Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

9018665848

FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2017**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **23** printed pages and **1** blank page.

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1 A particle P is moving in a circle of radius 0.8 m. At time t s its velocity is $(8 - pt + t^2)$ m s^{-1}, where p is a constant. The magnitude of the transverse component of the acceleration of P when $t = 2$ is zero. Find the magnitude of the radial component of the acceleration of P when $t = 2$. [4]

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2 The piston in a large engine rises and falls in simple harmonic motion. When the piston is 1.6 m below its highest level, the rate of change of its height is $\frac{3}{5}\pi$ metres per second. When the piston is 0.2 m below its highest level, the rate of change of its height is $\frac{1}{4}\pi$ metres per second. Find the amplitude and period of the motion. [7]

3 Three uniform small smooth spheres A, B and C have equal radii and masses m, km and m respectively, where k is a constant. The spheres are moving in the same direction along a straight line on a smooth horizontal surface, with B between A and C. The speeds of A, B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $\frac{1}{2}$. After sphere A has collided with sphere B, sphere B collides with sphere C.

(i) Find an inequality satisfied by k. [5]

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(ii) Given that $k = 2$, show that after B has collided with C there are no further collisions between any of the three spheres. [5]

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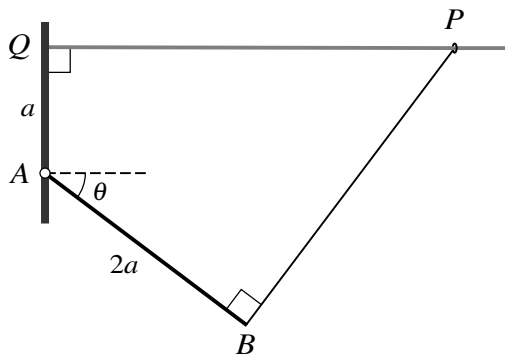

A small ring P of weight W is free to slide on a rough horizontal wire, one end of which is attached to a vertical wall at Q. The end A of a thin uniform rod AB of length $2a$ and weight $\frac{5}{2}W$ is freely hinged to the wall at the point A which is a distance a vertically below Q. A light elastic string of natural length $2a$ has one end attached to the ring P and the other end attached to the rod at B. The string is at right angles to the rod and A, B, P and Q lie in a vertical plane. The system is in limiting equilibrium with AB making an angle θ with the horizontal, where $\sin\theta = \frac{3}{5}$ (see diagram).

(i) Find the tension in the string in terms of W. [2]

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(ii) Find the coefficient of friction between the ring and the wire. [2]

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(iii) Find the magnitude of the resultant force on the rod at the hinge in terms of W. [3]

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(iv) Find the modulus of elasticity of the string in terms of W. [3]

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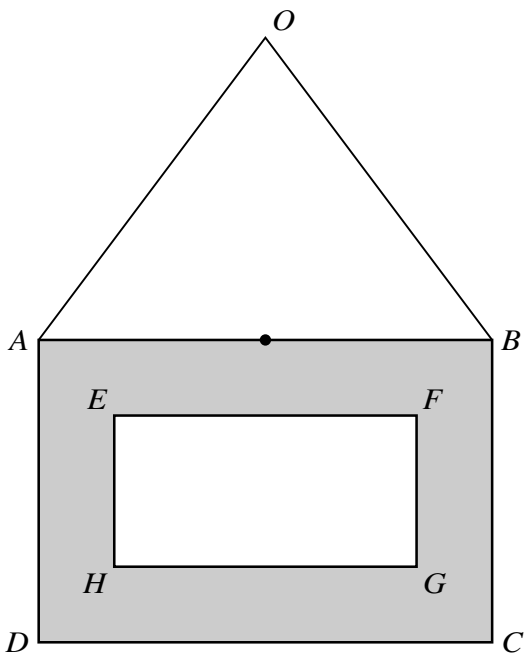

A uniform picture frame of mass m is made by removing a rectangular lamina $EFGH$ in which $EF = 4a$ and $FG = 2a$ from a larger rectangular lamina $ABCD$ in which $AB = 6a$ and $BC = 4a$. The side EF is parallel to the side AB. The point of intersection of the diagonals AC and BD coincides with the point of intersection of the diagonals EG and FH. One end of a light inextensible string of length $10a$ is attached to A and the other end is attached to B. The frame is suspended from the mid-point O of the string. A small object of mass $\frac{11}{12}m$ is fixed to the mid-point of AB (see diagram).

(i) Show that the moment of inertia of the system, consisting of frame and small object, about an axis through O perpendicular to the plane of the frame, is $\frac{169}{3}ma^2$. [7]

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(ii) Show that small oscillations of the system about this axis are approximately simple harmonic and state their period. [5]

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6 A pair of fair dice is thrown repeatedly until a pair of sixes is obtained. The number of throws taken is denoted by the random variable X.

(i) Find the mean value of X. [2]

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(ii) Find the probability that exactly 12 throws are required to obtain a pair of sixes. [2]

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(iii) Find the probability that more than 12 throws are required to obtain a pair of sixes. [2]

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7 The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} 0.2\mathrm{e}^{-0.2x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [2]

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(ii) Find $\mathrm{P}(X > 2)$. [2]

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(iii) Find the median of X. [3]

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8 Members of a Statistics club are voting to elect a new president of the club. Members must choose to vote either by post or by text or by email. The method of voting chosen by a random sample of 60 male members and 40 female members is given in the following table.

		Method of voting		
		Post	Text	Email
Gender	Male	10	12	38
	Female	5	21	14

Test, at the 1% significance level, whether there is an association between method of voting and gender. [8]

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9 The land areas x (in suitable units) and populations y (in millions) for a sample of 8 randomly chosen cities are given in the following table.

Land area (x)	1.0	4.5	2.4	1.6	3.8	8.6	7.5	6.5
Population (y)	0.8	8.4	4.2	1.6	2.2	10.2	4.2	5.2

[$\Sigma x = 35.9$, $\Sigma x^2 = 216.47$, $\Sigma y = 36.8$, $\Sigma y^2 = 244.96$, $\Sigma xy = 212.62$.]

(i) Find, showing all necessary working, the value of the product moment correlation coefficient for this sample. [3]

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(ii) Using a 1% significance level, test whether there is positive correlation between land area and population of cities. [4]

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The land areas and populations for another randomly chosen sample of cities, this time of size n, give a product moment correlation coefficient of 0.651. Using a test at the 1% significance level, there is evidence of non-zero correlation between the variables.

(iii) Find the least possible value of n, justifying your answer. [2]

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10 A factory produces bottles of an energy juice. Two different machines are used to fill empty bottles with the juice. The manager chooses a random sample of 50 bottles filled by machine X and a random sample of 60 bottles filled by machine Y. The volumes of juice, x and y respectively, measured in appropriate units, are summarised by

$$\Sigma x = 45.5, \quad \Sigma(x - \bar{x})^2 = 19.56, \quad \Sigma y = 72.3, \quad \Sigma(y - \bar{y})^2 = 30.25,$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the volume of juice in the bottles filled by X and Y respectively.

(i) Find a 90% confidence interval for the difference between the mean volume of juice in bottles filled by machine X and the mean volume of juice in bottles filled by machine Y. [7]

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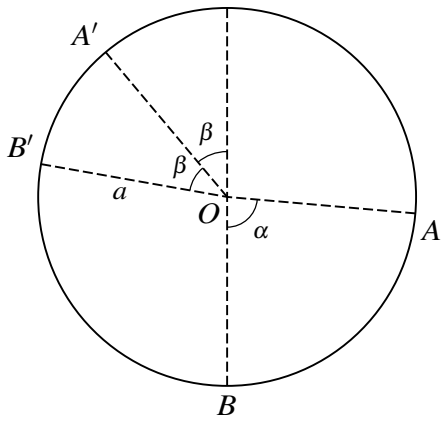
A test at the $\alpha\%$ significance level does not provide evidence that there is any difference in the means of the volume of juice in bottles filled by machine X and the volume of juice in bottles filled by machine Y.

(ii) Find the set of possible values of α. [6]

11 Answer only **one** of the following two alternatives.

EITHER



A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O. The points A and A' are on the inner surface of the sphere, on opposite sides of the vertical through O; the radius OA makes an angle α with the downward vertical and the radius OA' makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O. The point B' is on the inner surface of the sphere and such that OB' makes an angle 2β with the upward vertical through O (see diagram). It is given that $\cos\alpha = \frac{1}{16}$.

(i) P is projected from A with speed u along the surface of the sphere downwards towards B. Subsequently it loses contact with the sphere at A'. Show that $u^2 = \frac{1}{8}ag(1 + 24\cos\beta)$. [5]

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(ii) P is now projected from B with speed u along the surface of the sphere towards B'. Subsequently it loses contact with the sphere at B'. Find $\cos\beta$. [6]

(iii) In part **(i)**, the reaction of the sphere on P when it is initially projected at A is R. Find R in terms of m and g. [3]

OR

A large number of people attended a course to improve the speed of their logical thinking. The times taken to complete a particular type of logic puzzle at the beginning of the course and at the end of the course are recorded for each person. The time taken, in minutes, at the beginning of the course is denoted by x and the time taken, in minutes, at the end of the course is denoted by y. For a random sample of 9 people, the results are summarised as follows.

$$\Sigma x = 45.3 \quad \Sigma x^2 = 245.59 \quad \Sigma y = 40.5 \quad \Sigma y^2 = 195.11 \quad \Sigma xy = 218.72$$

Ken attended the course, but his time to complete the puzzle at the beginning of the course was not recorded. His time to complete the puzzle at the end of the course was 4.2 minutes.

(i) By finding, showing all necessary working, the equation of a suitable regression line, find an estimate for the time that Ken would have taken to complete the puzzle at the beginning of the course. [5]

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The values of $x - y$ for the sample of 9 people are as follows.

0.2 0.8 0.5 1.0 0.2 0.6 0.2 0.5 0.8

The organiser of the course believes that, on average, the time taken to complete the puzzle decreases between the beginning and the end of the course by more than 0.3 minutes.

(ii) Stating suitable hypotheses and assuming a normal distribution, test the organiser's belief at the $2\frac{1}{2}\%$ significance level. [9]

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