A Level Physics CIE

23. Nuclear Physics

CONTENTS

23.1 Mass Defect & Nuclear Binding Energy

23.1.1 Energy & Mass Equivalence

Energy & Mass Equivalence

- Einstein showed in his **theory of relativity** that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

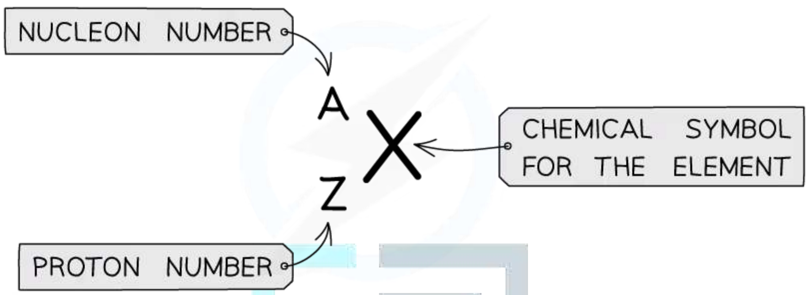
$$E = mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})

- Some examples of mass-energy equivalence are:
  - The **fusion** of hydrogen into helium in the centre of the sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

Representing Simple Nuclear Reactions

- Nuclear reactions can be represented by balanced equations of nuclei in the AZX form



AZX notation for atomic nuclei

- The top number A represents the **nucleon** number or the **mass** number
  - **Nucleon number (A)** = total number of **protons and neutrons** in the nucleus
- The lower number Z represents the **proton** or **atomic** number
  - **Proton number (Z)** = total number of **protons** in the nucleus

Worked Example

When a neutron is captured by a uranium-235 nucleus, the outcome may be represented by the nuclear equation:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{95}_{42}Mo + ^{139}_{57}La + x^{1}_{0}n + 7^{0}_{-1}e$$

What is the value of x?

Step 1: Balance the nucleon numbers (the top number)

235 + 1 = 95 + 139 + x(1) + 7(0)

Step 2: Rearrange to find the value of x

x = 235 + 1 − 95 − 139 = 2

Mass Defect & Binding Energy

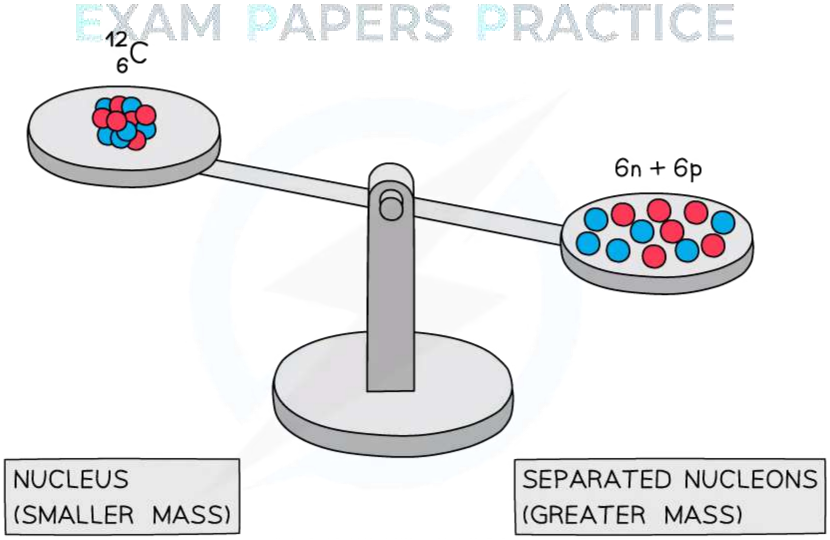
- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
- This difference in mass is known as the **mass defect**
- Mass defect is defined as:

The difference between an atom's mass and the sum of the masses of its protons and neutrons

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)



A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to the equivalence of mass and energy, this decrease in mass implies that energy is released in the process
- Since nuclei are made up of neutrons and protons, there are forces of repulsion between the positive protons
  - Therefore, it takes energy, ie. the binding energy, to hold nucleons together as a nucleus
- Binding energy is defined as:

The energy required to break a nucleus into its constituent protons and neutrons

- Energy and mass are proportional, so, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons
- The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction – meaning that it releases energy
- This can be calculated using the equation:

$$E = \Delta mc^2$$

Exam Tip

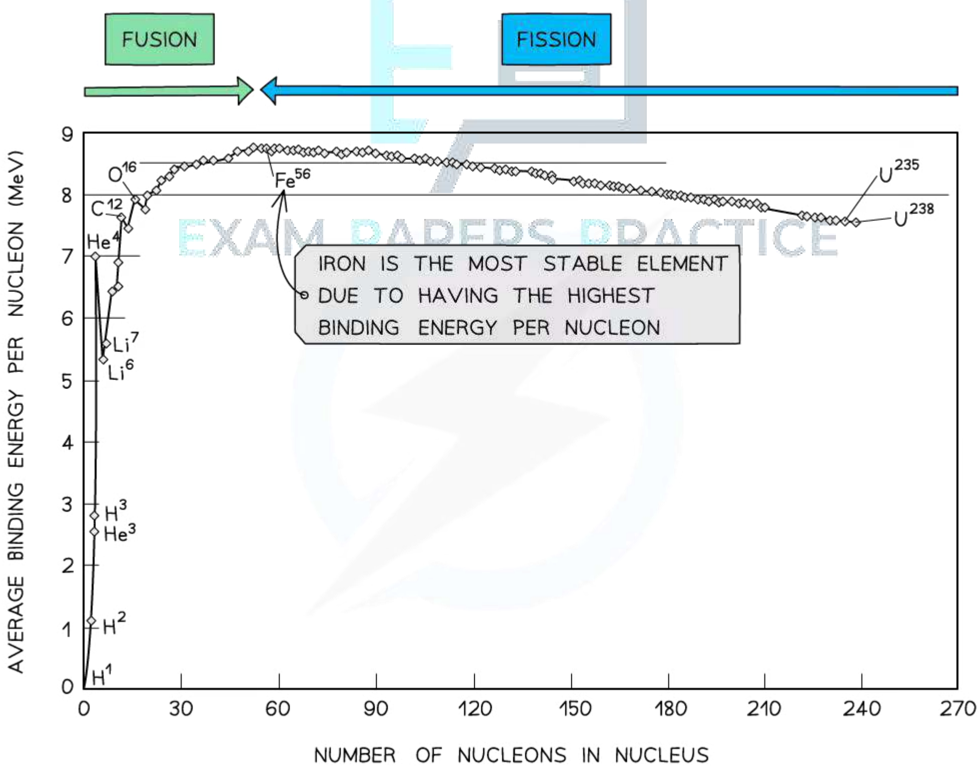
Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put into the nucleus to pull it apart.

Binding Energy per Nucleon

- In order to compare nuclear stability, it is more useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability
  - In other words, it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the most stable of all the elements



By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable
  - This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**
- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend

- Helium-4 is a particularly stable nucleus hence it has a high binding energy per nucleon
- Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together

- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

Worked Example

What is the binding energy per nucleon of iron-56 ($^{56}_{26}Fe$) in MeV?

Mass of a neutron = 1.675×10^{-27} kg
Mass of a proton = 1.673×10^{-27} kg
Mass of $^{56}_{26}Fe$ nucleus = 9.288×10^{-26} kg

Step 1: Calculate the mass defect

Number of protons, Z = 26

Number of neutrons, A – Z = 56 – 26 = 30

Mass defect, $\Delta m = Zm_p + (A - Z)m_n - m_{total}$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

Binding energy, $E = \Delta mc^2$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

$$\text{Binding energy per nucleon} = \frac{E}{A}$$

$$\frac{E}{A} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12} \text{ J}$$

Step 4: Convert to MeV

J → eV: divide by 1.6×10^{-19}

eV → MeV: divide by 10^6

$$\text{Binding energy per nucleon} = \frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}} = 8\,718\,750 \text{ eV} = \mathbf{8.7\ MeV} \text{ (2 s.f.)}$$

Exam Tip

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

- You will be expected to draw the best fit curve AND a cross to show the anomaly that is helium
- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy per nucleon
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

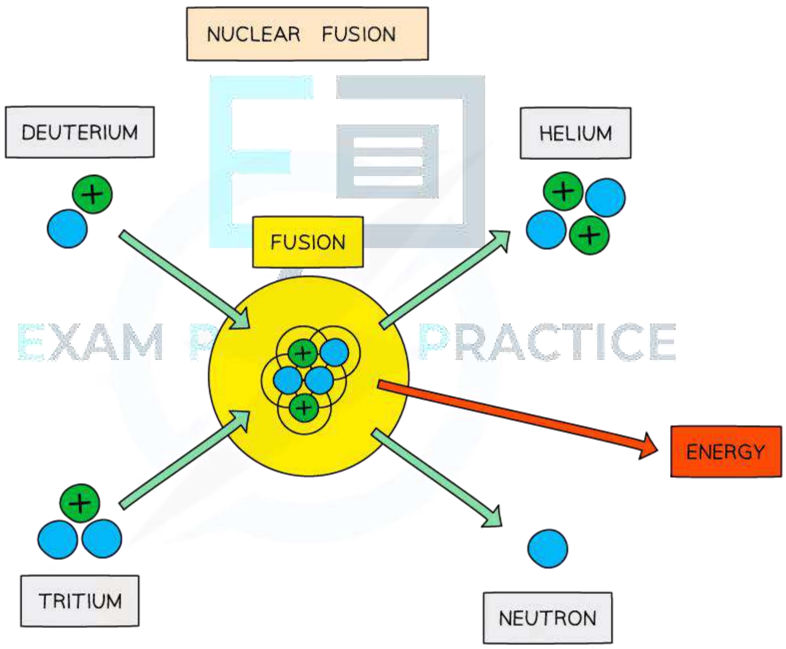
Nuclear Fusion & Fission

Nuclear Fusion

- Fusion is defined as:

The fusing together of two small nuclei to produce a larger nucleus

- Low mass nuclei (such as hydrogen and helium) can undergo fusion and release energy



The fusion of deuterium and tritium to form helium with the release of energy

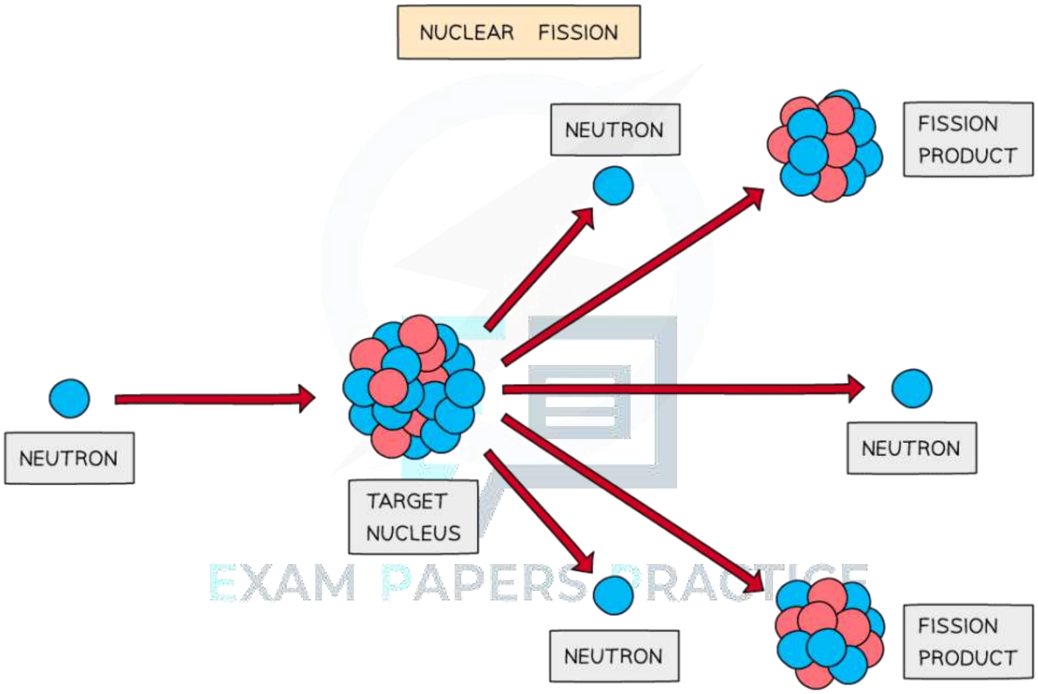
- For two nuclei to fuse, both nuclei must have high kinetic energy
  - This is because the protons inside the nuclei are positively charged, which means that they repel one another
- It takes a great deal of energy to overcome the electrostatic force, so this is why it is can only be achieved in an extremely high-energy environment, such as star's core
- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

Nuclear Fission

- Fission is defined as:

The splitting of a large atomic nucleus into smaller nuclei

- High mass nuclei (such as uranium) can undergo fission and release energy



The fission of a target nucleus, such as uranium, to produce smaller daughter nuclei with the release of energy

- Fission must first be induced by firing neutrons at a nucleus
- When the nucleus is struck by a neutron, it splits into two, or more, daughter nuclei
- During fission, neutrons are ejected from the nucleus, which in turn, can collide with other nuclei which triggers a cascade effect
- This leads to a chain reaction which lasts until all of the material has undergone fission, or the reaction is halted by a moderator
- Nuclear fission is the process which produces energy in nuclear power stations, where it is well controlled
- When nuclear fission is not controlled, the chain reaction can cascade to produce the effects of a nuclear bomb

> **Exam Tip**
>
> When an atom undergoes nuclear fission, take note that extra neutrons are ejected by the **nucleus** and not from the fission products

Significance of Binding Energy per Nucleon

- At low values of A:
  - Attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons
  - In the right conditions, nuclei undergo **fusion**
- In fusion, the mass of the nucleus that is created is slightly **less** than the total mass of the original nuclei
  - The mass defect is equal to the binding energy that is released, since the nucleus that is formed is more stable
- At high values of A:
  - Repulsive electrostatic forces between forces begin to dominate, and these forces tend to break apart the nucleus rather than hold it together
  - In the right conditions, nuclei undergo **fission**
- In fission, an unstable nucleus is converted into more stable nuclei with a smaller total mass
  - This difference in mass, the mass defect, is equal to the binding energy that is released

23.1.5 Calculating Energy Released in Nuclear Reactions

Calculating Energy Released in Nuclear Reactions

- The binding energy is equal to the amount of energy released in forming the nucleus, and can be calculated using:

$$E = (\Delta m)c^2$$

- Where:
  - E = Binding energy released (J)
  - Δm = mass defect (kg)
  - c = speed of light (m s^{-1})

- The daughter nuclei produced as a result of both fission and fusion have a higher binding energy per nucleon than the parent nuclei
- Therefore, energy is released as a result of the mass difference between the parent nuclei and the daughter nuclei

Worked Example

When Uranium−235 nuclei are fissioned by slow-moving neutrons, two possible reactions are:

Reaction 1: $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2^{1}_{0}n$ + energy

Reaction 2: $^{235}_{92}U + ^{1}_{0}n \rightarrow 2\,^{116}_{46}Pd$ + xc + energy

a) For reaction 2, identify the particle c and state the number x of such particles generated in the reaction.

b) The binding energy per nucleon *E* for a number of nuclides is given by the table below. Use the table to show that the energy produced in reaction 1 is about 210 MeV.

c) The energy produced in reaction 2 is 163 MeV. Suggest, with supporting reason, which one of the two reactions is more likely to happen.

nuclide	E / MeV
$^{95}_{38}Sr$	8.74
$^{139}_{54}Xe$	8.39
$^{235}_{92}U$	7.60

Part (a)

Step 1: Balance the number of protons on each side (bottom number)

$$92 = (2 \times 46) + xn_p \text{ (where } n_p \text{ is the number of protons in c)}$$

$$xn_p = 92 - 92 = 0$$

Therefore, c must be a neutron

Step 2: Balance the number of nucleons on each side

$$235 + 1 = (2 \times 116) + x$$

$$x = 235 + 1 - 232 = 4$$

Therefore, 4 neutrons are generated in the reaction

Part (b)

Step 1: Find the binding energy of each nucleus

Total binding energy of each nucleus = Binding energy per nucleon × Mass number

$$\text{Binding energy of } {}^{95}Sr = 8.74 \times 95 = 830.3 \text{ MeV}$$

$$\text{Binding energy of } {}^{139}Xe = 8.39 \times 139 = 1166.21 \text{ MeV}$$

$$\text{Binding energy of } {}^{235}U = 7.60 \times 235 = 1786 \text{ MeV}$$

Step 2: Calculate the difference in energy between the products and reactants

$$\text{Energy released in reaction 1} = E_{Sr} + E_{Xe} - E_U$$

$$\text{Energy released in reaction 1} = 830.3 + 1166.21 - 1786$$

$$\text{Energy released in reaction 1} = 210.5 \text{ MeV}$$

Part (c)

- Since reaction 1 releases more energy than reaction 2, its end products will have a higher binding energy per nucleon
  - Hence they will be more stable
- This is because the more energy is released, the further it moves up the graph of binding energy per nucleon against nucleon number (A)
  - Since at high values of A, binding energy per nucleon gradually decreases with A
- Nuclear reactions will tend to favour the more stable route, therefore, reaction 1 is more likely to happen

23.2 Radioactive Decay

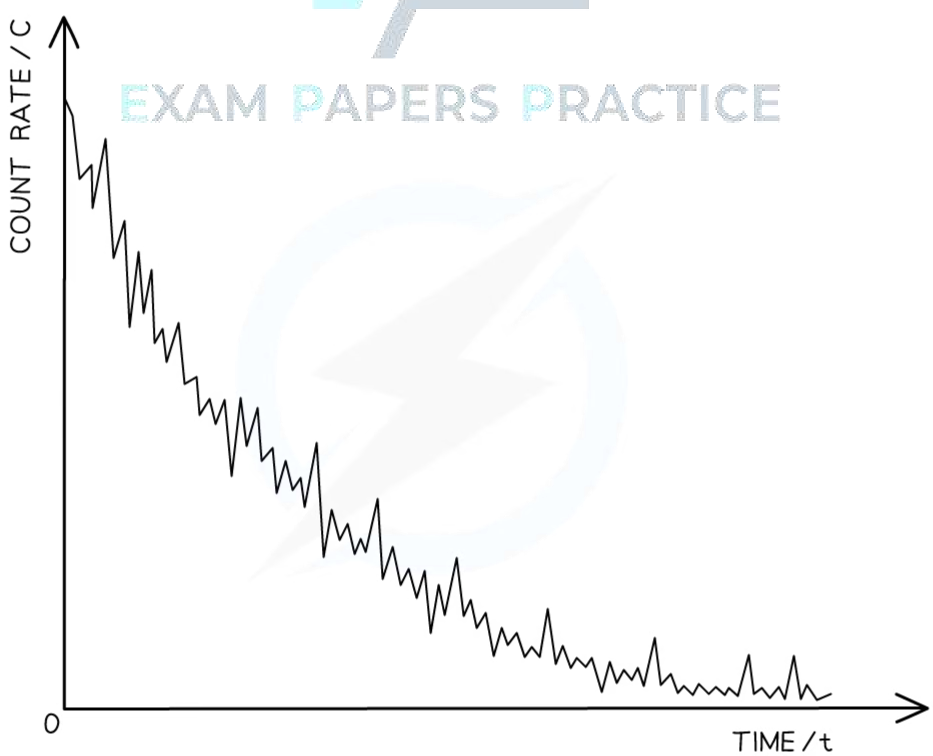
23.2.1 The Random Nature of Radioactive Decay

The Random Nature of Radioactive Decay

- Radioactive decay is defined as:

  The spontaneous disintegration of a nucleus to form a more stable nucleus, resulting in the emission of an alpha, beta or gamma particle

- The random nature of radioactive decay can be demonstrated by observing the count rate of a Geiger-Muller (GM) tube
  - When a GM tube is placed near a radioactive source, the counts are found to be irregular and cannot be predicted
  - Each count represents a decay of an unstable nucleus
  - These fluctuations in count rate on the GM tube **provide evidence for the randomness of radioactive decay**



The variation of count rate over time of a sample radioactive gas. The fluctuations show the randomness of radioactive decay

Characteristics of Radioactive Decay

- Radioactive decay is both **spontaneous** and **random**
- A spontaneous process is defined as:

A process which cannot be influenced by environmental factors

- This means radioactive decay cannot be affected by environmental factors such as:
  - Temperature
  - Pressure
  - Chemical conditions

- A random process is defined as:

A process in which the exact time of decay of a nucleus cannot be predicted

- Instead, the nucleus has a constant probability, ie. the same chance, of decaying in a given time
- Therefore, with large numbers of nuclei, it is possible to statistically predict the behaviour of the entire group

Exam Tip

Make sure you can define what constitutes a radioactive decay, a random process and a spontaneous decay – these are all very common exam questions!

Activity & The Decay Constant

- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei which are expected to decay per unit time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability that an individual nucleus will decay per unit of time

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**
- Activity, or the number of decays per unit time can be calculated using:

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample
- The activity of a sample is measured in **Becquerels** (Bq)
  - An activity of 1 Bq is equal to one decay per second, or 1 s^{-1}
- This equation shows:
  - The greater the decay constant, the greater the activity of the sample
  - The activity depends on the number of undecayed nuclei remaining in the sample
  - The minus sign indicates that the number of nuclei remaining decreases with time – however, for calculations it can be omitted

Worked Example

Americium-241 is an artificially produced radioactive element that emits α-particles. A sample of americium-241 of mass 5.1 μg is found to have an activity of 5.9×10^5 Bq.

(a)

Determine the number of nuclei in the sample of americium-241.

(b)

Determine the decay constant of americium-241.

Part (a)

Step 1: Write down the known quantities

- Mass = 5.1 μg = 5.1×10^{-6} g
- Molecular mass of americium = 241
- N_A = Avogadro constant

Step 2: Write down the equation relating number of nuclei, mass and molecular mass

$$\text{Number of nuclei} = \frac{\text{mass} \times N_A}{\text{molecular mass}}$$

Step 3: Calculate the number of nuclei

$$\text{Number of nuclei} = \frac{(5.1 \times 10^{-6}) \times (6.02 \times 10^{23})}{241} = 1.27 \times 10^{16}$$

Part (b)

Step 1: Write the equation for activity

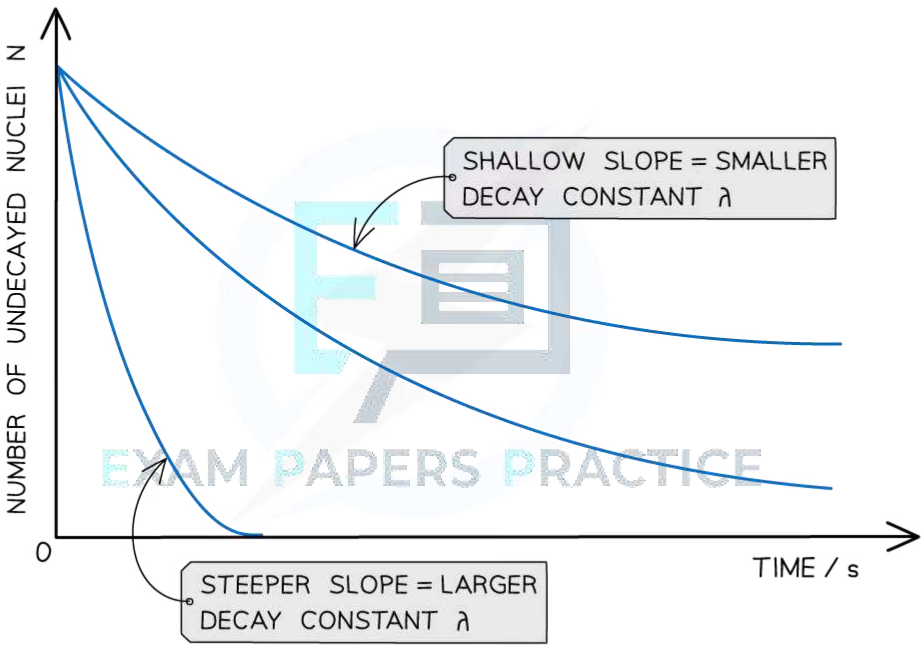
Activity, A = λN

Step 2: Rearrange for decay constant λ and calculate the answer

$$\lambda = \frac{A}{N} = \frac{5.9 \times 10^5}{1.27 \times 10^{16}} = 4.65 \times 10^{-11}\ s^{-1}$$

The Exponential Nature of Radioactive Decay

- In radioactive decay, the number of nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei and time has a very distinctive shape



Radioactive decay follows an exponential pattern. The graph shows three different isotopes each with a different rate of decay

Equations for Radioactive Decay

- The number of undecayed nuclei N can be represented in exponential form by the equation:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)
- The number of nuclei can be substituted for other quantities, for example, the activity A is directly proportional to N, so it can be represented in exponential form by the equation:

$$A = A_0 e^{-\lambda t}$$

- The received count rate C is related to the activity of the sample, hence it can also be represented in exponential form by the equation:

$$C = C_0 e^{-\lambda t}$$

The exponential function *e*

- The symbol e represents the exponential constant
  - It is approximately equal to e = 2.718
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then x = ln(y)

Worked Example

Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The decay constant of Strontium-90 is 0.025 $year^{-1}$.

Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0

Step 1: Write out the known quantities

Decay constant, λ = 0.025 $year^{-1}$

Time interval, t = 5.0 years

Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between A and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

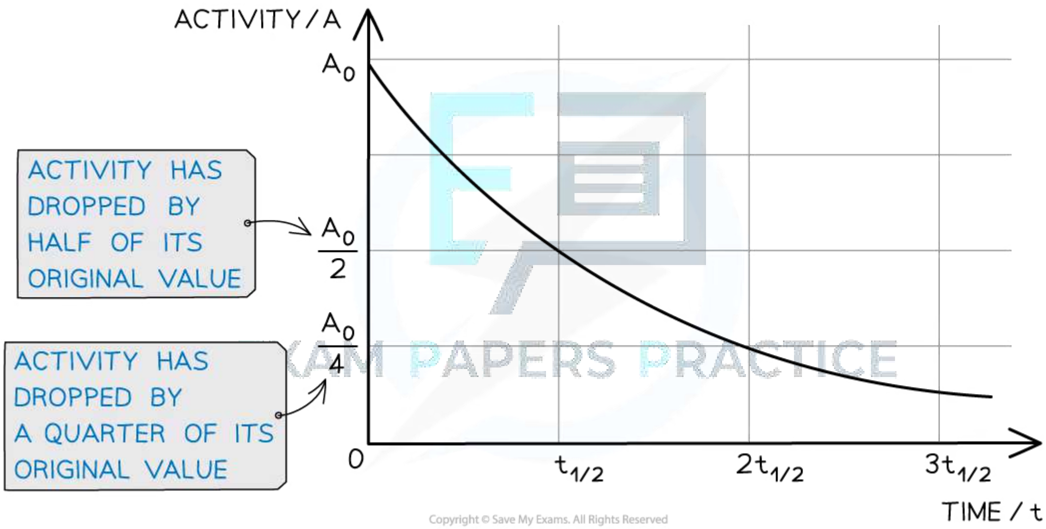
Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

Half-Life Definition

- Half life is defined as:

The time taken for the initial number of nuclei to reduce by half

- This means when a time equal to the half-life has passed, the activity of the sample will also half
- This is because activity is proportional to the number of undecayed nuclei, $A \propto N$



When a time equal to the half-life passes, the activity falls by half, when two half-lives pass, the activity falls by another half (which is a quarter of the initial value)

Calculating Half-Life

- To find an expression for half-life, start with the equation for exponential decay:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N = number of nuclei remaining in a sample
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- When time t is equal to the half-life $t_{½}$, the activity N of the sample will be half of its original value, so N = ½ N_0

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula can then be derived as follows:

Divide both sides by N_0: $\frac{1}{2} = e^{-\lambda t_{½}}$

Take the natural log of both sides: $\ln\left(\frac{1}{2}\right) = -\lambda t_{½}$

Apply properties of logarithms: $\lambda t_{½} = \ln(2)$

- Therefore, half-life $t_{½}$ can be calculated using the equation:

$$t_{½} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- This equation shows that half-life $t_{½}$ and the radioactive decay rate constant λ are inversely proportional
- Therefore, the shorter the half-life, the larger the decay constant and the **faster** the decay

Worked Example

Strontium-90 is a radioactive isotope with a half-life of 28.0 years. A sample of Strontium-90 has an activity of 6.4 × 10^9 Bq.Calculate the decay constant λ, in s^{-1}, of Strontium-90.

Step 1: Convert the half-life into seconds

$$\mathbf{28 \text{ years} = 28 \times 365 \times 24 \times 60 \times 60 = 8.83 \times 10^8 \text{ s}}$$

Step 2: Write the equation for half-life

$$t_{½} = \frac{\ln 2}{\lambda}$$

Step 3: Rearrange for λ and calculate

$\lambda = \frac{ln\,2}{t_{½}} = \frac{ln\,2}{8.83 \times 10^{8}}$ = **7.85×10^{-10} s^{-1}**