EXAM PAPERS PRACTICE

A Level Physics CIE

21. Alternating Currents

CONTENTS

21.1 Properties and Uses of Alternating Current

21.1.1 Alternating Current & Voltage

Properties of Alternating Current & Voltage

- An alternating current (a.c) is defined as:

  A current which periodically varies from positive to negative and changes its magnitude continuously with time

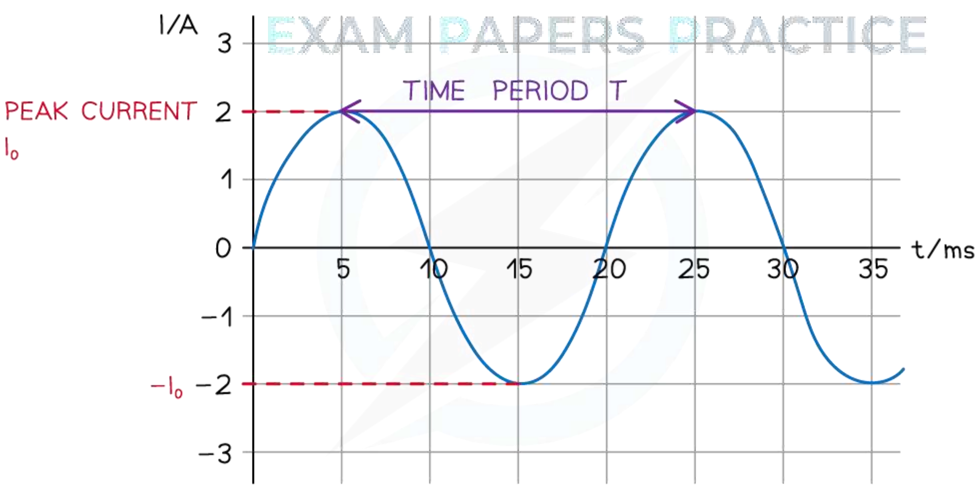
- This means the direction of an alternating current varies every **half cycle**
- The variation of current, or p.d., with time can be described as a sine curve ie. **sinusoidal**
  - Therefore, the electrons in a wire carrying a.c. move back and forth with simple harmonic motion
- As with SHM, the relationship between time period T and frequency f of an alternating current is given by:

$$T = \frac{1}{f}$$

- Peak current (I_0), or peak voltage (V_0), is defined as:

  The maximum value of the alternating current or voltage

- Peak current, or voltage, can be determined from the **amplitude** of the graph

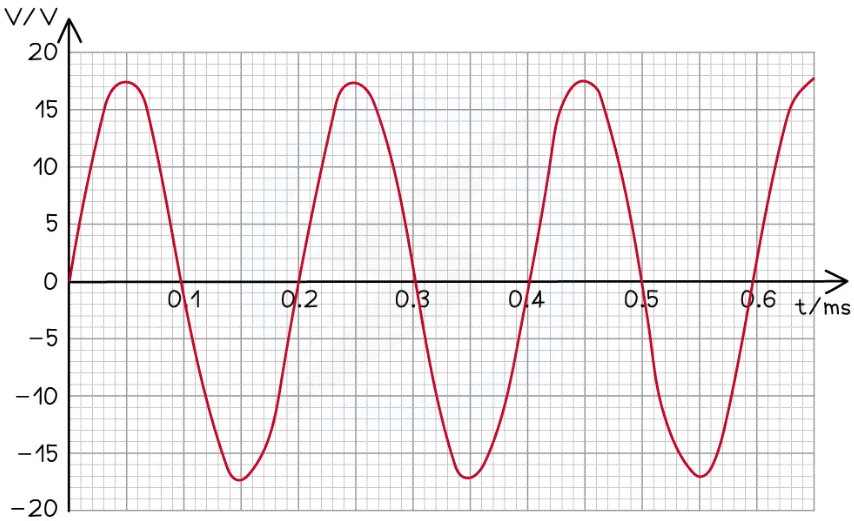


Graph of alternating current against time with a time period of 20 ms and peak current of 2 A

- Mains electricity is supplied as alternating current
  - Power stations produce alternating current
  - This is the type of current supplied when devices are plugged into sockets

Worked Example

The variation with time *t* of the output voltage *V* of an alternating voltage supply is shown in the graph below.



Use the graph to calculate the frequency of the supply.

Step 1:

Write down the period-frequency relation

$$f = \frac{1}{T}$$

Step 2:

Calculate the time period from the graph

- The time period is the time taken for one complete cycle
- From the graph, this is equal to 0.2 ms
- **Therefore,** the time period, T = 0.2 ms = $\mathbf{0.2 \times 10^{-3}}$ **s**

Step 3:

Substitute into frequency equation

$$f = \frac{1}{0.2 \times 10^{-3}} = 5000 \text{ Hz} = \mathbf{5 \text{ kHz}}$$

Exam Tip

Remember to double check the units on the alternating current and voltage graphs. These are often shown in the range of milli-seconds (ms) instead of seconds (s) on the x axis.

Using Sinusoidal Representations

- The equation representing alternating current which gives the value of the current I at any time t is:

$$I = I_0 \sin(\omega t)$$

- Where:
  - I = current (A)
  - I_0 = peak current (A)
  - ω = angular frequency of the supply (rad s^{-1})
  - t = time (s)
- **Note:** this a sine function since the alternative current graph is sinusoidal
- A similar equation can be used for representing alternating voltage:

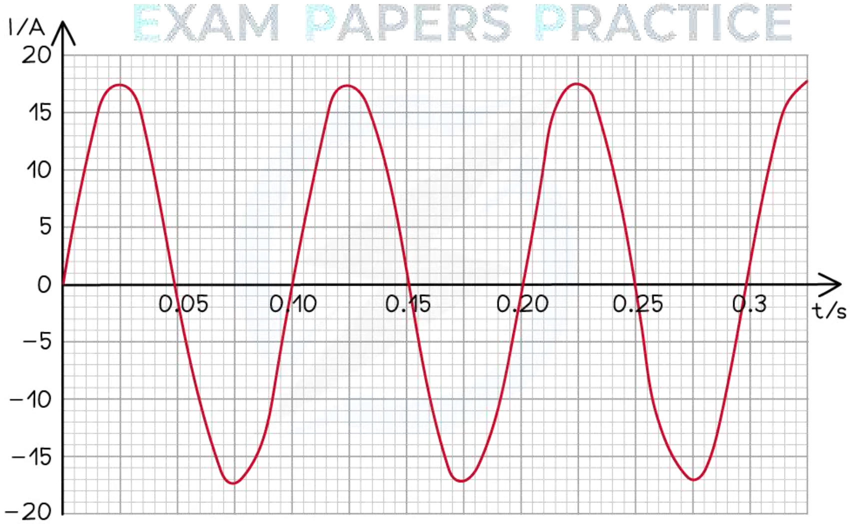
$$V = V_0 \sin(\omega t)$$

- Where:
  - V = voltage (V)
  - V_0 = peak voltage (V)
- Recall the relation the equation for angular frequency ω:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Worked Example

An alternating current I varies with time t as shown in the graph below.



Using the graph and the equation for alternating current, calculate the value of the current at a time 0.48 s.

Step 1: Write out the equation for alternating current

$$I = I_0 \sin(\omega t)$$

Step 2: Write out the equation for angular frequency

$$\omega = \frac{2\pi}{T}$$

Step 3: Measure the time period T and peak current I_0 from the graph

The time period is the time taken for one full cycle, T = 0.10 s

Peak current (amplitude), I_0 = 17 A

Step 4: Substitute values into alternating current equation at time t

Using the time given in the question, t = 0.48 s

$$I = I_0 \sin(\omega t) = I_0 \sin\left(\frac{2\pi t}{T}\right)$$

$$I = 17 \sin\left(\frac{2\pi(0.48)}{0.1}\right) = -16.168 = \mathbf{-16\ A} \text{ (2 s.f)}$$

Exam Tip

Remember to check that your calculator is in **radians** mode when using any of these equations. This is because angular frequency ω is measured in rad s^{-1}

Root-Mean-Square Current & Voltage

- Root-mean-square (r.m.s) values of current, or voltage, are a useful way of **comparing** a.c current, or voltage, to its equivalent direct current, or voltage
- The r.m.s values represent the d.c current, or voltage, values that will produce the same **heating effect**, or power dissipation, as the alternating current, or voltage
- The r.m.s value of an alternating current is defined as:

The value of a constant current that produces the same power in a resistor as the alternating current

- The r.m.s current $I_{r.m.s}$ is defined by the equation:

$$I_{r.m.s} = \frac{I_0}{\sqrt{2}}$$

- So, r.m.s current is equal to $0.707 \times I_0$, which is about 70% of the peak current I_0
- The r.m.s value of an alternating voltage is defined as:

The value of a constant voltage that produces the same power in a resistor as the alternating voltage

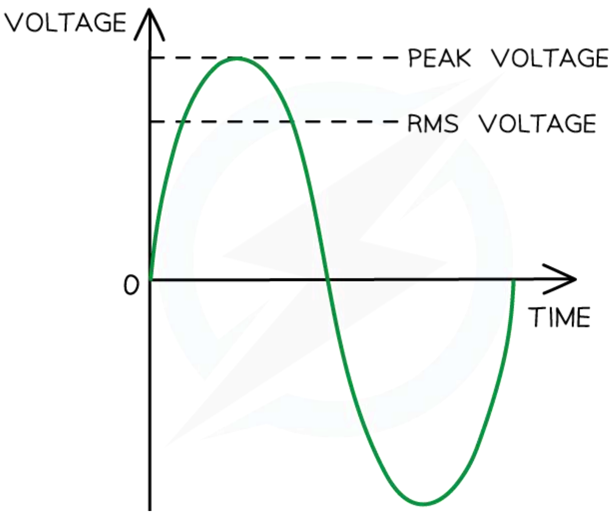
- The r.m.s voltage $V_{r.m.s}$ is defined by the equation:

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$$

- Where:
  - I_0 = peak current (A)
  - V_0 = peak voltage (V)
- The r.m.s value is therefore defined as:

The steady direct current, or voltage, that delivers the same average power in a resistor as the alternating current, or voltage

- A resistive load is any electrical component with resistance eg. a lamp



$V_{r.m.s}$ and peak voltage. The r.m.s voltage is about 70% of the peak voltage

Worked Example

An alternating current is I is represented by the equation

I = 410 sin(100πt)

where I is measured in amperes and t is in seconds.For this alternating current, determine the r.m.s current.

Step 1: Write out the equation for r.m.s current

$$I_{r.m.s} = \frac{I_0}{\sqrt{2}}$$

Step 2: Determine the peak voltage I_0

- The alternating current equation is in the form: **I = I₀ sin(ωt)**
- Comparing this to I = 410 sin(100πt) means the peak current is **I_0 = 410 A**

Step 3: Substitute into the $I_{r.m.s}$ equation

$$I_{r.m.s} = \frac{410}{\sqrt{2}} = 289.91 = \mathbf{290\ A}\ (2\ s.f.)$$

21.1.3 Mean Power

Mean Power

- In mains electricity, current and voltage are varying all the time
- This also means the **power** varies constantly, recall the equations for power:

$$P = IV = I^2R = \frac{V^2}{R}$$

- Where:
  - I = **direct** current (A)
  - V = **direct** voltage (A)
  - R = resistance (Ω)

- The r.m.s values means equations used for direct current and voltage can now be applied to alternating current and voltage
- These are also used to determine an **average** current or voltage for alternating supplies
- Recall the equation for peak current:

$$I_0 = \sqrt{2} I_{r.m.s}$$

- Therefore, the peak (maximum) power is related to the mean (average) power by:

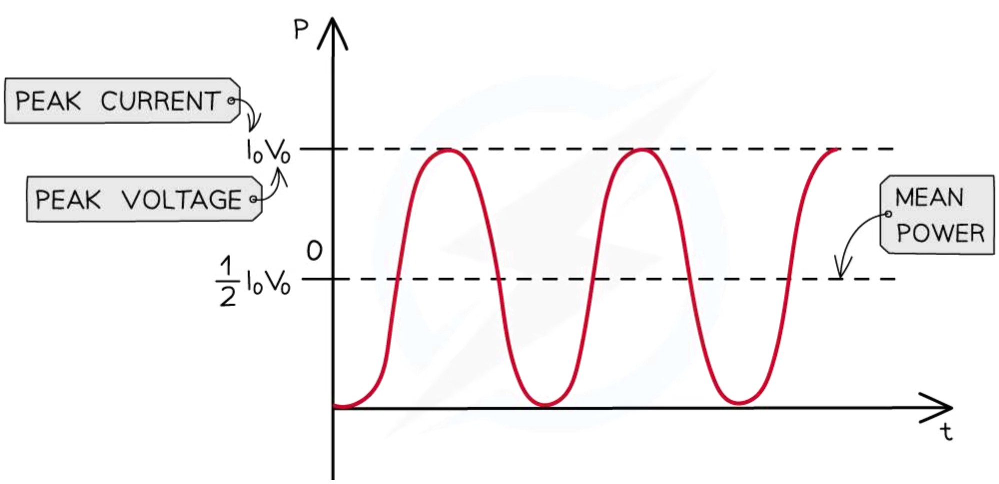
$$P_{mean} = I_{r.m.s}R$$

$$P_{peak} = I_0^2R = (\sqrt{2} I_{r.m.s})^2R = 2(I_{r.m.s})^2R = 2P_{mean}$$

$$P_{mean} = \frac{P_{peak}}{2}$$

- Therefore, it can be concluded that:

The mean power in a resistive load is half the maximum power for a sinusoidal alternating current or voltage



Mean power is exactly half the maximum power

Worked Example

An alternating voltage supplied across a resistor of 40 Ω has a peak voltage V_0 of 240 V.Calculate the mean power of this supply.

Step 1: Write down the known quantities

Resistance, R = 40 Ω

Peak voltage, V_0 = 240 V

Step 2: Write out the equation for the peak power and calculate

$$\text{Peak power, P} = \frac{V_0^{\,2}}{R}$$

$$\text{P} = \frac{(240)^2}{40} = 1440 \text{ W}$$

Step 3: Calculate the mean power

- The mean power is **half** of the maximum (peak) power

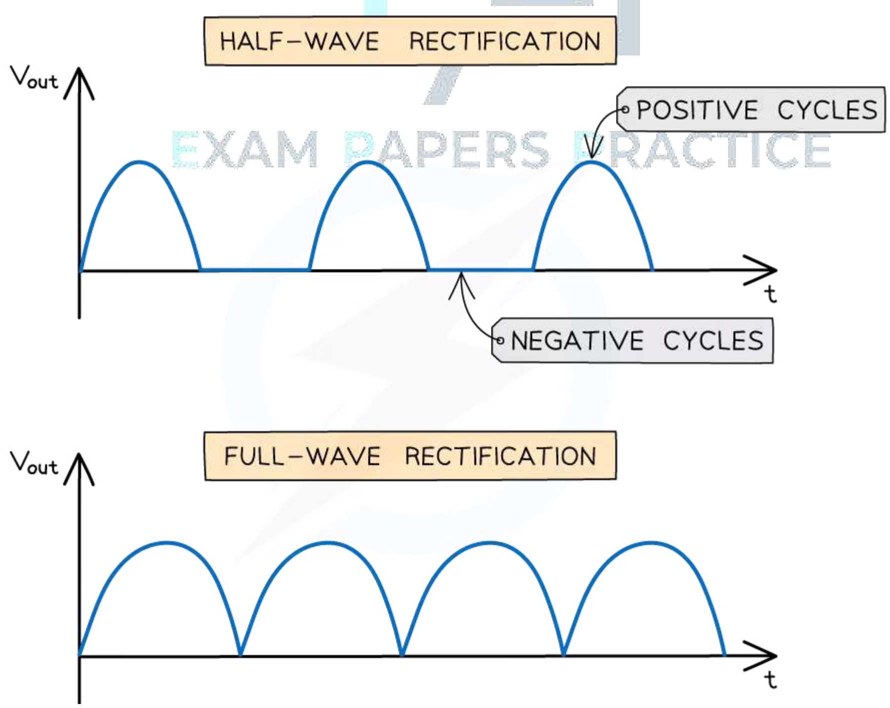
Mean power = 1440 / 2 = 720 W

Rectification Graphs

- Rectification is defined as:

  The process of converting alternating current and voltage into direct current and voltage

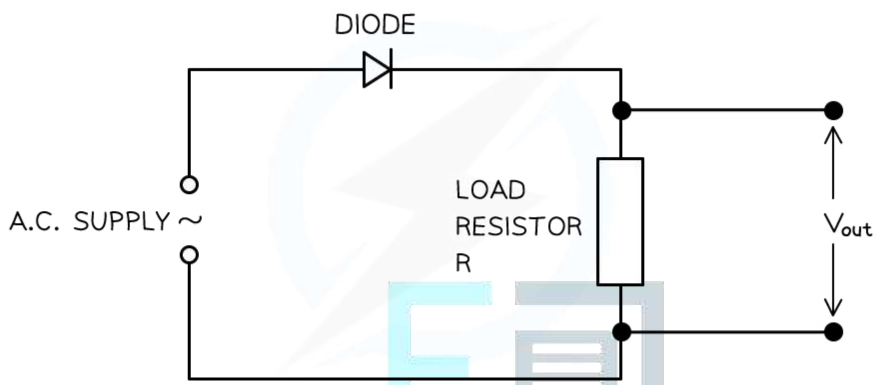
- Rectification is used in electronic equipment which requires a direct current
  - For example, mains voltage must be rectified from the alternating voltage produced at power stations
- There are two types of rectification:
  - Half-wave rectification
  - Full-wave rectification
- For **half-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve with the positive cycles and a flat line (V_{out} = 0) on the negative cycle
  - This is because the diode only conducts in the positive direction
- For **full-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve where the positive cycles and the negative cycles are both curved 'bumps'



The difference between the graphs of full-wave and half-wave rectification

Half-Wave Rectification

- Half-wave rectification consists of a single **diode**
  - An alternating input voltage is connected to a circuit with a load resistor and diode in series
- The diode will only conduct during the positive cycles of the input alternating voltage,
  - Hence there is only current in the load resistor during these positive cycles
- The output voltage V_{out} across the resistor will fluctuate against against time in the same way as the input alternating voltage **except** there are no negative cycles

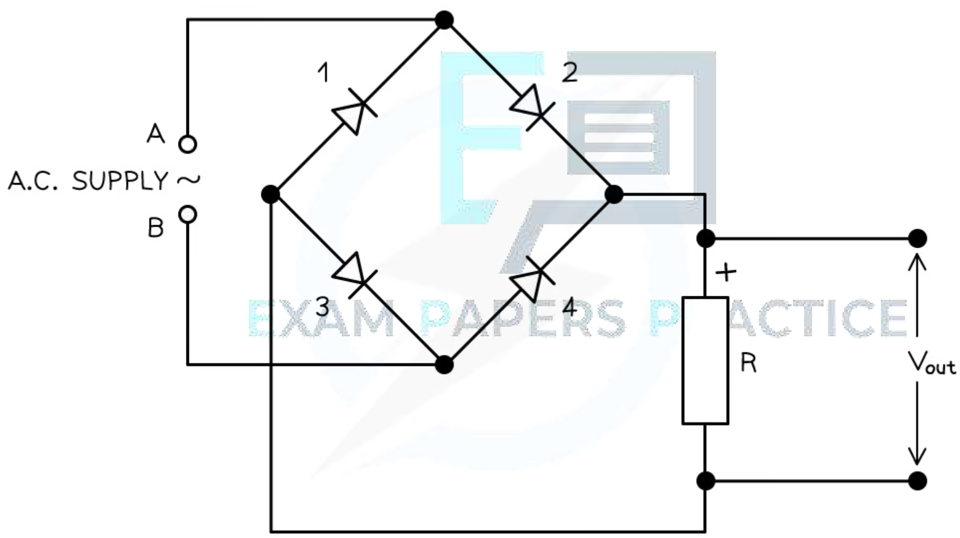


Half-wave rectification requires a single diode and the graph is represented by only the positive cycles

- This type of rectification means half of the time the voltage is zero
- So, the power available from a half-wave rectified supply is reduced

Full-Wave Rectification

- Full-wave rectification requires a bridge rectifier circuit
  - This consists of **four** diodes connected across an input alternating voltage supply
- The output voltage V_{out} is taken across a load resistor
- During the **positive** cycles of the input voltage, one terminal if the voltage supply is positive and the other negative
  - Two diodes opposite each other that are in forward bias will conduct
  - The other two in reverse bias will not conduct
  - A current will flow in the load resistor with the positive terminal at the top of the resistor
- During the **negative** cycles of the input voltage, the positive and negative terminals of the input alternating voltage supply will swap
  - The two diodes that were forward bias will now be in reverse bias and not conduct
  - The other two in reverse bias will now be in forward bias and will conduct
  - The current in the load resistor will still flow in the same direction as before

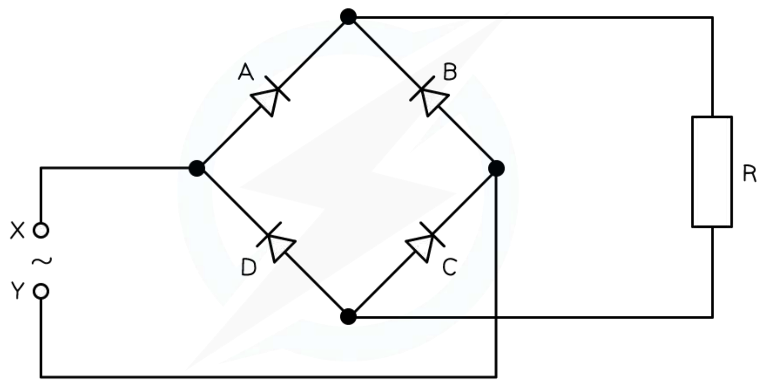


When A is positive and B is negative, diodes 2 and 3 will conduct and 1 and 4 will not. When A is negative and B is positive, diodes 1 and 4 will conduct and diodes 2 and 3 will not. The current in the load resistor R will flow downwards

- In both the positive and negative cycles, the current in the load resistor is the **same**
- Each diode pair is the same as in half-wave rectification
  - Since there are two pairs, this equates to full-wave rectification overall
- The main advantage of full-wave rectification compared to half-wave rectification is that there is **more power** available
  - Therefore, a greater power is supplied on every half cycle

Worked Example

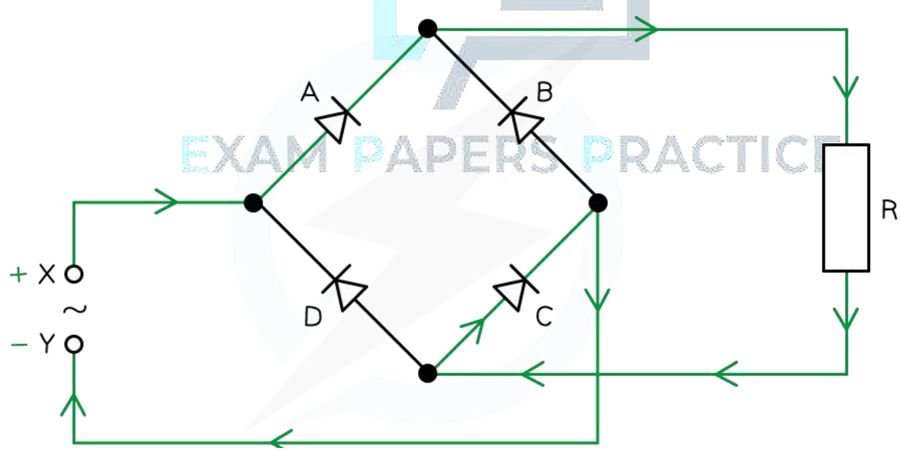
A bridge rectifier consists of four ideal diodes A, B, C and D as connected in the figure shown below



An alternating supply is applied between the terminal X and Y

State which diodes are conducting when terminal X of the supply is positive

- Draw path of the current direction with diodes in **forward bias**
- Remember that conventional current flow is from **positive to negative** and only travels through the paths with diodes in **forward bias**



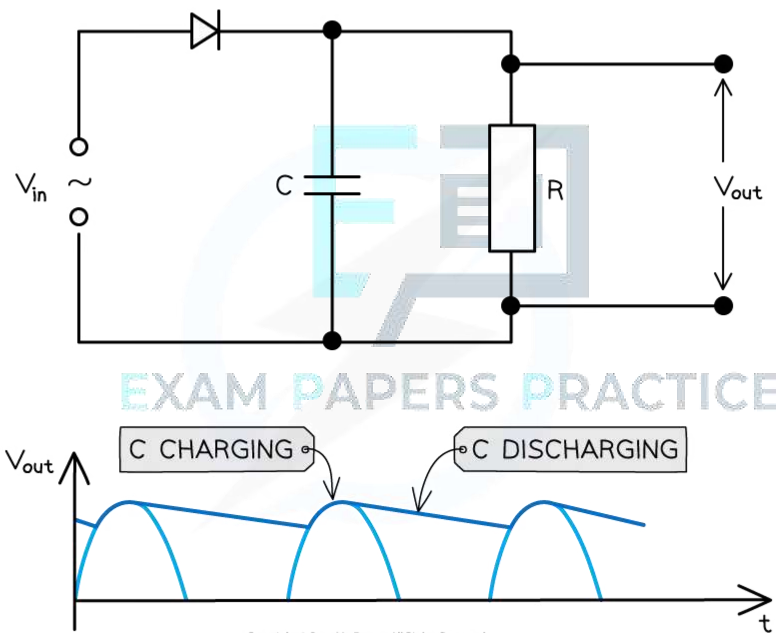
Therefore, the answer is: diodes A and C

Smoothing

- In rectification, to produce a steady direct current or voltage from an alternating current or voltage, a **smoothing capacitor** is necessary
- Smoothing is defined as:

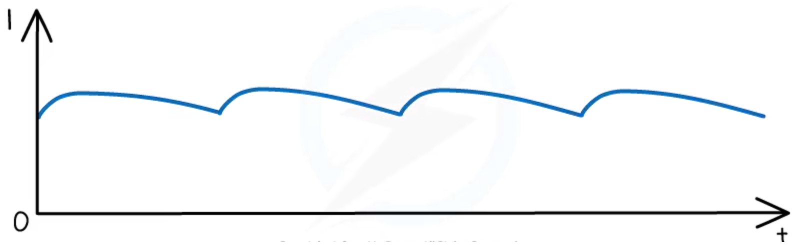
The reduction in the variation of the output voltage or current

- This works in the following ways:
  - A single capacitor with capacitance C is connected in parallel with a load resistor of resistance R
  - The capacitor charges up from the input voltage and maintains the voltage at a high level
  - As it discharges gradually through the resistor when the rectified voltage drops but the voltage then rises again and the capacitor charges up again



A smoothing capacitor connected in parallel with the load resistor. The capacitor charges as the output voltage increases and discharges as it decreases

- The resulting graph of a smoothed output voltage V_{out} and output current against time is a 'ripple' shape

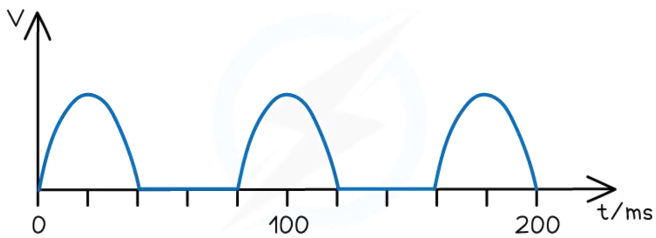


A smooth, rectified current graph creates a 'rippling' shape against time

- The amount of smoothing is controlled by the capacitance C of the capacitor and the resistance R of the load resistor
  - The less the rippling effect, the smoother the rectified current and voltage output
- The slower the capacitor discharges, the more the smoothing that occurs ie. smaller ripples
- This can be achieved by using:
  - A capacitor with **greater** capacitance *C*
  - A resistance with larger resistor *R*
- Recall that the product RC is the **time constant** τ of a resistor
- This means that the time constant of the capacitor must be **greater than the time interval** between the adjacent peaks of the output signal

Worked Example

The graph below shows the output voltage from a half-wave rectifier. The load resistor has a resistance of 2.6 kΩ. A student wishes to smooth the output voltage by placing a capacitor in parallel across the load resistor



Suggest if a capacitor of 60 pF or 800 μF would be suitable for this task

Step 1:

Calculate the time constant with the 60 pF capacitor

$$\tau = RC = (2.6 \times 10^3) \times (60 \times 10^{-12}) = 1.56 \times 10^{-7} \text{ s} = 156 \text{ ns}$$

Step 2:

Compare time constant of 60 pF capacitor with interval between adjacent peaks of the output signal

- The time interval between adjacent peaks is 80 ms
- The time constant of 156 ns is too small and the 60 pF capacitor will discharge far too quickly
- There would be no smoothing of the output voltages
- Therefore, the 60 pF capacitor is **not suitable**

Step 3:

Calculate the time constant with the 800 μF capacitor

$$\tau = RC = (2.6 \times 10^3) \times (800 \times 10^{-6}) = 2.08 \text{ s}$$

Step 4:

Compare time constant of 60 pF capacitor with interval between adjacent peaks of the output signal

- The time constant of 2.08 s is much larger than 80 ms
- The capacitor will not discharge completely between the positive cycles of the half-wave rectified signal
- Therefore, the **800 μF capacitor** would be suitable for the smoothing task