* Edwin Hubble investigated the light spectra emitted from a large number of galaxies
* He used redshift data to determine the recession velocities of these galaxies, and standard candles to determine the distances
* From these measurements, he formulated a relationship, now known as **Hubble's Law**
* Hubble's Law states:

   The recession speed of galaxies moving away from Earth is proportional to their distance from the Earth

* This can be calculated using:

$$v = H_0 d$$

* Where:
   ◦ v = the galaxy's recessional velocity (m s^{-1})
   ◦ d = distance between the galaxy and Earth (m)
   ◦ H_0 = Hubble's constant, or the rate of expansion of the universe (s^{-1})

* This equation tells us:
   ◦ The **further away** a galaxy, the **faster** it's recession velocity
   ◦ The gradient of a graph of recession velocity against distance is equal to the **Hubble constant**
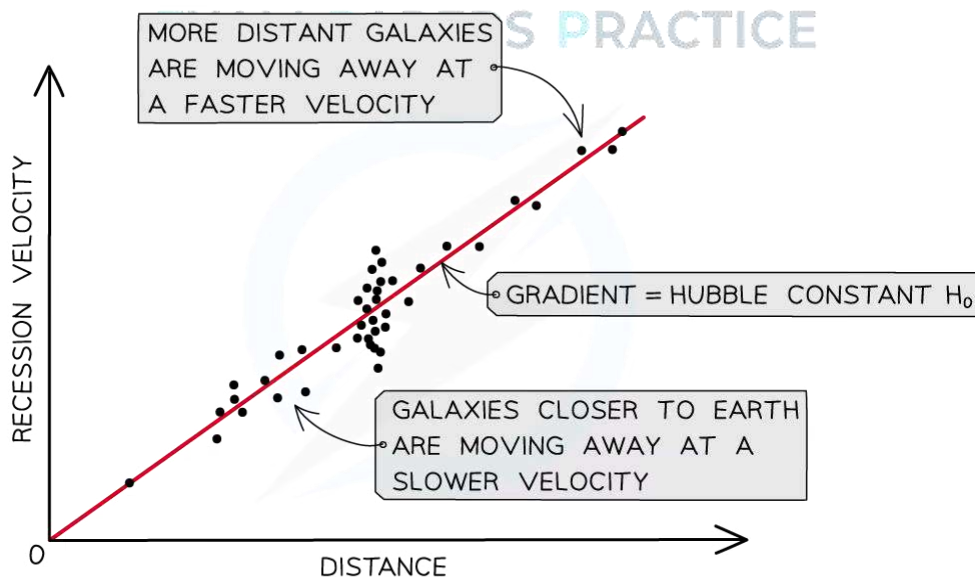

A key aspect of Hubble's law is that the furthest galaxies appear to move away the fastest

Age of the Universe

* If the galaxies are moving away from each other, then they must've started from the same point at some time in the past
* If this is true, the universe likely began in an extremely hot, dense singular point which subsequently began to expand very quickly
  ○ This idea is known as the **Big Bang theory**

* Redshift of galaxies and the expansion of the universe are now some of the most prominent pieces of evidence to suggest this theory is true
* The data from Hubble's law can be extrapolated back to the point that the universe started expanding ie. the beginning of the universe
* Therefore, the age of the universe T_0 is equal to:

$$T_0 = \frac{1}{H_0}$$

* Current estimates of the age of the universe range from **13 – 14 billion years**
* There is still some discussion about the exact age of the universe, therefore, obtaining accurate measurements for the Hubble constant is a top priority for cosmologists
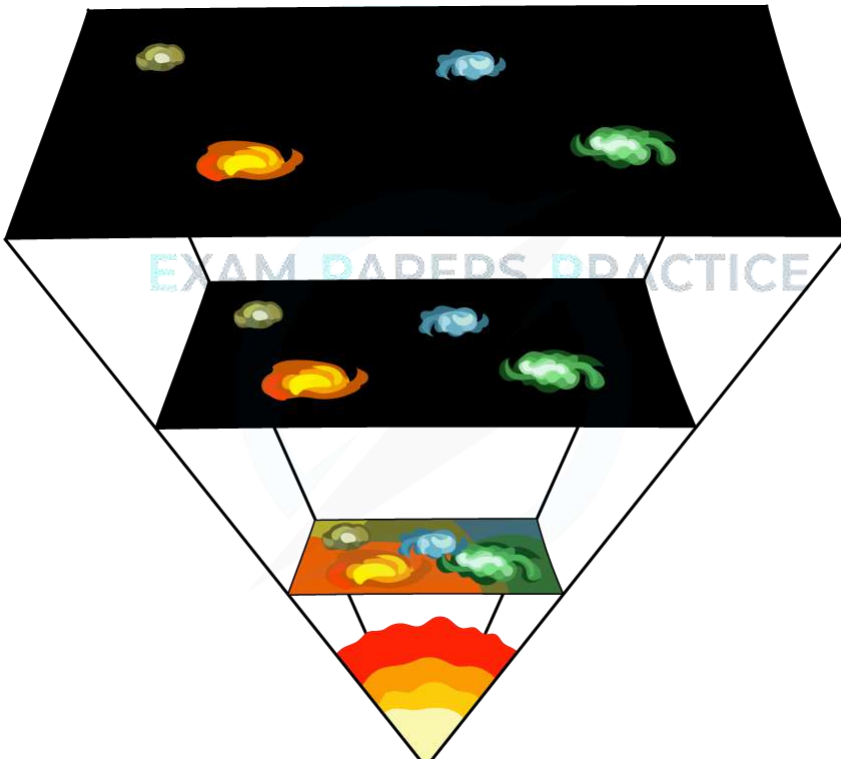
Tracing the expansion of the universe back to the beginning of time leads to the idea the universe began with a "big bang"

? Worked Example

A galaxy is found to be moving away with a speed of 2.1×10^7 m s^{-1}. The galaxy is at a distance of 9.5×10^{24} m from Earth. Assuming the speed has remained constant, what is the age of the universe in years?

Step 1: Write down Hubble's Law

$$v = H_0 d$$

Step 2: Rearrange for the Hubble constant H_0, and calculate

$$H_0 = \frac{v}{d} = \frac{2.1 \times 10^7}{9.5 \times 10^{24}} = 2.2 \times 10^{-18} \text{ s}^{-1}$$

Step 3: Write the equation for the age of the universe T_0, and calculate

$$T_0 = \frac{1}{H_0} = \frac{1}{2.2 \times 10^{-18}} = 4.52 \times 10^{17} \text{ s}$$

Step 4: Convert from seconds into years

$$T_0 = \frac{4.52 \times 10^{17}}{(365 \times 24 \times 60 \times 60)} = 1.43 \times 10^{10} \text{ years}$$

Therefore, the age of the universe is estimated to be about **14.3 billion years**