A Level Physics CIE

24. Medical Physics

CONTENTS

EXAM PAPERS PRACTICE

24.1 Ultrasound & X-rays

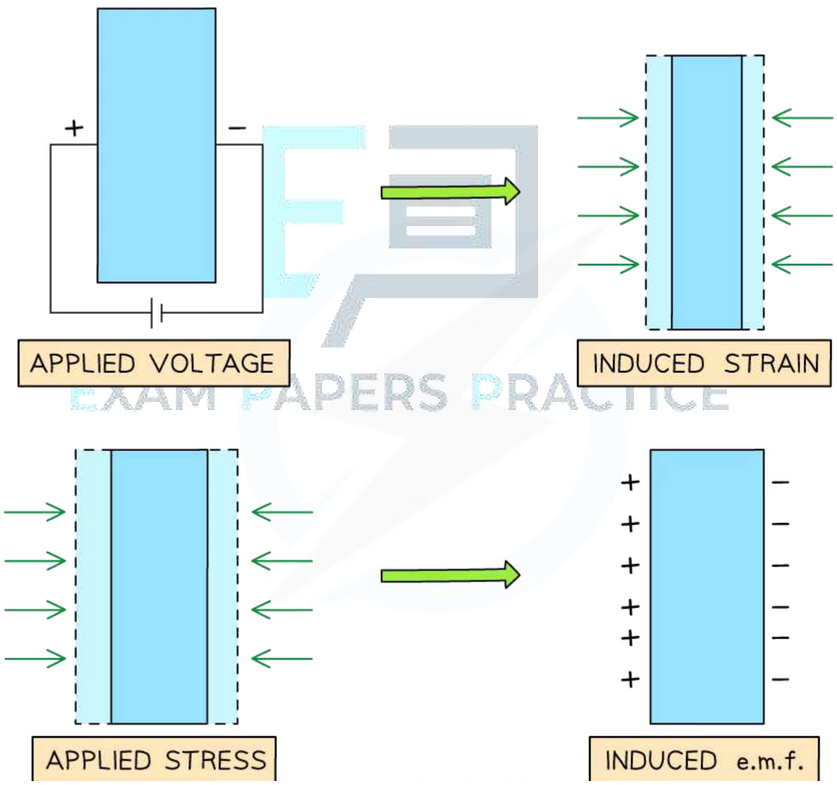
24.1.1 The Piezoelectric Transducer

The Piezoelectric Transducer

- The piezoelectric effect is defined as:

    The ability of particular materials to generate a potential difference (p.d.) by transferring mechanical energy to electrical energy

- A **transducer** is any device that converts energy from one form to another

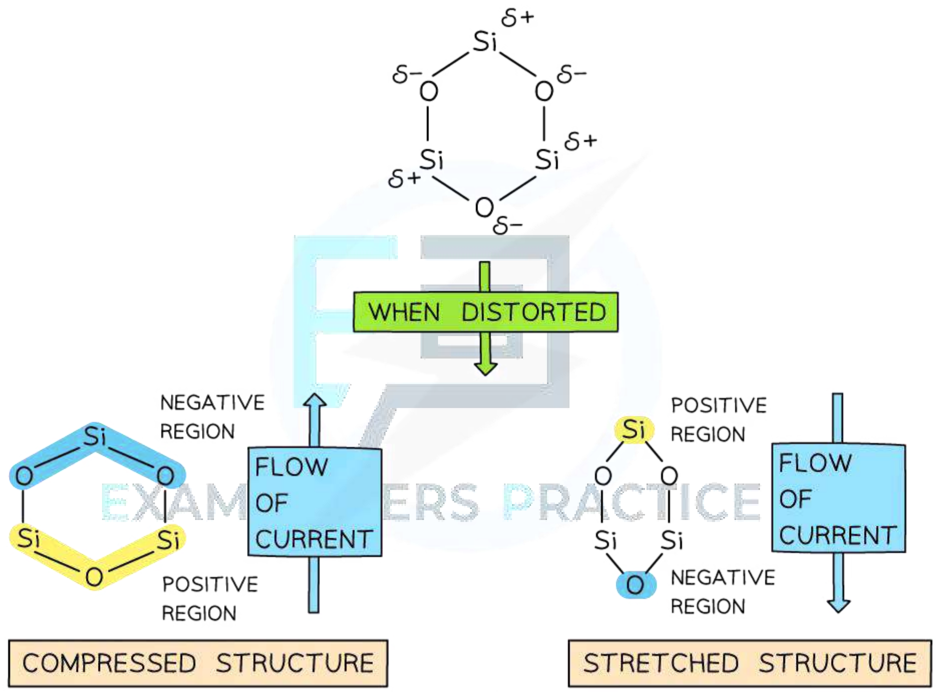


In the piezoelectric effect, an applied voltage causes a piezo-crystal to contract or expand, and vice versa

Piezoelectric Crystals

- At the heart of a piezoelectric transducer is a **piezoelectric crystal**
- Piezoelectric crystals are materials which produce a p.d. when they are deformed
    - This deformation can be by compression or stretching
- If a p.d. is applied to a piezoelectric crystal, then it deforms, and if the p.d. is reversed, then it expands
    - If this is an alternating p.d. then the crystal will vibrate at the same frequency as the alternating voltage
    - Crystals must be cut to a certain size in order to induce resonance

- One of the most common piezoelectric crystals is **quartz**, which is made from a lattice of silicon dioxide atoms
  - When the lattice is distorted, the structure becomes charged creating an electric field and, as a result, an electric current
  - If an electric current is applied to the crystal, then this causes the shape of the lattice to alternate which produces a sound wave
  - Due to the conventional direction of electric current, it will flow from the positive to the negative region of the crystal



A molecule in a quartz crystal. When the compression and stretching alternates, an alternating e.m.f. is induced

Applications of the Piezoelectric Transducer

- Microphone
  - A piezoelectric microphone detects pressure variations in sound waves
  - These can then be converted to an electrical signal for processing

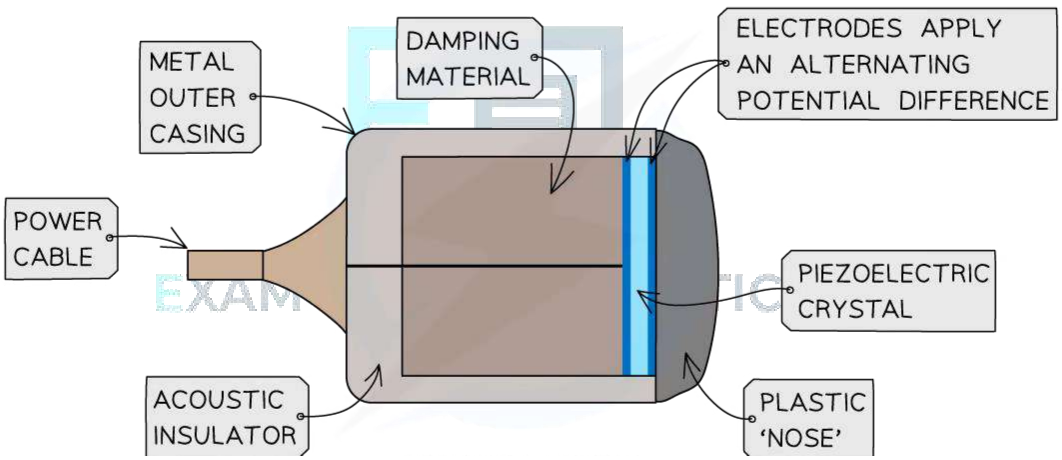
- Ultrasound
  - In a piezoelectric transducer, an alternating p.d. is applied to produce ultrasound waves and sent into the patient's body
  - The returning ultrasound waves induce a p.d. in the transducer for analysis by a healthcare professional

Generating Ultrasound

- An ultrasound is defined as:

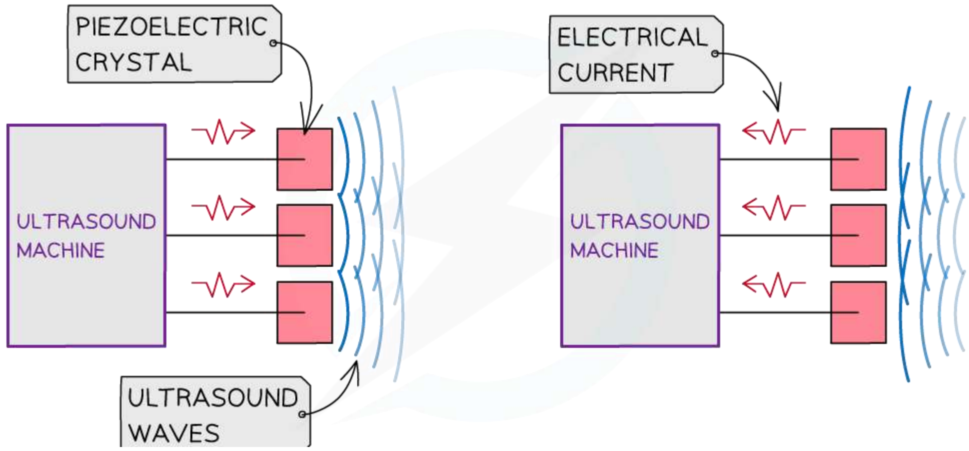
  A high frequency sound above the range of human hearing

- This is above 20 kHz, although in medical applications the frequencies can be up to the MHz range
- An ultrasound transducer is made up of a piezoelectric crystal and electrodes which produce an alternating p.d.
- The crystal is heavily damped, usually with epoxy resin, to stop the crystal from vibrating too much
  - This produces short pulses and increases the resolution of the ultrasound device



The structure of an ultrasound transducer

- A piezoelectric crystal can act as both a receiver or transmitter of ultrasound
  - When it is **receiving** ultrasound, it converts the sound waves into an alternating p.d.
  - When it is **transmitting** ultrasound, it converts an alternating p.d. into sound waves



A piezoelectric crystal can act as both a receiver or transmitter of ultrasound

Worked Example

Explain the principles of the generation and detection of ultrasound waves.

Generation:

- An alternating p.d. is applied across a piezo-electric crystal, causing it to change shape
- The alternating p.d. causes the crystal to vibrate and produce ultrasound waves
- The crystal vibrates at the frequency of the alternating p.d., so, the crystal must be cut to a specific size in order to produce resonance

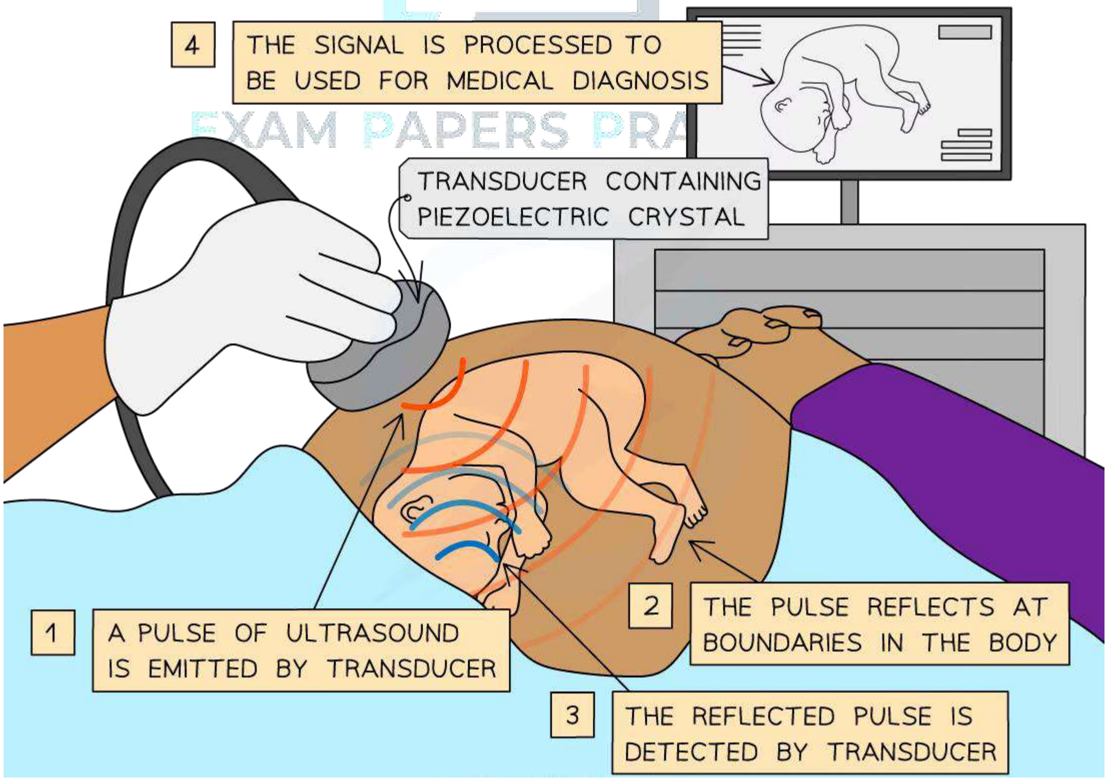
Detection:

- When the ultrasound wave returns, the crystal vibrates which produces an alternating p.d. across the crystal
- This received signal can then be processed and used for medical diagnosis

Using Ultrasound in Medical Imaging

- In an ultrasound scanner, the transducer sends out a beam of sound waves into the body
- The sound waves are reflected back to the transducer by boundaries between tissues in the path of the beam
    - For example, the boundary between fluid and soft tissue or tissue and bone
- When these echoes hit the transducer, they generate electrical signals that are sent to the ultrasound scanner
- Using the speed of sound and the time of each echo's return, the scanner calculates the distance from the transducer to the tissue boundary
- These distances can be used to generate two-dimensional images of tissues and organs

- The frequency of the ultrasound is important because:
    - The higher the frequency of the ultrasound, the higher the resolution and the smaller structures that can be distinguished



Using ultrasound to obtain information about an unborn child

- The ultrasound gives two main pieces of information about the boundary:
    - **Depth**: the time between transmission and receipt of the pulse (the time delay)
    - **Nature**: amount of transmitted intensity received (will vary depending on the type of tissue)

Worked Example

Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal body structures.

ANSWER:

- A pulse of ultrasound is emitted by the piezo-electric crystal
- This is reflected from the boundaries between media
- The reflected pulse is detected by the ultrasound transmitter
- The signal is then processed and displayed on the screen for the healthcare worker to analyse and use for medical diagnosis
- The **intensity of the reflection** gives information about the nature of the boundary
- The time between transmission and receipt of the pulse (**the time delay**) gives information about the depth of the boundary

Exam Tip

6 mark exam questions about this topic are very common, make sure you practice writing about using and detecting ultrasounds in full, coherent sentences with correct spelling and grammar. Writing short or vague answers could lose you marks, as well as misspelling words!

Specific Acoustic Impedance

- The **acoustic impedance**, Z, of a medium is defined as:

  The product of the speed of the ultrasound in the medium and the density of the medium

- This quantity describes how much resistance an ultrasound beam encounters as it passes through a tissue
- Acoustic impedance can be calculated using the equation:

$$Z = \rho c$$

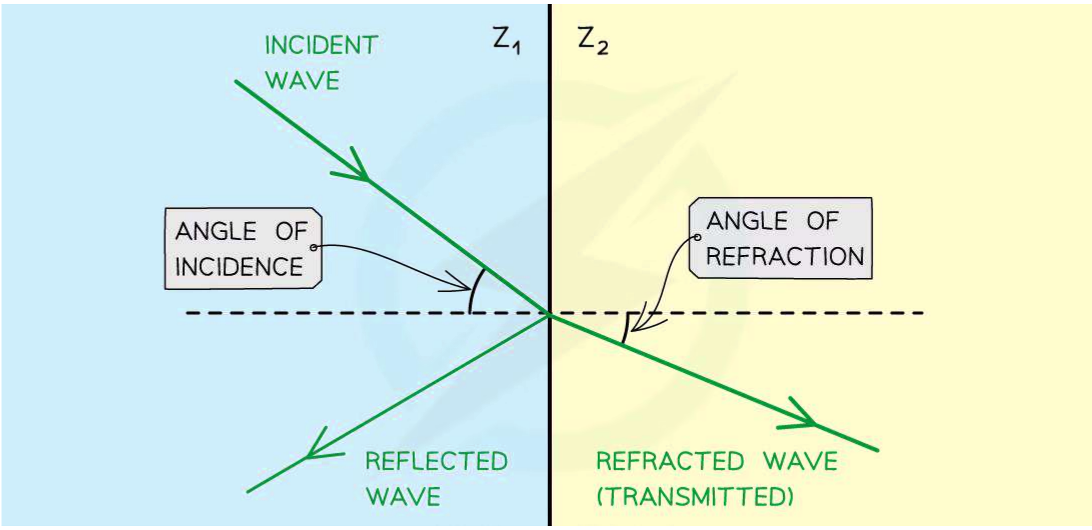
- Where:
  - Z = acoustic impedance (kg m^{-2} s^{-1})
  - ρ = the density of the material (kg m^{-3})
  - c = the speed of sound in the material (m s^{-1})

- This equation tells us:
  - The higher the density of a tissue, the greater the acoustic impedance
  - The faster the ultrasound travels through the material, the greater the acoustic impedance also

- This is because sound travels faster in denser materials
  - Sound is fastest in solids and slowest in gases
  - The closer the particles in the material, the faster the vibrations can move through the material

- At the boundary between media of different acoustic impedances, some of the wave energy is **reflected** and some is **transmitted**
- The greater the **difference** in acoustic impedance between the two media, the greater the reflection and the smaller the transmission
  - Two materials with the same acoustic impedance would give no reflection
  - Two materials with a large difference in values would give much larger reflections

- Air has an acoustic impedance of Z_{air} = 400 kg m^{-2} s^{-1}
- Skin has an acoustic impedance of Z_{skin} = 1.7×10^6 kg m^{-2} s^{-1}
  - The large difference means ultrasound would be significantly reflected, hence a coupling gel is necessary
  - The coupling gel used has a similar Z value to skin, meaning that very little ultrasound is reflected



Refraction and reflection of ultrasound waves at a boundary between two materials with different acoustic impedances (in this case, $Z_1 < Z_2$)

Worked Example

The table shows the speed of sound acoustic impedance in four different materials.Use this to calculate the value for the density of bone.

medium	speed of ultrasound / m s⁻¹	acoustic impedance / kg m⁻² s⁻¹
air	330	4.3×10^2
gel	1500	1.5×10^6
soft tissue	1600	1.6×10^6
bone	4100	7.0×10^6

Step 1: Write down known quantities

Acoustic impedance of bone, Z = $\mathbf{7.0 \times 10^6}$ **kg m^{-2} s^{-1}**

Speed of ultrasound in bone, c = **4100 m s^{-1}**

Step 2: Write out the equation for acoustic impedance

$$Z = \rho c$$

Step 3: Rearrange for density and calculate

$$\rho = \frac{Z}{c} = \frac{7.0 \times 10^6}{4100} = \mathbf{1700\ kg\ m^{-3}}$$

Exam Tip

A common mistake is to confuse the c in the acoustic impedance equation for the speed of light – don't do this!

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Intensity Reflection Coefficient

- The intensity reflection coefficient α is defined as:

 The ratio of the intensity of the reflected wave relative to the incident (transmitted) wave

- This can be calculated using the fraction:

$$\alpha = \frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

- Where:
  - α = intensity reflection coefficient
  - I_R = intensity of the reflected wave (W m^{-2})
  - I_0 = intensity of the incident wave (W m^{-2})
  - Z_1 = acoustic impedance of one material (kg m^{-2} s^{-1})
  - Z_2 = acoustic impedance of a second material (kg m^{-2} s^{-1})
- This equation will be provided on the datasheet for your exam
- This ratio shows:
  - If there is a large difference between the impedance of the two materials, then most of the energy will be reflected
  - If the impedance is the same, then there will be **no reflection**

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Coupling Medium

- When ultrasound is used in medical imaging, a coupler is needed between the transducer and the body
- The soft tissues of the body are much denser than air
- If air is present between the transducer and the body, then almost all the ultrasound energy will be reflected
- The coupling gel is placed between the transducer and the body, as skin and the coupling gel have a similar density, so little ultrasound is reflected
  - This is an example of impedance matching

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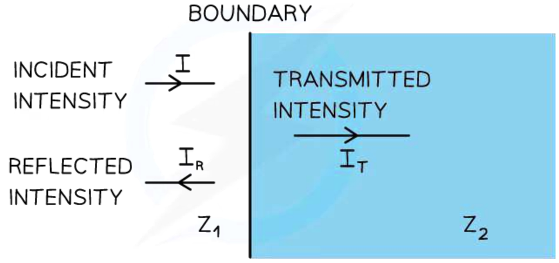
Worked Example

A beam of ultrasound is incident at right-angles to a boundary between two materials as shown in the diagram.



The materials have acoustic impedances of Z_1 and Z_2. The intensity of the transmitted ultrasound beam is I_T, and the reflected intensity is I_R.

a) What is the relationship between I, I_T and I_R?

b) Use the data from the table to determine the reflection coefficient α for a boundary between

(i) gel and soft tissue

(ii) air and soft tissue

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c) Explain why gel is usually put on the skin during medical diagnosis using ultrasound.

medium	speed of ultrasound / m s^{-1}	acoustic impedance / kg m^{-2} s^{-1}
air	330	4.3×10^2
gel	1500	1.5×10^6
soft tissue	1600	1.6×10^6
bone	4100	7.0×10^6

Part (a)

Incident intensity = Transmitted intensity + Reflected intensity

$$I = I_T + I_R$$

Part (b)(i)

Step 1: Write down the equation for intensity reflection coefficient α

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

Step 2: Write down the acoustic impedances for gel and soft tissue

Gel, Z_1 = **1.5×10^6 kg m^{-2} s^{-1}**

Soft tissue, Z_2 = **1.6×10^6 kg m^{-2} s^{-1}**

Step 3: Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.6\times10^6 - 1.5\times10^6)^2}{(1.6\times10^6 + 1.5\times10^6)^2} = \frac{(0.1)^2}{(3.1)^2} = 0.001$$

This result means that only **0.1%** of the incident intensity will be reflected, with the remaining being transmitted

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Part (b)(ii)

Step 1: Write down the acoustic impedances for air and soft tissue

Air, Z_1 = 4.3×10^2 kg m^{-2} s^{-1}

Soft tissue, Z_2 = 1.6×10^6 kg m^{-2} s^{-1}

Step 2: Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.6\times10^6 - 4.3\times10^2)^2}{(1.6\times10^6 + 4.3\times10^2)^2} \approx \frac{(1.6\times10^6)^2}{(1.6\times10^6)^2} \approx 1$$

This result means that **100%** of the incident intensity will be reflected, with none being transmitted

Part (c)

- At the air-soft tissue boundary, the intensity reflection coefficient is $\alpha \approx 1$
  - Therefore, without gel, there is almost complete reflection - no ultrasound is transmitted through the skin
- At the gel-soft tissue boundary, the intensity reflection coefficient is $\alpha = 0.001$
  - Therefore, the gel enables almost complete transmission of the ultrasound through the skin, with very little reflection

Attenuation of Ultrasound in Matter

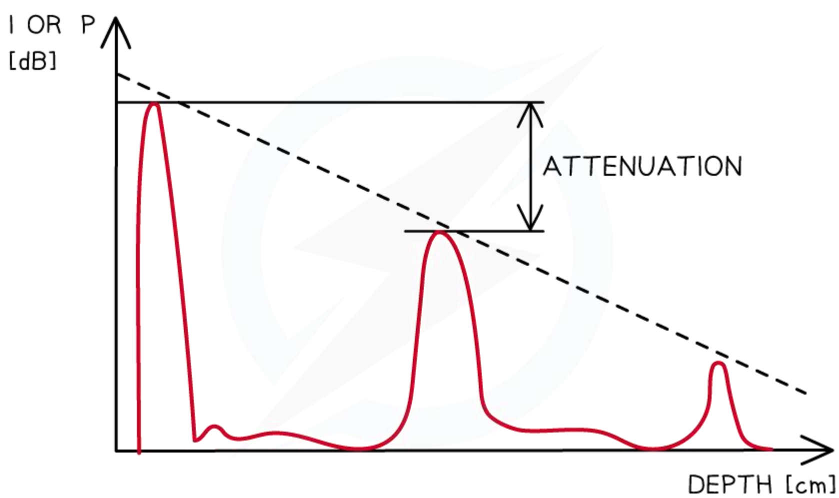
- Attenuation of ultrasound is defined as:

  The reduction of energy due to the absorption of ultrasound as it travels through a material

- The attenuation coefficient of the ultrasound is expressed in **decibels per centimetre** lost for every incremental increase in megahertz frequency
  - Generally, 0.5 dB/cm is lost for every 1MHz
- The intensity I of the ultrasound decreases with distance x, according to the equation:

$$I = I_0 e^{-\mu x}$$

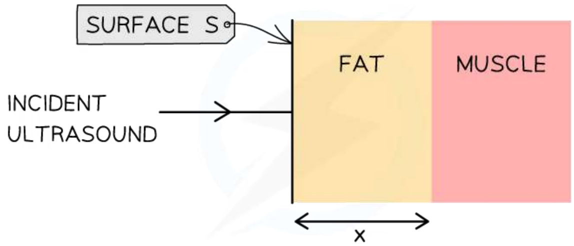
- Where:
  - I_0 = the intensity of the incident beam (W m^{-2})
  - I = the intensity of the reflected beam (W m^{-2})
  - μ = the absorption coefficient (m^{-1})
  - x = distance travelled through the material (m)
- The absorption coefficient μ, will vary from material to material
- Attenuation is not a major problem in ultrasound scanning as the scan relies on the reflection of the ultrasounds at boundaries of materials



When the intensity is expressed in decibels, the amplitudes of the echoes can be seen to decrease linearly

Worked Example

The thickness x of the layer of fat on an animal, as shown in the diagram, is to be investigated using ultrasound.



The intensity of the parallel ultrasound beam entering the surface S of the layer of fat is I.

The beam is reflected from the boundary between fat and muscle.

The intensity of the reflected ultrasound detected at the surface S of the fat is 0.012I.Using the table below, calculate:a) The intensity reflection coefficient at the boundary between the fat and the muscle.b) The thickness x of the layer of fat.

Medium	Z / kg m^{-2} s^{-1}	μ / m^{-1}
Fat	1.3×10^6	48
Muscle	1.7×10^6	23

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Part (a)

Step 1:

Write down the equation for intensity reflection coefficient α

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

Step 2:

Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.7\times10^6 - 1.3\times10^6)^2}{(1.7\times10^6 + 1.3\times10^6)^2} = \frac{(0.4)^2}{(3)^2} = 0.018$$

This means that 0.018 of the intensity is reflected at the interface between fat and muscle. This reflected intensity will move back through the fat towards surface S.

Part (b)

Step 1:

Write out the known quantities

The intensity of the ultrasound pulse is affected 3 times:

1. Attenuation from the surface S to the fat-muscle boundary
2. Reflection at the boundary
3. Attenuation from the boundary back to the surface S

After being transmitted in the fat, the intensity at surface S is given to be 0.012 I.

Therefore, the intensity is 0.018 I at the fat-muscle boundary, and as the ultrasound moves through the fat, it gets attenuated and the new intensity at the surface S is now 0.012 I

Incident intensity, equal to the intensity of the reflected pulse, $I_0 = 0.018\ I \times e^{-\mu x}$

Transmitted intensity, I = 0.012 I

Absorption coefficient, $\mu = 48\ m^{-1}$

Thickness of fat = x

Step 2:

Write out the equation for attenuation

$$I = I_0 e^{-\mu x}$$

Step 3:

Substitute in values for intensity and simplify

$$0.012\ I = [0.018\ I \times e^{-\mu x}] \times e^{-\mu x}$$

$$0.012 = 0.018\ e^{-2\mu x}$$

Step 4:

Rearrange and take the natural log of both sides

$$\frac{0.012}{0.018} = e^{-2\mu x}$$

$$\ln\left(\frac{0.012}{0.018}\right) = -2\mu x$$

Step 5:

Rearrange and calculate the thickness x

$$x = \frac{\ln\left(\frac{0.012}{0.018}\right)}{-2\mu} = \frac{\ln\left(\frac{0.012}{0.018}\right)}{-2 \times 48} = 4.22 \times 10^{-3}\ \text{m} = \mathbf{0.42\ cm}$$

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Exam Tip

The intensity equation will not be provided for you on your exam datasheet, so make sure you remember this!

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Production of X-rays

- X-rays are short wavelength, high-frequency part of the electromagnetic spectrum
  - They have wavelengths in the range 10^{-8} to 10^{-13} m
- X-rays are produced when fast-moving electrons rapidly decelerate and transfer their kinetic energy into photons of EM radiation

Producing X-rays

- At the cathode (negative terminal), the electrons are released by thermionic emission
- The electrons are accelerated towards the anode (positive terminal) at high speed
- When the electrons bombard the metal target, they lose some of their kinetic energy by transferring it to photons
- The electrons in the outer shells of the atoms (in the metal target) move into the spaces in the lower energy levels
- As they move to lower energy levels, the electrons release energy in the form of **X-ray photons**
- When an electron is accelerated, it gains energy equal to the electronvolt; this energy can be calculated using:

$$E_{max} = eV$$

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- This is the **maximum energy** that an X-ray photon can have
- Therefore, the maximum X-ray frequency f_{max}, or the minimum wavelength λ_{min}, that can be produced is calculated using the equation:

$$E_{max} = eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\text{Maximum frequency: } f_{max} = \frac{eV}{h}$$

$$\text{Minimum wavelength: } \lambda_{min} = \frac{hc}{eV}$$

- Where:
  - e = charge of an electron (C)
  - V = voltage across the anode (V)
  - h = Planck's constant (J s)
  - c = speed of light (m s^{-1})

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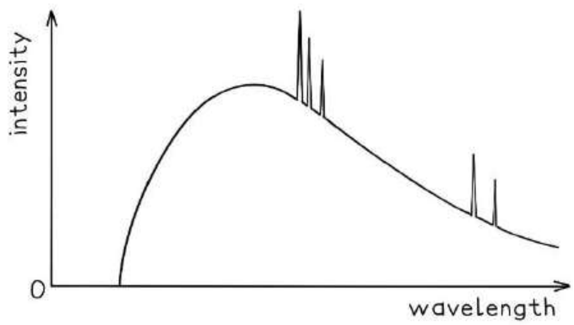
Worked Example

A typical spectrum of the X-ray radiation produced by electron bombardment of a metal target is shown below.



Explain why:

a) A continuous spectrum of wavelengths is produced.

b) The spectrum has a sharp cut-off at short wavelengths.

Part (a)

- Photons are produced whenever a charged particle is accelerated towards a metal target
- The wavelength of the photons depends on the magnitude of the acceleration
- The electrons which hit the target have a distribution of accelerations, therefore, a continuous spectrum of wavelengths is observed

Part (b)

- The minimum wavelength is equal to

$$\lambda_{min} = \frac{hc}{E_{max}}$$

- This equation shows the maximum energy of the electron corresponds to the minimum wavelength
  - Therefore, the higher the acceleration, the shorter the wavelength
- At short wavelengths, the sharp cut-off occurs as each electron produces a single photon, so, all the electron energy is given up in one collision

Using X-rays in Medical Imaging

- X-rays have been highly developed to provide detailed images of soft tissue and even blood vessels
- When treating patients, the aims are to:
  - Reduce the exposure to radiation as much as possible
  - Improve the **contrast** of the image

Reducing Exposure

- X-rays are **ionising**, meaning they can cause damage to living tissue and can potentially lead to cancerous mutations
- Therefore, healthcare professionals must ensure patients receive the minimum dosage possible
- In order to do this, **aluminium filters** are used
  - This is because many wavelengths of X-ray are emitted
  - Longer wavelengths of X-ray are more penetrating, therefore, they are more likely to be absorbed by the body
  - This means they do not contribute to the image and pose more of a health hazard
  - The aluminium sheet **absorbs** these long wavelength X-rays making them safer

Contrast & Sharpness

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- Contrast is defined as:

  The difference in degree of blackening between structures

- Contrast allows a clear difference between tissues to be seen
- Image contrast can be improved by:
  - Using the correct level of X-ray hardness: **hard X-rays** for bones, **soft X-rays** for tissue
  - Using a contrast media

- Sharpness is defined as:

  How well defined the edges of structures are

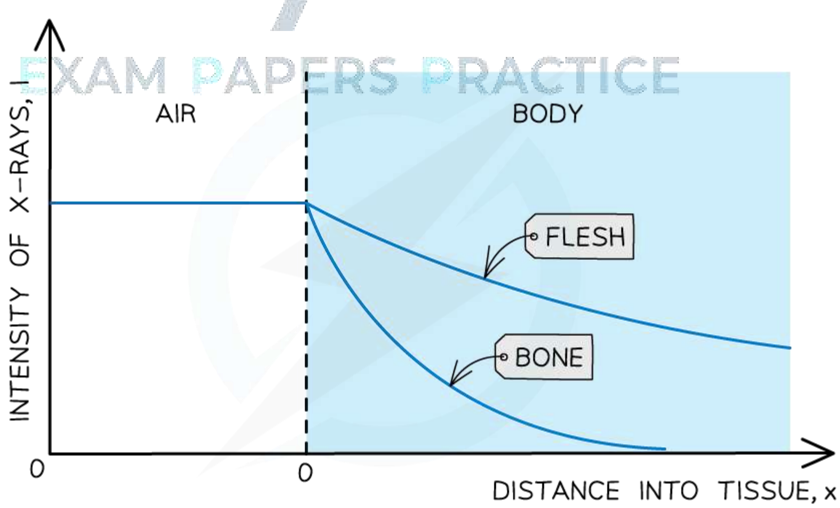
- Image sharpness can be improved by:
  - Using a narrower X-ray beam
  - Reducing X-ray scattering by using a collimator or lead grid
  - Smaller pixel size

Attenuation of X-rays in Matter

- Bones **absorb** X-ray radiation
  - This is why they appear white on the X-ray photograph
- When the collimated beam of X-rays passes through the patient's body, they are **absorbed** and **scattered**
- The attenuation of X-rays can be calculated using the equation:

$$I = I_0 e^{-\mu x}$$

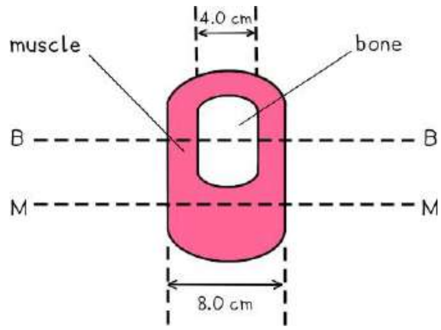
- Where:
  - I_0 = the intensity of the incident beam (W m^{-2})
  - I = the intensity of the reflected beam (W m^{-2})
  - μ = the linear absorption coefficient (m^{-1})
  - x = distance travelled through the material (m)
- The attenuation coefficient also depends on the energy of the X-ray photons
- The intensity of the X-ray decays exponentially
- The thickness of the material that will reduce the X-ray beam or a particular frequency to half its original value is known as the **half thickness**



Absorption of X-rays by different materials

Worked Example

A student investigates the absorption of X-ray radiation in a model arm. A cross-section of the model arm is shown in the diagram.



Parallel X-ray beams are directed along the line MM and along the line BB. The linear absorption coefficients of the muscle and the bone are 0.20 cm^{-1} and 12 cm^{-1} respectively.Calculate the ratio:

$$\frac{\textit{intensity of emergent X-ray beam from model}}{\textit{intensity of incident X-ray beam on model}}$$

for a parallel X-ray beam directed along the line

a) MM

b) BB

and state whether the X-ray images are sharp, or have good contrast.

Part (a)

Step 1:

Write out the known quantities

Linear absorption coefficient for muscle, μ = **0.20 cm^{-1}**

Distance travelled through the muscle, x = **8.0 cm**

Step 2:

Write out the equation for attenuation and rearrange

$$I = I_0 e^{-\mu x}$$

$$\frac{\textit{intensity of emergent X-ray beam from model}}{\textit{intensity of incident X-ray beam on model}} = \frac{I}{I_0} = e^{-\mu x}$$

Step 3:

Substitute in values and calculate the ratio

$$\frac{I}{I_0} = e^{-(0.20 \times 8)} = 0.2$$

Part (b)

Step 1:

Write out the known quantities

Linear absorption coefficient for muscle, μ_m = **0.20 cm^{-1}**

Linear absorption coefficient for bone, μ_b = **12 cm^{-1}**

Distance travelled through the muscle, x_m = **4.0 cm**

Distance travelled through the bone, x_b = **4.0 cm**

Step 2:

Write out the equation for attenuation for two media and rearrange

$$\frac{I}{I_0} = e^{-\mu_m x_m} \times e^{-\mu_b x_b}$$

Step 3:

Substitute in values and calculate the ratio

$$\frac{I}{I_0} = e^{-(0.20 \times 4)} \times e^{-(12 \times 4)} = 6.4 \times 10^{-22} \approx 0$$

Step 4:

Write a concluding statement

Each ratio gives a measure of the amount of transmission of the beam

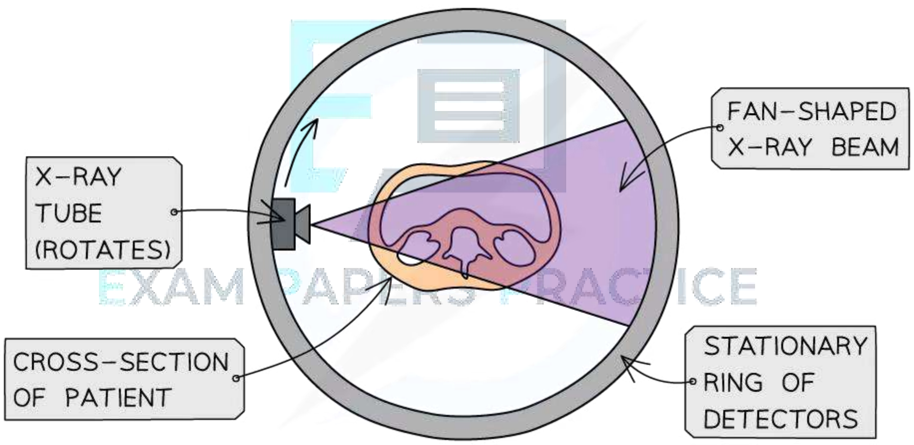
A good contrast is when:

- There is a large difference between the intensities
- The ratio is much less than 1.0

Therefore, both images have a good contrast

Computed Tomography Scanning

- A simple X-ray image can provide useful, but limited, information about internal structures in a 2D image
- When a more comprehensive image is needed, a **computerised axial tomography** (CAT or CT) scan is used
- The main features of the operation of a CT scan are as follows:
  - An X-ray tube rotates around the stationary patient
  - A CT scanner takes X-ray images of the **same slice**, at many different angles
  - This process is **repeated**, then images of successive slices are combined together
  - A computer pieces the images together to build a **3D image**
  - This 3D image can be **rotated** and viewed from different angles



CAT scans take 2D images from multiple positions to create a 3D image

Advantages & Disadvantages of CAT Scans

- Advantages:
  - Produces much more detailed images
  - Can distinguish between tissues with similar attenuation coefficients
  - Produces a 3D image of the body by combining the images at each direction
- Disadvantages:
  - The patient receives a much higher dose than a normal X-ray
  - Possible side effects from the contrast media

Worked Example

An X-ray image is taken of the skull of a patient. Another patient has a CT scan of his head.By reference to the formation of the image in each case, suggest why the exposure to radiation differs between the two imaging techniques.

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X-ray

- The simple X-ray image involves taking a single exposure
- This produces a single 2D image

CT scan

- The CT scan requires taking several exposures of a slice from many different angles
- This is then repeated for different slices before being combined together to build a 3D image
- This involves taking a much greater exposure than the simple X-ray

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24.2 PET Scanning

24.2.1 Radioactive Tracers

Radioactive Tracers

- A **radioactive tracer** is defined as:

  A radioactive substance that can be absorbed by tissue in order to study the structure and function of organs in the body

- Radioactive **isotopes**, such as **technetium-99m** or **fluorine-18**, are suitable for this purpose because:
  - They both bind to organic molecules, such as glucose or water, which are readily available in the body
  - They both emit gamma (γ) radiation and decay into stable isotopes
  - **Technetium-99m** has a short half-life of 6 hours (it is a short-lived form of Technetium-99)
  - **Fluorine-18** has an even shorter half-life of 110 minutes, so the patient is exposed to radiation for a shorter time

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Using Tracers in PET Scanning

- Positron Emission Tomography (PET) is:

  A type of nuclear medical procedure that images tissues and organs by measuring the metabolic activity of the cells of body tissues

- A common tracer used in PET scanning is a glucose molecule with radioactive fluorine attached called **fluorodeoxyglucose**
  - The fluorine nuclei undergoes **β^+ decay** – emitting a **positron** (β^+ particle)

- The radioactive tracer is injected or swallowed into the patient and flows around the body
- Once the tissues and organs have absorbed the tracer, then they appear on the screen as a bright area for a diagnosis
  - This allows doctors to determine the progress of a disease and how effective any treatments have been

- Tracers are used not only for the diagnosis of cancer but also for the heart and detecting areas of decreased blood flow and brain injuries, including Alzheimer's and dementia

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Worked Example

Write a nuclear decay for the decay of fluorine-18 by β^+ emission.

Step 1:

Work out what will be on the reactants and products side

Reactant:

Fluorine = ${}^{18}_{9}F$

Products:

Beta-plus β+ (positron) = ${}^{0}_{1}\beta$

Oxygen = ${}^{18}_{8}O$

Gamma-ray = γ

Step 2:

Write the nuclear equation

$$ {}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{1}\beta + \gamma $$

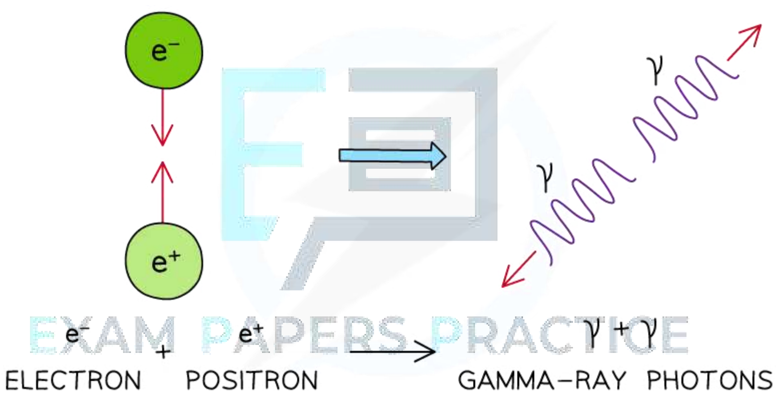
24.2.2 Annihilation in PET Scanning

The Process of Annihilation

- When a positron is emitted from a tracer in the body, it travels less than a millimetre before it collides with an electron
- The positron and the electron will **annihilate**, and their mass becomes pure energy in the form of two gamma rays which move apart in opposite directions
- Annihilation doesn't just happen with electrons and positrons, annihilation is defined as:

When a particle meets its equivalent antiparticle they are both destroyed and their mass is converted into energy

- As with all collisions, the mass, energy and momentum are conserved



Annihilation of a positron and electron to form two gamma-ray photons

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Positron Emission Tomography (PET) Scanning

- Once the tracer is introduced to the body it has a **short half-life**, so, it begins emitting positrons (β^+) immediately
  - This allows for a short exposure time to the radiation
  - A short half-life does mean the patient needs to be scanned quickly and not all hospitals have access to expensive PET scanners
- In PET scanning:
  - Positrons are emitted by the decay of the tracer
  - They travel a small distance and annihilate when they interact with electrons in the tissue
  - This annihilation produces a pair of gamma-ray photons which travel in opposite directions

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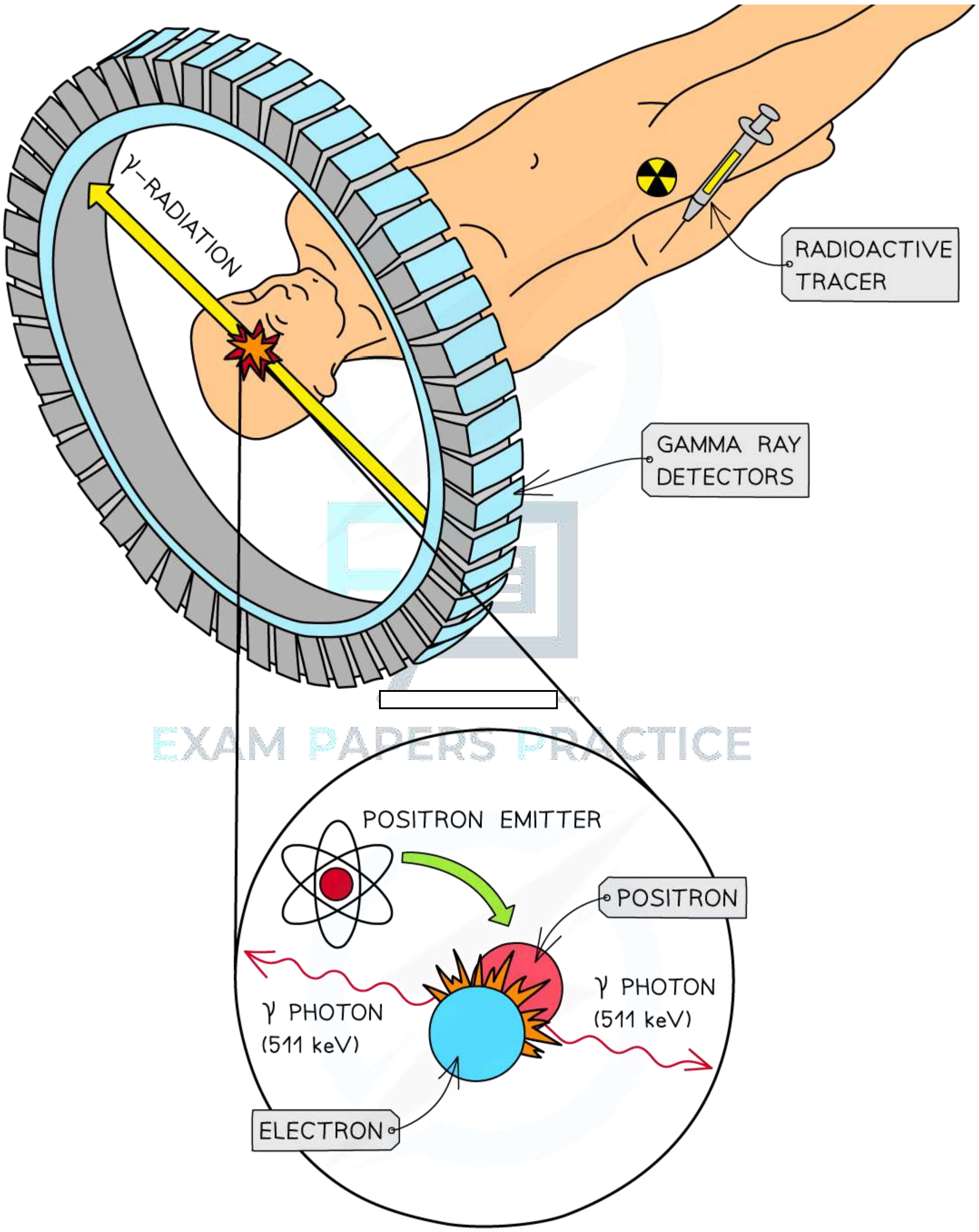
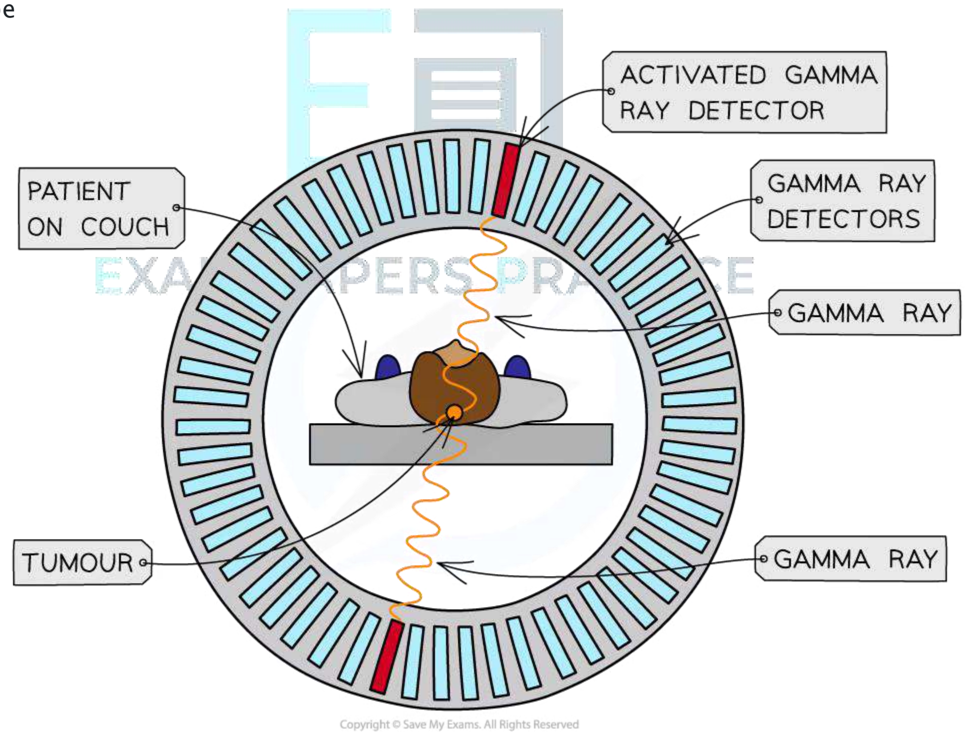


Annihilation of a positron and an electron is the basis of PET Scanning

Detecting Gamma-Rays from PET Scanning

- The patient lays stationary in a tube surrounded by a ring of detectors
- Images of slices of the body can be taken to show the position of the **radioactive tracers**
- The detector consists of two parts:
- **Crystal Scintillator** - when the gamma-ray (γ-ray) photon is incident on a crystal, an electron in the crystal is excited to a higher energy state
  - As the excited electron travels through the crystal, it excites more electrons
  - When the excited electrons move back down to their original state, the lost energy is transmitted as visible light photons
- **Photomultiplier** -The photons produced by the scintillator are very faint, so they need to be amplified and converted to an electrical signal by a photomultiplier tube



Detecting gamma rays with a PET scanner

Creating an Image from PET Scanning

- The γ rays travel in straight lines in opposite directions when formed from a positron-electron annihilation
  - This happens in order to **conserve momentum**
- They hit the detectors in a line - known as the **line of response**
- The tracers will emit lots of γ rays simultaneously, and the computers will use this information to create an image

- The more photons from a particular point, the more tracer that is present in the tissue being studied, and this will appear as a bright point on the image
- An image of the **tracer concentration** in the tissue can be created by **processing the arrival times** of the gamma-ray photons

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Calculating Energy of Gamma-Ray Photons

- In the annihilation process, both mass-energy and momentum are conserved
- The gamma-ray photons produced have an energy and frequency that is determined solely by the mass-energy of the positron-electron pair
- The energy E of the photon is given by

$$E = hf = m_e c^2$$

- The momentum p of the photon is given by

$$p = \frac{E}{c}$$

- Where:
  - m_e = mass of the electron or positron (kg)
  - h = Planck's constant (J s)
  - f = frequency of the photon (Hz)
  - c = the speed of light in a vacuum (m s^{-1})

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Worked Example

Fluorine-18 decays by β+ emission. The positron emitted collides with an electron and annihilates producing two γ-rays.

(a) Calculate the energy released when a positron and an electron annihilate.

(b) Calculate the frequency of the γ-rays emitted.

(c) Calculate the momentum of one of the γ-rays.

Part (a)

Step 1: Write down the known quantities

- Mass of an electron = mass of a positron, $m_e = 9.11 \times 10^{-31}$ kg
- Total mass is equal to the mass of the electron and positron = $2m_e$

Step 2: Write out the equation for mass-energy equivalence

$$E = m_e c^2$$

Step 3: Substitute in values and calculate energy E

$$E = 2 \times (9.11 \times 10^{-31}) \times (3.0 \times 10^8)^2 = 1.6 \times 10^{-13} \text{ J}$$

Part (b)

Step 1: Determine the energy of one photon

- Planck's constant, $h = 6.63 \times 10^{-34}$ J s
- Two photons are produced, so, the energy of one photon is equal to half of the total energy from part (a):

$$E = \frac{1.6 \times 10^{-13}}{2} = 0.8 \times 10^{-13} \text{ J}$$

Step 2: Write out the equation for the energy of a photon

$$E = hf$$

Step 3: Rearrange for frequency *f*, and calculate

$$f = \frac{E}{h} = \frac{0.8 \times 10^{-13}}{6.63 \times 10^{-34}} = \mathbf{1.2 \times 10^{20} \text{ Hz}}$$

Part (c)

Step 1: Write out the equation for the momentum of a photon

$$p = \frac{E}{c}$$

Step 2: Substitute in values and calculate momentum, *p*

$$p = \frac{E}{c} = \frac{0.8 \times 10^{-13}}{3.0 \times 10^{8}} = \mathbf{2.7 \times 10^{-22} \text{ N s}}$$

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