



A Level Physics CIE

22. Quantum Physics

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22.1 The Photoelectric Effect

22.1.1 The Photon

The Particle Nature of Light

- In classical wave theory, electromagnetic (EM) radiation is assumed to behave as a wave
- This is demonstrated by the fact EM radiation exhibits phenomena such as **diffraction** and **interference**
- However, experiments from the last century, such as the photoelectric effect and atomic line spectra, can only be explained if EM radiation is assumed to behave as particles
- These experiments have formed the basis of **quantum theory**, which will be explored in detail in this section

The Photon

- Photons are fundamental particles which make up all forms of electromagnetic radiation
- A photon is a massless “packet” or a “quantum” of electromagnetic energy
- What this means is that the energy is not transferred continuously, but as discrete packets of energy
- In other words, each photon carries a specific amount of energy, and transfers this energy all in one go, rather than supplying a consistent amount of energy



Exam Tip

Make sure you learn the definition for a photon: *discrete quantity / packet / quantum of electromagnetic energy* are all acceptable definitions



Calculating Photon Energy

- The energy of a photon can be calculated using the formula:

$$E = hf$$

- Using the wave equation, energy can also be equal to:

$$E = h \frac{c}{\lambda}$$

- Where:

- E = energy of the photon (J)
- h = Planck's constant (J s)
- c = the speed of light (m s^{-1})
- f = frequency in Hertz (Hz)
- λ = wavelength (m)

- This equation tells us:

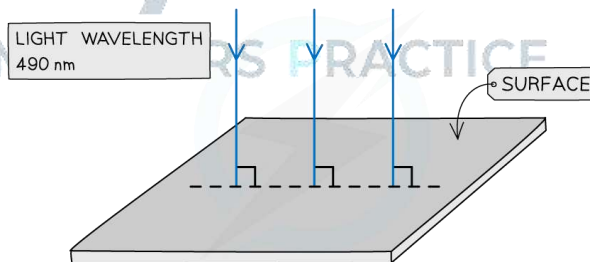
- The higher the frequency of EM radiation, the higher the energy of the photon
- The energy of a photon is inversely proportional to the wavelength
- A long-wavelength photon of light has a lower energy than a shorter-wavelength photon



Worked Example

Light of wavelength 490 nm is incident normally on a surface, as shown in

the diagram.



The power of the light is 3.6 mW. The light is completely absorbed by the surface. Calculate the number of photons incident on the surface in 2.0 s.

Step 1: Write down the known quantities

$$\text{Wavelength, } \lambda = 490 \text{ nm} = 490 \times 10^{-9} \text{ m}$$

$$\text{Power, } P = 3.6 \text{ mW} = 3.6 \times 10^{-3} \text{ W}$$

$$\text{Time, } t = 2.0 \text{ s}$$

Step 2: Write the equations for wave speed and photon energy

$$\text{wave speed: } c = f\lambda \rightarrow f = \frac{c}{\lambda}$$



photon energy: $E = hf \rightarrow E = \frac{hc}{\lambda}$

Step 3: Calculate the energy of one photon

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.0 \times 10^8)}{490 \times 10^{-9}} = 4.06 \times 10^{-19} \text{ J}$$

Step 4: Calculate the number of photons hitting the surface every second

$$\frac{\text{Power of light source}}{\text{Energy of one photon}} = \frac{3.6 \times 10^{-3}}{4.06 \times 10^{-19}} = 8.9 \times 10^{15} \text{ s}^{-1}$$

Step 5: Calculate the number of photons that hit the surface in 2 s

$$(8.9 \times 10^{15}) \times 2 = 1.8 \times 10^{16}$$



Exam Tip

The values of Planck's constant and the speed of light will always be given to you in an exam, however, it helps to memorise them to speed up calculation questions!





Photon Momentum

- Einstein showed that a photon travelling in a vacuum has momentum, despite it having no mass
- The momentum (p) of a photon is related to its energy (E) by the equation:

$$p = \frac{E}{c}$$

- Where c is the speed of light



Worked Example

A 5.0 mW laser beam is incident normally on a fixed metal plate. The cross-sectional area of the beam is $8.0 \times 10^{-6} \text{ m}^2$. The light from the laser has frequency $5.6 \times 10^{14} \text{ Hz}$. Assuming that all the photons are absorbed by the plate, calculate the momentum of the photon, and the pressure exerted by the laser beam on the metal plate.

Step 1: Write down the known quantities

$$\text{Power, } P = 5.0 \text{ mW} = 5.0 \times 10^{-3} \text{ W}$$

$$\text{Frequency, } f = 5.6 \times 10^{14} \text{ Hz}$$

$$\text{Cross-sectional area, } A = 8.0 \times 10^{-6} \text{ m}^2$$

Step 2: Write the equations for photon energy and momentum

$$\text{photon energy: } E = hf$$

$$\text{photon momentum: } p = \frac{E}{c} \rightarrow p = \frac{hf}{c}$$

Step 3: Calculate the photon momentum

$$p = \frac{hf}{c} = \frac{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})}{3.0 \times 10^8} = 1.24 \times 10^{-27} \text{ N s}$$

Step 4: Calculate the number of photons incident on the plate every second

$$\frac{\text{Power of light source}}{\text{Energy of one photon}} = \frac{5.0 \times 10^{-3}}{hf} = \frac{5.0 \times 10^{-3}}{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})} = 1.35 \times 10^{16} \text{ s}^{-1}$$

Step 5: Calculate the force exerted on the plate in a 1.0 s time interval

$$\begin{aligned} \text{Force} &= \text{rate of change of momentum} \\ &= \text{number of photons per second} \times \text{momentum of each photon} \\ &= (1.35 \times 10^{16}) \times (1.24 \times 10^{-27}) \\ &= 1.67 \times 10^{-11} \text{ N} \end{aligned}$$



Step 6: Calculate the pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{1.67 \times 10^{-11}}{8.0 \times 10^{-6}} = 2.1 \times 10^{-6} \text{ Pa}$$





22.1.2 The Electronvolt

The Electronvolt

- The electronvolt is a unit which is commonly used to express very small energies
- This is because quantum energies tend to be much smaller than 1 Joule
- The electronvolt is derived from the definition of potential difference:

$$V = \frac{E}{Q}$$

- When an electron travels through a potential difference, energy is transferred between two points in a circuit, or electric field
- If an electron, with a charge of 1.6×10^{-19} C, travels through a potential difference of 1 V, the energy transferred is equal to:

$$E = QV = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$$

- Therefore, an electronvolt is defined as:

The energy gained by an electron travelling through a potential difference of one volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Relation to kinetic energy

- When a charged particle is accelerated through a potential difference, it gains kinetic energy
- If an electron accelerates from rest, an **electronvolt** is equal to the kinetic energy gained:

$$eV = \frac{1}{2} mv^2$$

- Rearranging the equation gives the speed of the electron:

$$v = \sqrt{\frac{2eV}{m}}$$

**Worked Example**

Show that the photon energy of light with wavelength 700nm is about 1.8 eV.

Step 1: Write the equations for wave speed and photon energy

wave speed: $c = f\lambda \rightarrow f = \frac{c}{\lambda}$

photon energy: $E = hf \rightarrow E = \frac{hc}{\lambda}$

Step 2: Calculate the photon energy in Joules



$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.0 \times 10^8)}{700 \times 10^{-9}} = 2.84 \times 10^{-19} \text{ J}$$

Step 3: Convert the photon energy into electronvolts

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

J \rightarrow eV: divide by 1.6×10^{-19}

$$E = \frac{2.84 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.78 \text{ eV}$$



Exam Tip

- To convert between eV and J:
- eV \rightarrow J: **multiply** by 1.6×10^{-19}
- J \rightarrow eV: **divide** by 1.6×10^{-19}

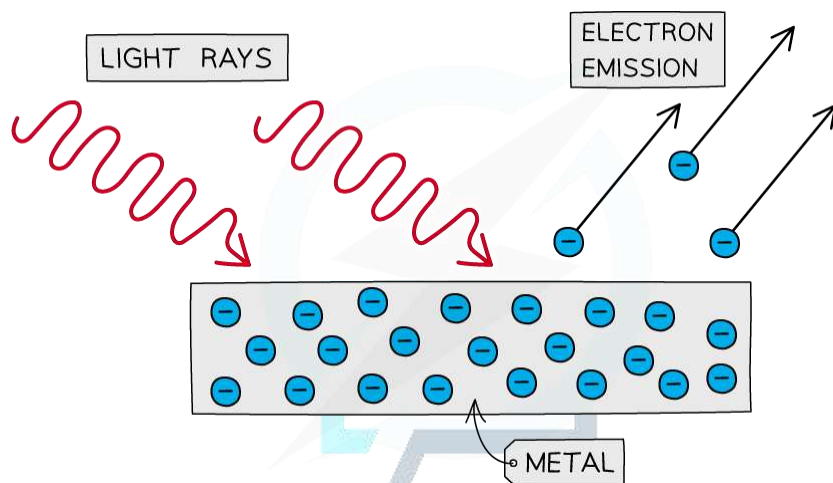




22.1.3 The Photoelectric Effect: Basics

The Photoelectric Effect: Basics

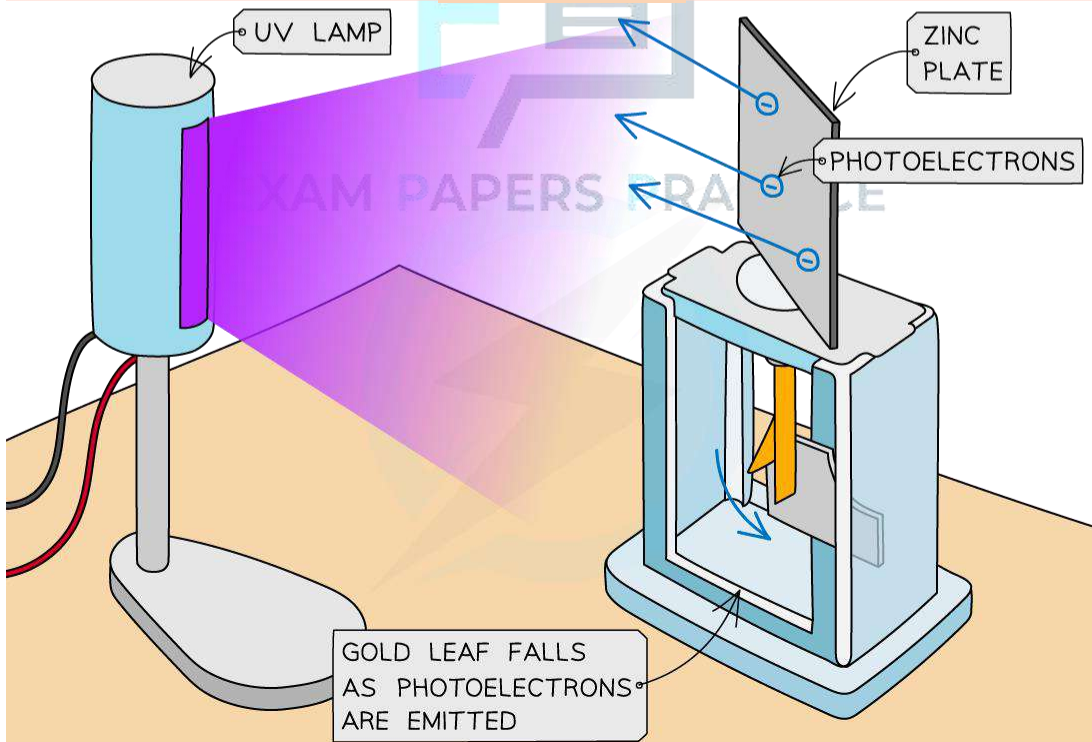
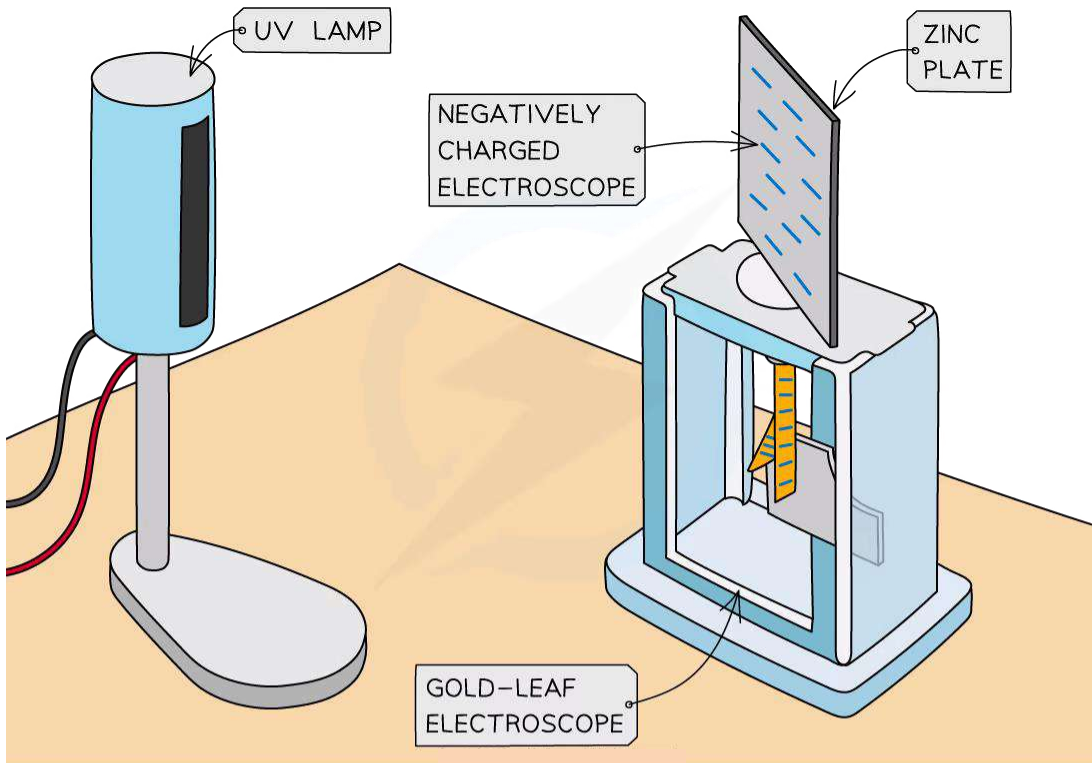
- The **photoelectric effect** is the phenomena in which electrons are emitted from the surface of a metal **upon the absorption of electromagnetic radiation**
- Electrons removed from a metal in this manner are known as **photoelectrons**
- The photoelectric effect provides important evidence that light is **quantised**, or carried in discrete packets
 - This is shown by the fact each electron can absorb only a single photon
 - This means only the frequencies of light above a **threshold frequency** will emit a photoelectron



Photoelectrons are emitted from the surface of metal when light shines onto it

Observing the Photoelectric Effect

- The photoelectric effect can be observed on a **gold leaf electroscope**
- A plate of metal, usually **zinc**, is attached to a gold leaf, which initially has a negative charge, causing it to be repelled by a central negatively charged rod
 - This causes negative charge, or electrons, to build up on the zinc plate
- **UV light** is shone onto the metal plate, leading to the **emission of photoelectrons**
- This causes the extra electrons on the central rod and gold leaf to be removed, so, the gold leaf begins to fall back towards the central rod
 - This is because they become less negatively charged, and hence repel less
- Some notable observations:
 - Placing the UV light source closer to the metal plate causes the gold leaf to fall more quickly
 - Using a higher frequency light source does not change the how quickly the gold leaf falls
 - Using a filament light source causes no change in the gold leaf's position
 - Using a positively charged plate also causes no change in the gold leaf's position



Typical set-up of the gold leaf electroscope experiment



22.1.4 Threshold Frequency

Threshold Frequency & Wavelength

- The concept of a threshold frequency is required in order to explain why a low frequency source, such as a filament lamp, was unable to liberate any electrons in the gold leaf experiment
- The **threshold frequency** is defined as:

The minimum frequency of incident electromagnetic radiation required to remove a photoelectron from the surface of a metal

- The **threshold wavelength**, related to threshold frequency by the wave equation, is defined as:

The longest wavelength of incident electromagnetic radiation that would remove a photoelectron from the surface of a metal

- Threshold frequency and wavelength are properties of a material, and vary from metal to metal

Threshold frequencies and wavelengths for different metals

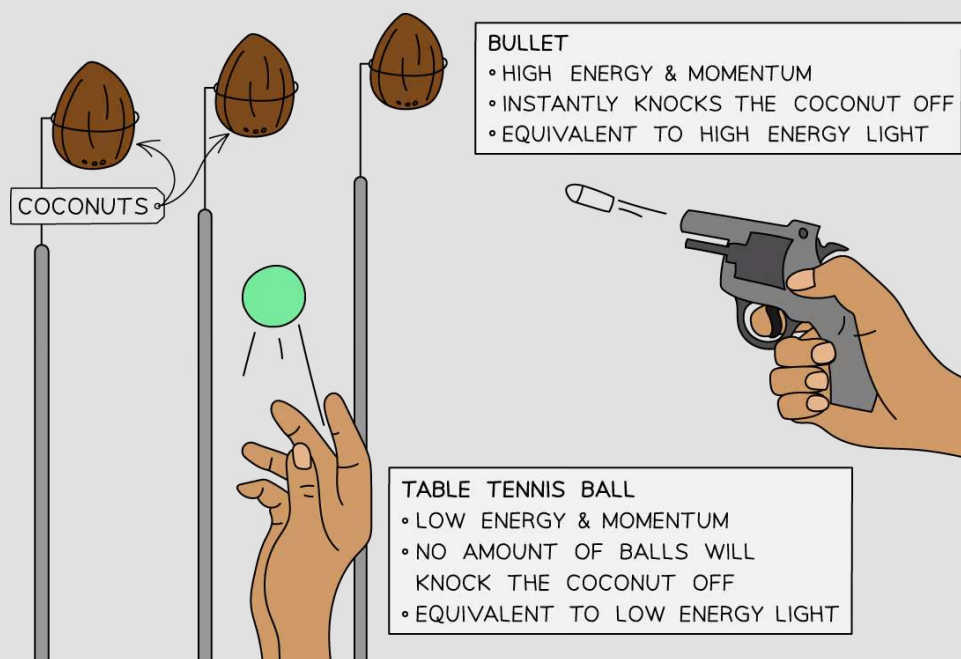
Metal	Threshold Frequency (f_0) / Hz	Threshold Wavelength (λ_0) / nm
Sodium	4.40×10^{14}	682
Potassium	5.56×10^{14}	540
Zinc	1.02×10^{15}	294
Iron	1.04×10^{15}	289
Copper	1.13×10^{15}	266
Gold	1.23×10^{15}	244
Silver	9.71×10^{15}	30.9



Exam Tip

A useful analogy for threshold frequency is a fairground coconut shy:

- One person is throwing table tennis balls at the coconuts, and another person has a pistol
- No matter how many of the table tennis balls are thrown at the coconut it will still stay firmly in place – this represents the **low frequency quanta**
- However, a single shot from the pistol will knock off the coconut immediately – this represents the **high frequency quanta**





The Photoelectric Equation

- Since energy is always conserved, the energy of an incident photon is equal to:

The threshold energy + the kinetic energy of the photoelectron

- The energy within a photon is equal to hf
- This energy is transferred to the electron to release it from a material (the work function) and gives the emitted photoelectron the remaining amount as kinetic energy
- This equation is known as the **photoelectric equation**:

$$E = hf = \Phi + \frac{1}{2}mv_{max}^2$$

- Symbols:
 - h = Planck's constant (J s)
 - f = the frequency of the incident radiation (Hz)
 - Φ = the work function of the material (J)
 - $\frac{1}{2}mv_{max}^2$ = the maximum kinetic energy of the photoelectrons (J)
- This equation demonstrates:
 - If the incident photons do not have a high enough frequency (f) and energy to overcome the work function (Φ), then no electrons will be emitted
 - When $hf_0 = \Phi$, where f_0 = threshold frequency, photoelectric emission only just occurs
 - E_{kmax} depends only on the frequency of the incident photon, and not the intensity of the radiation
 - The majority of photoelectrons will have kinetic energies less than E_{kmax}

Graphical Representation of Work Function

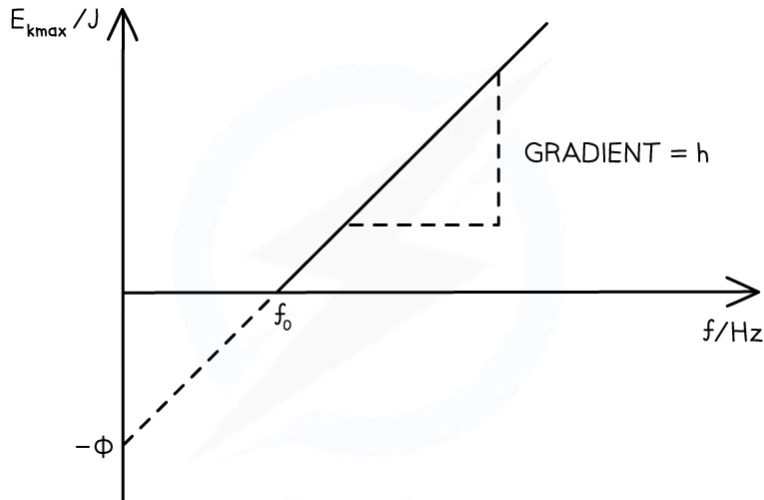
- The photoelectric equation can be rearranged into the straight line equation:

$$y = mx + c$$

- Comparing this to the photoelectric equation:

$$E_{kmax} = hf - \Phi$$

- A graph of maximum kinetic energy E_{kmax} against frequency f can be obtained



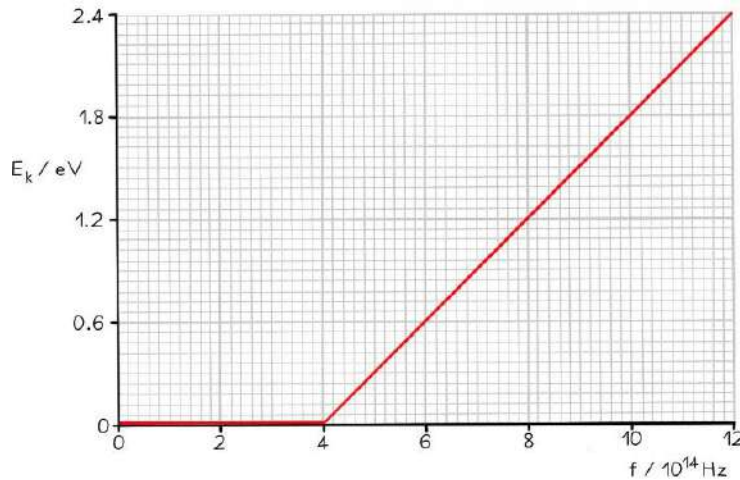
Graph of maximum kinetic energy of photoelectrons against photon frequency

- The key elements of the graph:
 - The work function ϕ is the y-intercept
 - The threshold frequency f_0 is the x-intercept
 - The gradient is equal to Planck's constant h
 - There are no electrons emitted below the threshold frequency f_0



Worked Example

The graph below shows how the maximum kinetic energy E_k of electrons emitted from the surface of sodium metal varies with the frequency f of the incident radiation.



Calculate the

work function of sodium in eV.

Step 1: Write out the photoelectric equation and rearrange to fit the equation of a

straight line

For more help, please visit www.exampaperspractice.co.uk



$$E = hf = \Phi + \frac{1}{2}mv_{\max}^2 \quad \rightarrow \quad E_{k\max} = hf - \Phi$$

$$y = mx + c$$

Step 2: Identify the threshold frequency from the x-axis of the graph

$$\text{When } E_k = 0, f = f_0$$

Therefore, the threshold frequency is $f_0 = 4 \times 10^{14}$ Hz

Step 3: Calculate the work function

$$\text{From the graph at } f_0, \frac{1}{2}mv_{\max}^2 = 0$$

$$\Phi = hf_0 = (6.63 \times 10^{-34}) \times (4 \times 10^{14}) = 2.652 \times 10^{-19} \text{ J}$$

Step 4: Convert the work function into eV

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \quad \text{J} \rightarrow \text{eV: divide by } 1.6 \times 10^{-19}$$

$$E = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$





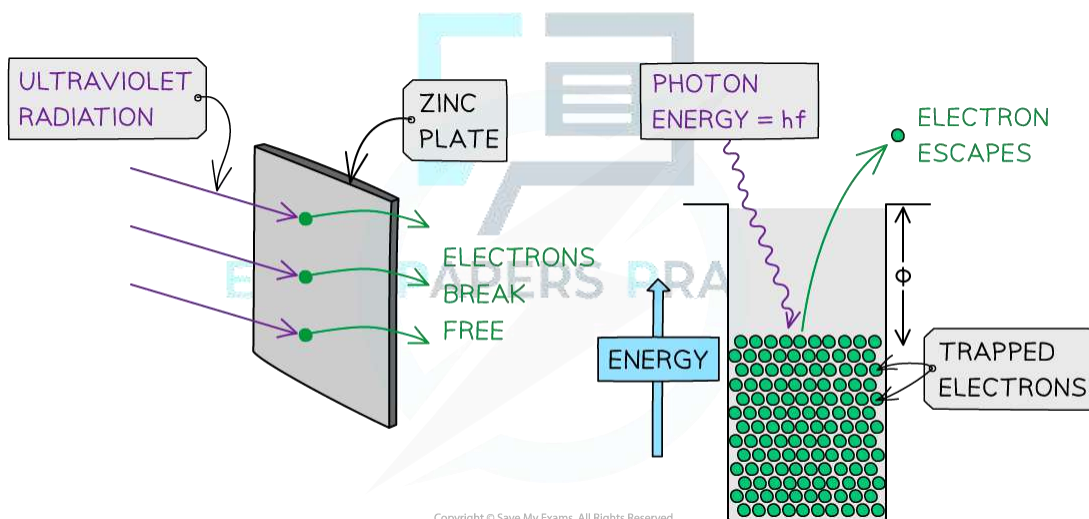
22.1.5 The Work Function

Photoelectric Emission

- The work function ϕ , or threshold energy, of a material is defined as:

The minimum energy required to release a photoelectron from the surface of a material

- Consider the electrons in a metal as trapped inside an 'energy well' where the energy between the surface and the top of the well is equal to the work function ϕ
- A single electron absorbs one photon
- Therefore, an electron can only escape the surface of the metal if it absorbs a photon which has an energy equal to ϕ or higher
- Different metals have different threshold frequencies, and hence different work functions
- Using the well analogy:
 - A more tightly bound electron requires more energy to reach the top of the well
 - A less tightly bound electron requires less energy to reach the top of the well



In the photoelectric effect, a single photon may cause a surface electron to be released if it has sufficient energy

- Alkali metals, such as sodium and potassium, have threshold frequencies in the **visible light region**
 - This is because the attractive forces between the surface electrons and positive metal ions are relatively weak
- Transition metals, such as manganese and iron, have threshold frequencies in the **ultraviolet region**
 - This is because the attractive forces between the surface electrons and positive metal ions are much stronger



Laws of Photoelectric Emission

- ♦ Observation:
 - Placing the UV light source **closer** to the metal plate causes the gold leaf to **fall more quickly**

- ♦ Explanation:
 - Placing the UV source closer to the plate increases the intensity incident on the surface of the metal
 - Increasing the intensity, or brightness, of the incident radiation increases the number of photoelectrons emitted per second
 - Therefore, the gold leaf loses negative charge more rapidly

- ♦ Observation:
 - Using a higher frequency light source **does not change** how quickly the gold leaf falls

- ♦ Explanation:
 - The maximum kinetic energy of the emitted electrons increases with the frequency of the incident radiation
 - In the case of the photoelectric effect, energy and frequency are **independent** of the intensity of the radiation
 - So, the intensity of the incident radiation affects how quickly the gold leaf falls, not the frequency

- ♦ Observation:
 - Using a filament light source causes **no change** in the gold leaf's position

- ♦ Explanation:
 - If the incident frequency is below a certain threshold frequency, no electrons are emitted, no matter the intensity of the radiation
 - A filament light source has a frequency below the threshold frequency of the metal, so, no photoelectrons are released

- ♦ Observation:
 - Using a positively charged plate causes **no change** in the gold leaf's position

- ♦ Explanation:
 - If the plate is positively charged, that means there is an excess of positive charge on the surface of the metal plate
 - Electrons are negatively charged, so they will not be emitted unless they are on the surface of the metal
 - Any electrons emitted will be attracted back by positive charges on the surface of the metal

- ♦ Observation:



- Emission of photoelectrons happens **as soon as the radiation is incident on the surface of the metal**
- ♦ Explanation:
 - A single photon interacts with a single electron
 - If the energy of the photon is equal to the work function of the metal, photoelectrons will be released instantaneously



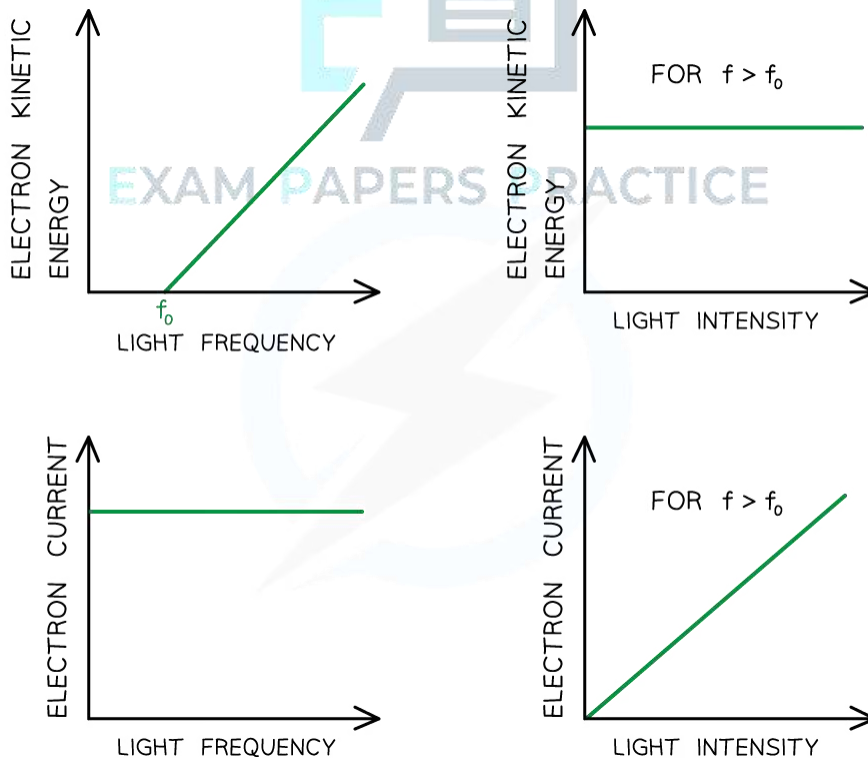


Intensity & Photoelectric Current

- The **maximum kinetic energy** of the photoelectrons is **independent of the intensity** of the incident radiation
- This is because **each electron can only absorb one photon**
- Kinetic energy is only dependent on the **frequency** of the incident radiation
- Intensity is a measure of the number of photons incident on the surface of the metal
- So, increasing the number of electrons striking the metal will not increase the kinetic energy of the electrons, it will increase the **number** of photoelectrons emitted

Photoelectric Current

- The photoelectric current is the number of photoelectrons emitted per second
- **Photoelectric current** is proportional to the **intensity** of the radiation incident on the surface of the metal
- This is because intensity is proportional to the number of photons striking the metal per second
- Since each photoelectron absorbs a single photon, the photoelectric current must be proportional to the intensity of the incident radiation



Kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity and independent of frequency



22.2 Wave-Particle Duality

22.2.1 Wave-Particle Duality

Wave-Particle Duality

- Light waves can behave like particles, i.e. photons, **and** waves
- This phenomenon is called the wave-particle nature of light or wave-particle duality
- Light interacts with matter, such as electrons, as a particle
 - The evidence for this is provided by the **photoelectric effect**
- Light propagates through space as a wave
 - The evidence for this comes from the diffraction and interference of light in **Young's Double Slit experiment**

Light as a Particle

- Einstein proposed that light can be described as a quanta of energy that behave as particles, called photons
- The photon model of light explains that:
 - Electromagnetic waves carry energy in discrete packets called photons
 - The energy of the photons are quantised according to the equation $E = hf$
 - In the photoelectric effect, each electron can absorb only a single photon – this means only the frequencies of light above the threshold frequency will emit a photoelectron
- The wave theory of light does not support a threshold frequency
 - The wave theory suggests any frequency of light can give rise to photoelectric emission if the exposure time is long enough
 - This is because the wave theory suggests the energy absorbed by each electron will increase gradually with each wave
 - Furthermore, the kinetic energy of the emitted electrons should increase with radiation intensity
 - However, in the photoelectric effect none of this is observed
- If the frequency is above the threshold and the intensity of the light is increased, more photoelectrons are emitted per second
- Although the wave theory provided good explanations for phenomena such as interference and diffraction, it failed to explain the photoelectric effect

Compare wave theory and particulate nature of light



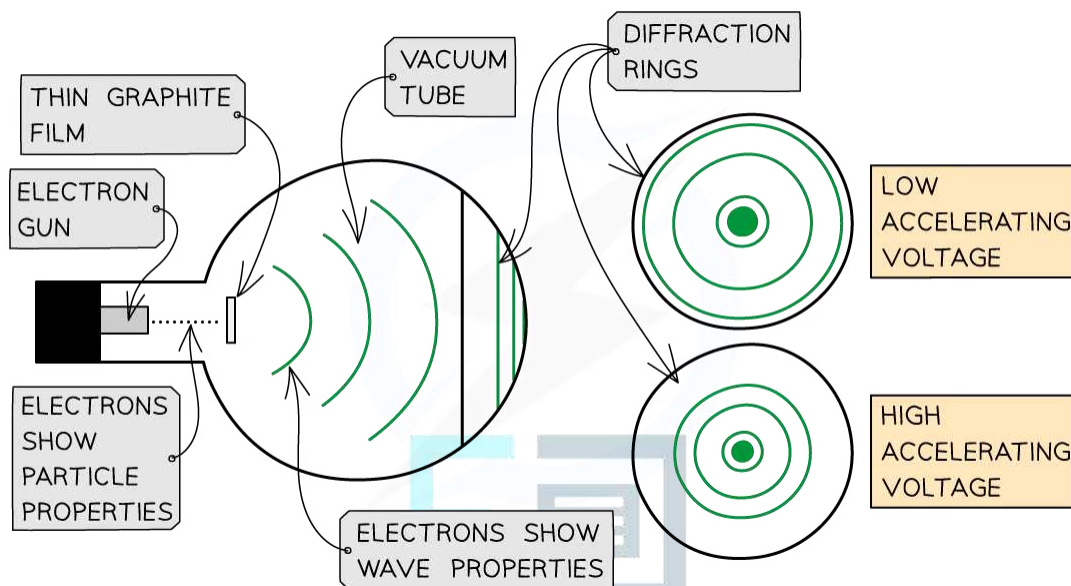
The wave theory of light suggests...	This is wrong because...
Any frequency of light can give rise to photoelectric emission if the exposure time is long enough	Photoelectrons will be released immediately if the frequency is above the threshold for that metal
The energy absorbed by each electron will increase gradually with each wave	Energy is absorbed instantaneously – photoelectrons are either emitted or not emitted after exposure to light
The kinetic energy of the emitted electrons should increase with radiation intensity	If the intensity of the light is increased, more photoelectrons are emitted per second





Wave-Particle Duality: Electron Diffraction

- ♦ Louis de Broglie discovered that matter, such as electrons, can behave as a wave
- ♦ He showed a diffraction pattern is produced when a beam of electrons is directed at a thin graphite film
- ♦ Diffraction is a property of waves, and cannot be explained by describing electrons as particles

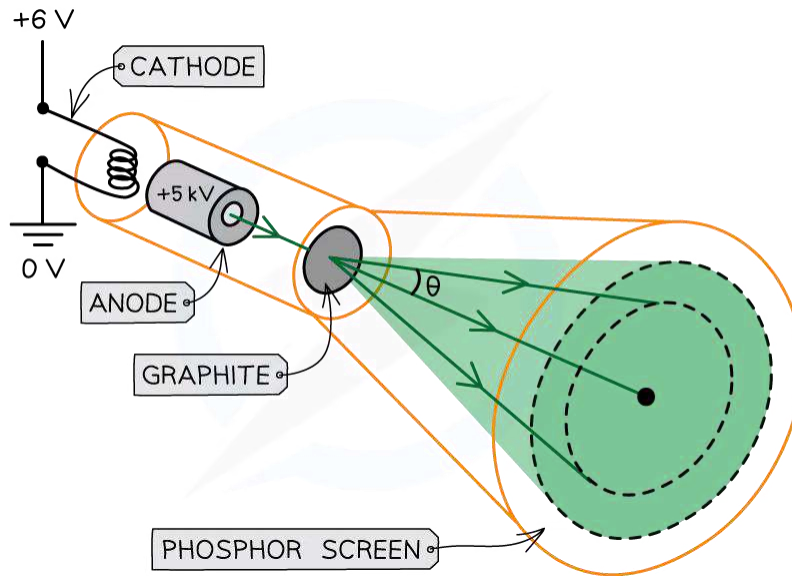


When an electron beam is focused through a crystalline structure, a diffraction pattern can be observed

- ♦ In order to observe the diffraction of electrons, they must be focused through a gap similar to their size, such as an atomic lattice
- ♦ Graphite film is ideal for this purpose because of its crystalline structure
 - The gaps between neighbouring planes of the atoms in the crystals act as slits, allowing the electron waves to spread out and create a diffraction pattern
- ♦ The diffraction pattern is observed on the screen as a series of concentric rings
 - This phenomenon is similar to the diffraction pattern produced when light passes through a diffraction grating
 - If the electrons acted as particles, a pattern would not be observed, instead the particles would be distributed uniformly across the screen
- ♦ It is observed that a larger accelerating voltage reduces the diameter of a given ring, while a lower accelerating voltage increases the diameter of the rings

Investigating Electron Diffraction

- ♦ Electron diffraction tubes can be used to investigate wave properties of electrons
- ♦ The electrons are accelerated in an electron gun to a high potential, such as 5000 V, and are then directed through a thin film of graphite
- ♦ The electrons diffract from the gaps between carbon atoms and produce a circular pattern on a fluorescent screen made from phosphor



Electrons are accelerated from the cathode (negative terminal) to the anode (positive terminal) before they are diffracted through a graphite film

- ♦ Increasing the voltage between the anode and the cathode causes the energy, and hence speed, of the electrons to increase
- ♦ The kinetic energy of the electrons is proportional to the voltage across the anode-cathode:

$$E_k = \frac{1}{2} mv^2 = eV$$



22.2.2 The de Broglie Wavelength

The de Broglie Wavelength

- ♦ De Broglie proposed that electrons travel through space as a wave
 - This would explain why they can exhibit behaviour such as diffraction
- ♦ He therefore suggested that electrons must also hold wave properties, such as wavelength
 - This became known as the de Broglie wavelength
- ♦ However, he realised **all particles** can show wave-like properties, not just electrons
- ♦ So, the de Broglie wavelength can be defined as:

The wavelength associated with a moving particle

- ♦ The majority of the time, and for everyday objects travelling at normal speeds, the de Broglie wavelength is far too small for any quantum effects to be observed
- ♦ A typical electron in a metal has a de Broglie wavelength of about 10 nm
- ♦ Therefore, quantum mechanical effects will only be observable when the width of the sample is around that value
- ♦ The electron diffraction tube can be used to investigate how the wavelength of electrons depends on their speed
 - The smaller the radius of the rings, the smaller the de Broglie wavelength of the electrons
- ♦ As the voltage is increased:
 - The energy of the electrons increases
 - The radius of the diffraction pattern decreases
- ♦ This shows as the speed of the electrons increases, the de Broglie wavelength of the electrons decreases



Calculating de Broglie Wavelength

- Using ideas based upon the quantum theory and Einstein's theory of relativity, de Broglie suggested that the momentum (p) of a particle and its associated wavelength (λ) are related by the equation:

$$\lambda = \frac{h}{p}$$

- Since momentum $p = mv$, the de Broglie wavelength can be related to the speed of a moving particle (v) by the equation:

$$\lambda = \frac{h}{mv}$$

- Since kinetic energy $E = \frac{1}{2} mv^2$
- Momentum and kinetic energy can be related by:

$$E = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mE}$$

- Combining this with the de Broglie equation gives a form which relates the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

- Where:
 - λ = the de Broglie wavelength (m)
 - h = Planck's constant (J s)
 - p = momentum of the particle (kg m s^{-1})
 - E = kinetic energy of the particle (J)
 - m = mass of the particle (kg)
 - v = speed of the particle (m s^{-1})



Worked Example

A proton and an electron are each accelerated from rest through the same potential difference.

Determine the ratio: $\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}}$

- Mass of a proton = 1.67×10^{-27} kg
- Mass of an electron = 9.11×10^{-31} kg



Step 1: Consider how the proton and electron can be related via their masses

The proton and electron are accelerated through the same p.d., therefore, they both have the same kinetic energy

Step 2: Write the equation relating the de Broglie wavelength of a particle to its kinetic energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Step 3: Calculate the ratio

$$\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}} = \frac{1}{\sqrt{m_p}} \div \frac{1}{\sqrt{m_e}}$$

$$\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2.3 \times 10^{-2}$$

This means the de Broglie wavelength of the proton is 0.023 times smaller than that of the electron **OR** the de Broglie wavelength of the electron is about 40 times larger than that of the proton

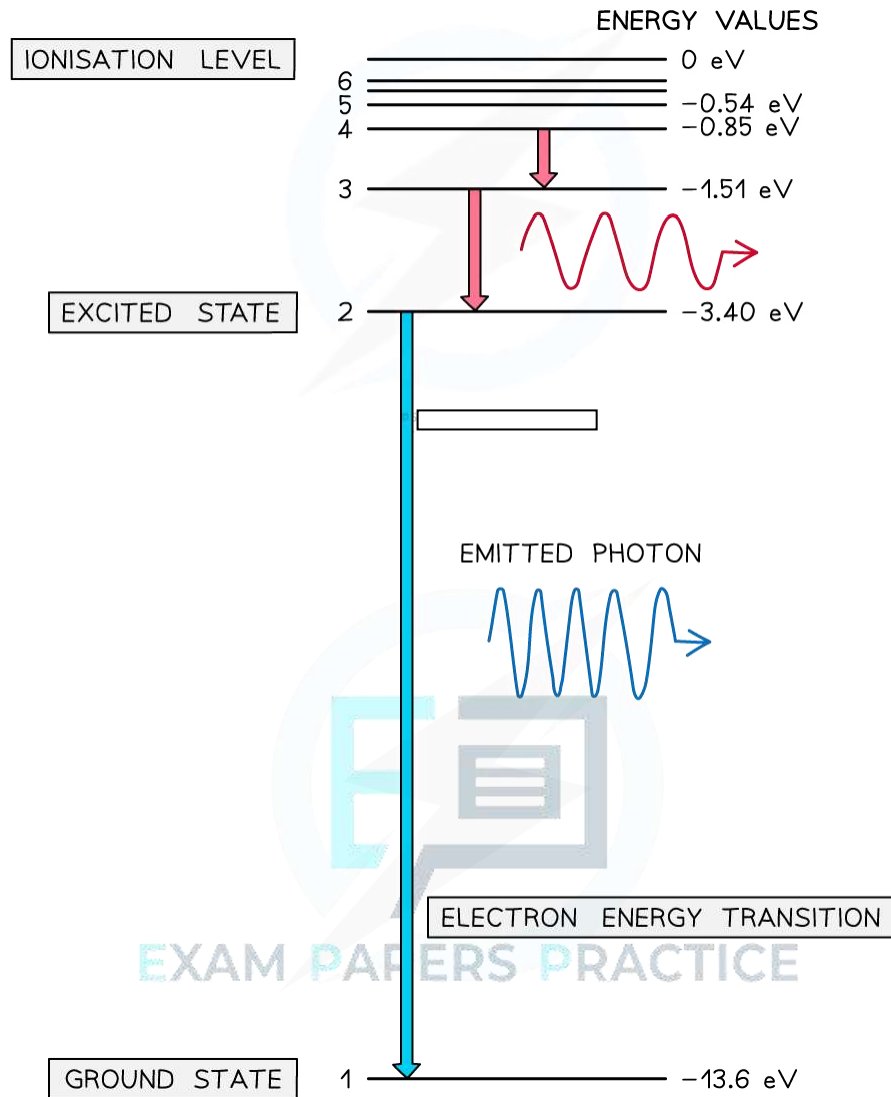


22.3 Quantisation of Energy

22.3.1 Atomic Energy Levels

Atomic Energy Levels

- Electrons in an atom can have only certain specific energies
 - These energies are called **electron energy levels**
- They can be represented as a series of stacked horizontal lines increasing in energy
- Normally, electrons occupy the lowest energy level available, this is known as the **ground state**
- Electrons can gain energy and move up the energy levels if it absorbs energy either by:
 - Collisions with other atoms or electrons
 - Absorbing a photon
 - A physical source, such as heat
- This is known as **excitation**, and when electrons move up an energy level, they are said to be in an **excited state**
- If the electron gains enough energy to be removed from the atom entirely, this is known as **ionisation**
- When an electron returns to a lower energy state from a higher excited state, it releases energy in the form of a photon



Electron energy levels in atomic hydrogen. Photons are emitted when an electron moves from a higher energy state to a lower energy state



22.3.2 Line Spectra

Line Spectra

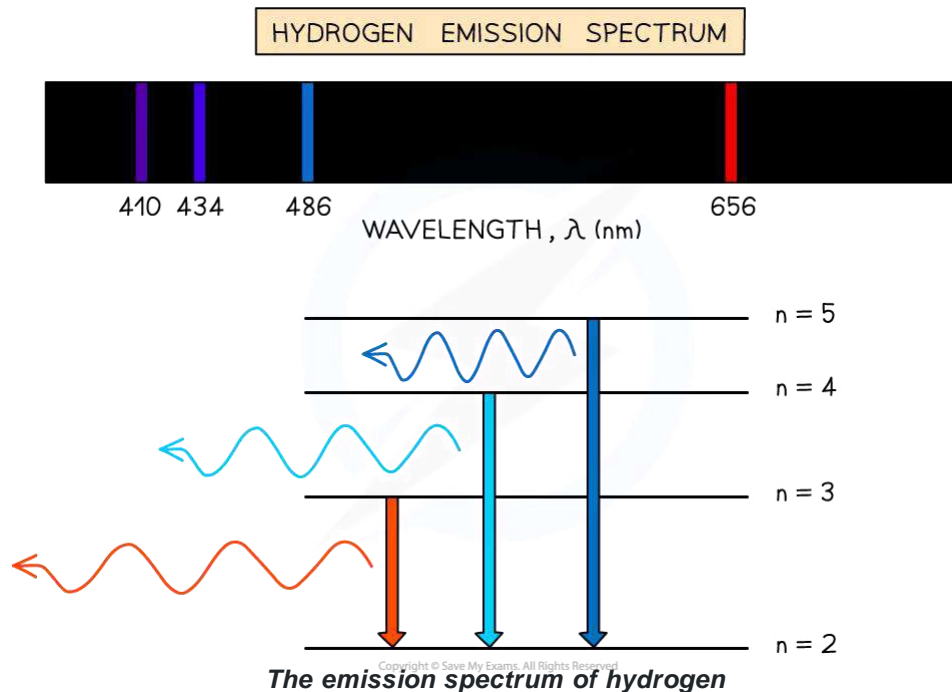
- Line spectra is a phenomenon which occurs when excited atoms emit light of certain wavelengths which correspond to different colours
- The emitted light can be observed as a series of coloured lines with dark spaces in between
 - These series of coloured lines are called **line or atomic spectra**
- Each element produces a unique set of spectral lines
- No two elements emit the same set of spectral lines, therefore, elements can be identified by their line spectrum
- There are two types of line spectra: **emission spectra** and **absorption spectra**

Emission Spectra

- When an electron transitions from a higher energy level to a lower energy level, this results in the **emission** of a photon
- Each transition corresponds to a different wavelength of light and this corresponds to a line in the spectrum
- The resulting emission spectrum contains a set of discrete wavelengths, represented by coloured lines on a black background
- Each emitted photon has a wavelength which is associated with a discrete change in energy, according to the equation:

$$\Delta E = hf = \frac{hc}{\lambda}$$

- Where:
 - ΔE = change in energy level (J)
 - h = Planck's constant (J s)
 - f = frequency of photon (Hz)
 - c = the speed of light (m s^{-1})
 - λ = wavelength of the photon (m)
- Therefore, this is evidence to show that electrons in atoms can only transition between **discrete energy levels**

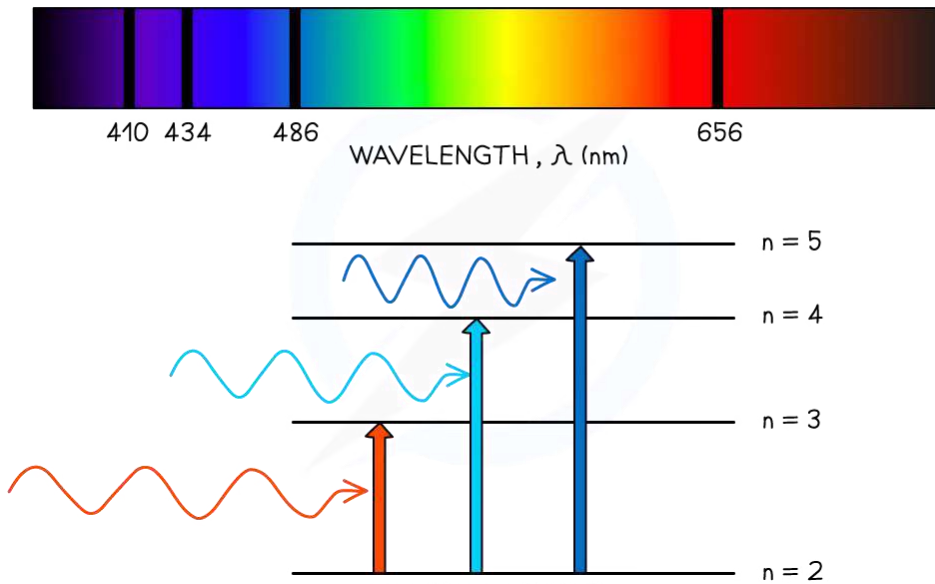


Absorption Spectra

- ♦ An atom can be raised to an excited state by the absorption of a photon
- ♦ When white light passes through a **cool, low pressure gas** it is found that light of certain wavelengths are missing
 - This type of spectrum is called an absorption spectrum
- ♦ An absorption spectrum consists of a continuous spectrum containing all the colours with dark lines at certain wavelengths
- ♦ These dark lines correspond exactly to the differences in energy levels in an atom
- ♦ When these electrons return to lower levels, the photons are emitted in all directions, rather than in the original direction of the white light
 - Therefore, some wavelengths appear to be missing
- ♦ The wavelengths missing from an absorption spectrum are the same as their corresponding emission spectra of the same element



HYDROGEN ABSORPTION SPECTRUM



The absorption spectrum of hydroge





22.3.3 Calculating Discrete Energies

Calculating Discrete Energies

- The difference between two energy levels is equal to a specific photon energy
- The energy (hf) of the photon is given by:

$$\Delta E = hf = E_2 - E_1$$

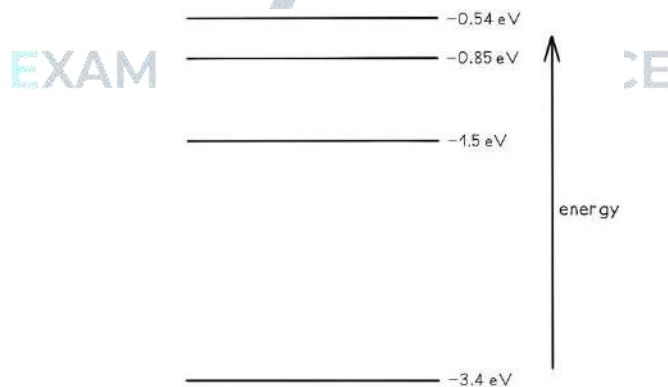
- Where,
 - E_1 = Energy of the higher level (J)
 - E_2 = Energy of the lower level (J)
 - h = Planck's constant (J s)
 - f = Frequency of photon (Hz)
- Using the wave equation, the wavelength of the emitted, or absorbed, radiation can be related to the energy difference by the equation:

$$\lambda = \frac{hc}{E_2 - E_1}$$

- This equation shows that the larger the difference in energy of two levels ΔE , the shorter the wavelength λ and vice versa

**Worked Example**

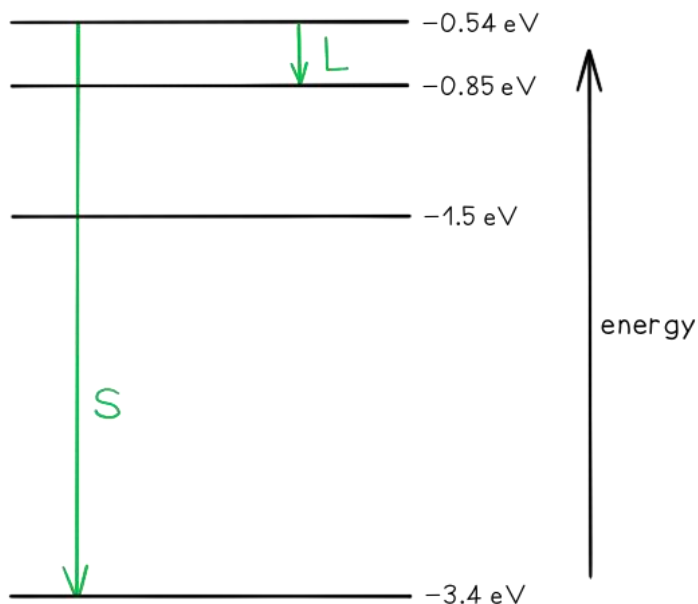
Some electron energy levels in atomic hydrogen are shown below.



The longest wavelength produced as a result of electron transitions between two of the energy levels is 4.0×10^{-6} m. a) Draw and mark:

- The transition giving rise to the wavelength of 4.0×10^{-6} m with letter **L**.
- The transition giving rise to the shortest wavelength with letter **S**.

b) Calculate the wavelength for the transition giving rise to the shortest wavelength.

**Part (a)**

Photon energy and wavelength are inversely proportional, so the largest energy change corresponds to the shortest wavelength (**line S**) and the smallest energy change corresponds to the longest wavelength (**line L**)

Part (b)

Step 1: Write down the equation linking the wavelength and the energy levels

$$\lambda = \frac{hc}{E_2 - E_1}$$

Step 2: Identify the energy levels giving rise to the shortest wavelength

$$E_1 = 0.54 \text{ eV}$$

$$E_2 = 3.4 \text{ eV}$$

Step 3: Calculate the wavelength

To convert from eV \rightarrow J: **multiply** by 1.6×10^{-19}

$$\lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{(3.4 - 0.54)(1.6 \times 10^{-19})} = 4.347 \times 10^{-7} \text{ m} = \mathbf{435 \text{ nm}}$$