CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9231 FURTHER MATHEMATICS

9231/21 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Question Number	Mark Scheme Details		Part Mark	Total
1	Equate impulse to momentum to find initial speed v and Newton's law of restitution to find new speed:	$v = 4u,\ v' = ev = [-]\,3u$ M1 A1	2	**2**
2	Find v^2 at both A and B:	$v_A{}^2 = \omega^2(a^2 - 0{\cdot}5^2)$ *and* $v_B{}^2 = \omega^2(a^2 - 0{\cdot}75^2)$ B1		
	Find amplitude a m from given K.E. ratio:	$\frac{1}{2} mv_A{}^2 = (12/11)\ \frac{1}{2} mv_B{}^2$		
		$11\,(a^2 - 0{\cdot}5^2) = 12\,(a^2 - 0{\cdot}75^2)$		
		$a^2 = \frac{1}{4}(27 - 11) = 4,\ \ a = 2$ M1 A1	3	
	Find ω from $v_{\max} = a\omega$:	$0{\cdot}6 = 2\omega,\ \ \omega = 0{\cdot}3$ B1		
	Find time (√ on a) at A	$\omega^{-1}\sin^{-1}(0{\cdot}5/2)$ *or* $\omega^{-1}\cos^{-1}(0{\cdot}5/2)$		
	or at B, e.g.:	$\omega^{-1}\sin^{-1}(0{\cdot}75/2)$ *or* $\omega^{-1}\cos^{-1}(0{\cdot}75/2)$ M1 A1√		
	Combine correctly to find time from A to B:	$\omega^{-1}\sin^{-1}(0{\cdot}75/2) - \omega^{-1}\sin^{-1}(0{\cdot}5/2)$		
		or $\omega^{-1}\cos^{-1}(0{\cdot}5/2) - \omega^{-1}\cos^{-1}(0{\cdot}75/2)$ M1		
	Evaluate to 3 d.p.:	$= \omega^{-1}(0{\cdot}3844 - 0{\cdot}2527)$ or $\omega^{-1}(1{\cdot}318 - 1{\cdot}186)$		
		$= 1{\cdot}2813 - 0{\cdot}8423$		
		$4{\cdot}3937 - 3{\cdot}9547 = 0{\cdot}439$ [s] A1	5	**8**

3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$	M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{4}mu^2$	M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10$, $v_B = (1 + e)u/10$			
		or v_A, $v_B = -u/2$, $u/6$ [or $7u/10$, $u/30$]	M1 A1		
	Use in 3rd eqn to find e, e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$			
	(A0 if finally $\pm\frac{2}{3}$)	$90e^2 = 40$, $e = \frac{2}{3}$	M1 A1	7	
	Use Newton's law of restitution with	$v_C = 2v_B'$, e.g.: $v_C - v_B' = ev_B$, $v_B' = \frac{2}{3}v_B$	B1		
		[$v_B = u/6$, $v_B = u/9$, $v_C = 2u/9$]			
	Use conservation of momentum to find k:	$9mv_B' + kmv_C = 9mv_B$			
		$9v_B' + 2kv_B' = 13{\cdot}5v_B'$, $k = 9/4$	M1 A1	3	**10**

4	(i)	Use conservation of energy at lowest point: Use $F = ma$ radially at lowest point: Eliminate v^2 to find R $[v^2 = 2{\cdot}3\,ga]$:	$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + mga$ B1 $R - mg = mv^2/a$ B1 $R = mu^2/a + 3mg = 3{\cdot}3\,mg$ B1	3	
	(ii)	Use conservation of energy at B to find V_B : (A.E.F.)	$\tfrac{1}{2}mV_B{}^2 = \tfrac{1}{2}mu^2 + mga\sin\theta$ M1A1 $V_B{}^2 = (0{\cdot}3 + 0{\cdot}5)\,ga,\ V_B = \surd(0{\cdot}8\,ga)$ *or* $2\surd(ga/5)$ *or* $0{\cdot}894\surd(ga)$ A1	3	
	(iii)	Use vertical component v_B of speed V_B at B: Find height h reached above B: Find height h reached above level of O:	$v_B = V_B\cos\theta$ $[= \tfrac{1}{4}\surd 15\, V_B = \surd(\tfrac{3}{4}ga)]$ M1 $h = v_B{}^2/2g = 3a/8$ M1 A1 $h - a\sin\theta = 3a/8 - \tfrac{1}{4}a = a/8$ **A.G.** A1	4	10
5		Find MI of components about A: (M1 for BC or CD) Find total MI about A: (OR can first find total MI about centre of mass) State or imply total mass acts at mid-point of AC Use eqn of circular motion to find $d^2\theta/dt^2$: Approximate $\sin\theta$ by θ and substitute for I: Find period $T = 2\pi/\omega$ with $\omega = \surd(49g/384a)$:	Glass $(3M/5)\{\tfrac{1}{3}(5a)^2 + 25a^2\} = 20Ma^2$ M1 A1 AB $M\{\tfrac{1}{3}(4a)^2 + (4a)^2\} = 64Ma^2/3$ B1 AD $\tfrac{1}{3}M\{\tfrac{1}{3}(3a)^2 + (3a)^2\} = 4Ma^2$ B1 BC $\tfrac{1}{3}M\{\tfrac{1}{3}(3a)^2 + 73a^2\} = 76Ma^{2/3}$ M1 A1 CD $M\{\tfrac{1}{3}(4a)^2 + 52a^2\} = 172Ma^{2/3}$ A1 $I = 128Ma^2$ A.G. A1 M1 $I\,d^2\theta/dt^2 = [-]\,(49Mg/15)\,5a\sin\theta$ M1 A1 $d^2\theta/dt^2 = -(49g/384a)\,\theta$ A1 $T = 2\pi\surd(384a/49g)$ *or* $(16\pi/7)\surd(6a/g)$ *or* $17{\cdot}6\surd(a/g)$ (A.E.F.) B1	8 5	13

6		State or find the expected value of X: using $p = ¼$:	$E(X) = 1/p = 1/¼ = 4$	B1	1	
	(i)	Find $P(X = 4)$:	$P(X = 4) = (¾)^3 ¼ = 27/256$ or $0·105$	M1 A1	2	
	(ii)	Find $P(X<6)$:	$P(X < 6) = 1 – (¾)^5$			
			or $\{1 + ¾ + (¾)^2 + (¾)^3 + (¾)^4\}¼$			
		S.R. Using $p = ½$ can earn B0 M1 A0 M0 A0	$= 781/1024$ or $0·763$	M1 A1	2	**5**
7	**(i)**	State probability density function of T:	$f(t) = 0·001 \exp(-0·001\, t) \quad (t \ge 0)$ $[\ = 0 \quad$ (otherwise *or* $t < 0$)]	B1	1	
	(ii)	Find $P(T > 2000)$:	$P(t > 2000) = 1 – F(2000)$	M1		
		S.R. $1 – e^{-2} = 0·865$ earns B1 only (max 1/3)	$= 1 – (1 – e^{-2}) = e^{-2}$ *or* $0·135$	M1 A1	3	
		State inequality for t (lose A1 if = or ≤):	$(\exp(-0·001\, t))^{10} \ge$ [*or* >] $0·9$	M1 A1		
		Solve for t_{max}: (Omitting power 10 earns 0/4; using $1 – (\exp(-0·001t))^{10}$ can earn M1 A0 M1 A0 only)	$t_{max} = (\ln 0·9) / (-0·01) = 10·5$	M1 A1	4	**8**

8	State hypotheses (B0 for $\bar{\chi}$ …): Estimate both popln. variances using two samples: (allow use of biased: $\sigma_{X,60}{}^2 = 236$ *or* $15{\cdot}36^2$) (allow use of biased: $\sigma_{Y,50}{}^2 = 265$ *or* $16{\cdot}28^2$) Estimate population variance for combined sample: (allow $\sigma_{X,60}{}^2/60 + \sigma_{Y,50}{}^2/50$: $9{\cdot}233$ *or* $3{\cdot}039^2$) Calculate value of z (to 2 d.p., either sign): State or use correct tabular z – value (to 2 d.p.): (or can compare 6 with e.g. $2{\cdot}326\,s = 7{\cdot}13$ *or* $7{\cdot}07$) Correct conclusion (A.E.F, √on z – values): **S.R.** Assuming equal population variances: Find pooled estimate of common variance s^2: Calculate value of z (to 2 d.p., either sign): Tabular value; conclusion	$H_0: \mu_X = \mu_Y$, $H_1: \mu_X \neq \mu_Y$ $S_x{}^2 = (626\,220 - 6060^2/60) / 59$ $[= 240$ *or* $15{\cdot}49^2]$ *And* $s_Y{}^2 = (464\,500 - 4750^2/50) / 49$ $[= 270{\cdot}4$ *or* $16{\cdot}44^2]$ $s^2 = s_X{}^2/60 + s_Y{}^2/50$ $= 9{\cdot}408$ *or* $3{\cdot}067^2$ $z = (101 - 95) / s$ $= 6/3{\cdot}067 = 1{\cdot}96$ (*or* $1{\cdot}97$) $z_{0.99} = 2{\cdot}326$ *or* $2{\cdot}33$ (allow $2{\cdot}36$) [Accept H_0] Claims are the same Hypotheses; Explicit assumption : $s^2 = (626\,220 - 6060^2/60 +$ $464\,500 - 4750^2/50) / 108$ $z = 6 / s\sqrt{(1/60+1/50)} = 1{\cdot}97$ $= 253{\cdot}8$ *or* $15{\cdot}93^2$ As above)	B1 M1 A1 M1 A1 M1 A1 B1 B1√ (B1; B1) (M1 A1) (M1 A1) (A1) (B1; B1√			9

9	Find expected frequency p:	$p = 200 \int_2^3 (1 / x \ln 8)\,dx$ $= (200 / \ln 8)\,[\ln x]_2^3$ $= 200 \times 0{\cdot}1950 = 39{\cdot}00$ **A.G.**	 M1A1		
	Find q by similar method *or* by using total of 200:	$q = 21{\cdot}46$ *or* $21{\cdot}45$	M1A1	4	
	State (at least) null hypothesis:	H $_0$: f (x) fits data (A.E.F.)	B1		
	Calculate χ^2 (to 3 s.f.):	$\chi^2 = 0{\cdot}202 + 0{\cdot}923 + 0{\cdot}678 + 0{\cdot}584$ $+ 1{\cdot}134 + 4{\cdot}134 + 3{\cdot}644 = 11{\cdot}3$	 M1A1		
	State or use correct tabular χ^2 value (to 3 s.f.)::	$\chi_{6,\,0.95}{}^2 = 12{\cdot}59$	B1		
	Valid method for reaching conclusion:	Accept H$_0$ if $\chi^2 \leqslant$ tabular value	M1		
	Conclusion consistent with correct values (A.E.F):	Distribution fits observations	A1	6	**10**
10	Find correlation coefficient r: $r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)\,(55\,991 - 639^2 / 10)\}}$		M1 A1		
	(A.E.F.; A0 if only 3 s.f. clearly used)	$= 18\,189{\cdot}6 / \sqrt{(46\,280{\cdot}4 \times 15\,158{\cdot}9)}$	A1		
		$= 0{\cdot}687$	*A1	4	
	State both hypotheses (B0 for r …):	H $_0$: $\rho = 0$, H $_1$: $\rho \neq 0$	B1		
	State or use correct tabular two-tail r-value:	$r_{10,\,5\%} = 0{\cdot}632$	*B1		
	Valid method for reaching conclusion:	Reject H$_0$ if $\mid r \mid >$ tabular value	M1		
	Correct conclusion (A.E.F, dep *A1, *B1):	There is non-zero correlation	A1	4	
	Calculate gradient p in $x - \bar{x} = p\,(y - \bar{y})$:	$p = 18\,189{\cdot}6 / 15\,158{\cdot}9 = 1{\cdot}20$	B1		
	Find regression line of x on y:	$x = 86{\cdot}6 + 1{\cdot}20\,(y - 63{\cdot}9)$ $= 1{\cdot}20\,y + 9{\cdot}92$	 M1 A1	3	**11**

11 A (i)	Use Pythagoras to find AB:	$AB = \sqrt{(4a^2 + 12a^2)} = 4a$	**A.G.**	M1 A1		
	Find $\angle SAB$:	$\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ *or* $\cos^{-1} 2a/4a$				
		or $\tan^{-1} 2a\sqrt{3}/2a$				
		$= 60°$ so $\angle SAB = 30°$	**A.G.**	M1 A1	4	
(ii)	*EITHER*					
	Resolve vertically and horizontally, e.g.:	$\frac{1}{2} N_A + \frac{1}{2}\sqrt{3}\, N_B + \frac{1}{2}\sqrt{3}\, F_A = W$				
	(F_A may be in either direction)	*and* $\frac{1}{2}\sqrt{3}\, N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$		M1 A1		
	Eliminate $N_B + F_A$ to find N_A:	$N_A = \frac{1}{2} W$	**A.G.**	A1		
	OR				3	
	Resolve in dirn. PQ to find N_A:	$N_A = \frac{1}{2} W$	**A.G.**	(M1 A1)		
(iii)	Second resolution, e.g. in dirn. PS:	$N_B + F_A = \frac{1}{2}\sqrt{3}\, W$		(A1)		
	Take moments, e.g. about A:	$\frac{1}{2}\sqrt{3}\, W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$				
	(A1 for each side of eqn)	$= N_B \times 2a$		M1 A1 A1		
	Solve to find N_B:	$N_B = \{(7\sqrt{3} - 6)/8\}\, W$		M1 A1		
	Use N_B to find F_A:	$F_A = \sqrt{3}\, N_A - N_B$ *or* $\frac{1}{2}\sqrt{3}\, W - N_B$				
		$= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.)		M1 A1	7	**14**

B	Estimate population variance:	$s_P^2 = (236{\cdot}0 - 42{\cdot}8^2/8)/7$			
	(allow biased here: $0{\cdot}8775$ or $0{\cdot}9367^2$)	$= 351/350$ or $1{\cdot}003$ or $1{\cdot}001^2$	M1		
	Find confidence interval (allow z in place of t) e.g.:	$42{\cdot}8/8 \pm t\sqrt{(s_P^2/8)}$	M1		
	Use correct tabular t-value:	$t_{7,\,0.975} = 2{\cdot}365$	A1		
	Evaluate C.I. correct to 2 d.p.:	$5{\cdot}35 \pm 0{\cdot}84$ or $[4{\cdot}51, 6{\cdot}19]$	A1	4	
	Formulate inequality for k (or equality for $k_{\max}$):	$(5{\cdot}35 - k)/\sqrt{(s_P^2/8)} \geqslant [\text{or} >]\ t$	M1		
	Use correct tabular t-value:	$t_{7,\,0.9} = 1{\cdot}415$	A1		
	Solve for $k_{\max}$ (A0 if = or ⩽ was used for k above):	$5{\cdot}35 - k \geqslant 0{\cdot}50,\ k_{\max} = 4{\cdot}85$	A1	3	
	State hypotheses (B0 for $\bar{x}$ …), e.g.:	$H_0: \mu_P = \mu_Q,\ H_1: \mu_P > \mu_Q$	B1		
	State assumption (A.E.F.):	Normal distns. for [P and] Q			
		and equal variances	B1		
	Estimate (pooled) common variance:	$s^2 = (7 \times 1{\cdot}003 + 11 \times 1{\cdot}962)/18$			
		$= 1{\cdot}589$ or $1{\cdot}261^2$	M1 A1		
	Calculate value of t (to 3 s.f.):	$t = (5{\cdot}35 - 4{\cdot}60)/(s\sqrt{(1/8 + 1/12)})$			
		$= 1{\cdot}30$	M1 A1		
	Correct conclusion (A.E.F., $\sqrt{}$ on t):	$t < t_{18,\,0.9} = 1{\cdot}33$ so Q's mean is not less than P's	B1$\sqrt{}$	7	**14**