A Level Physics CIE

20. Magnetic Fields

CONTENTS

20.1 Magnetic Fields

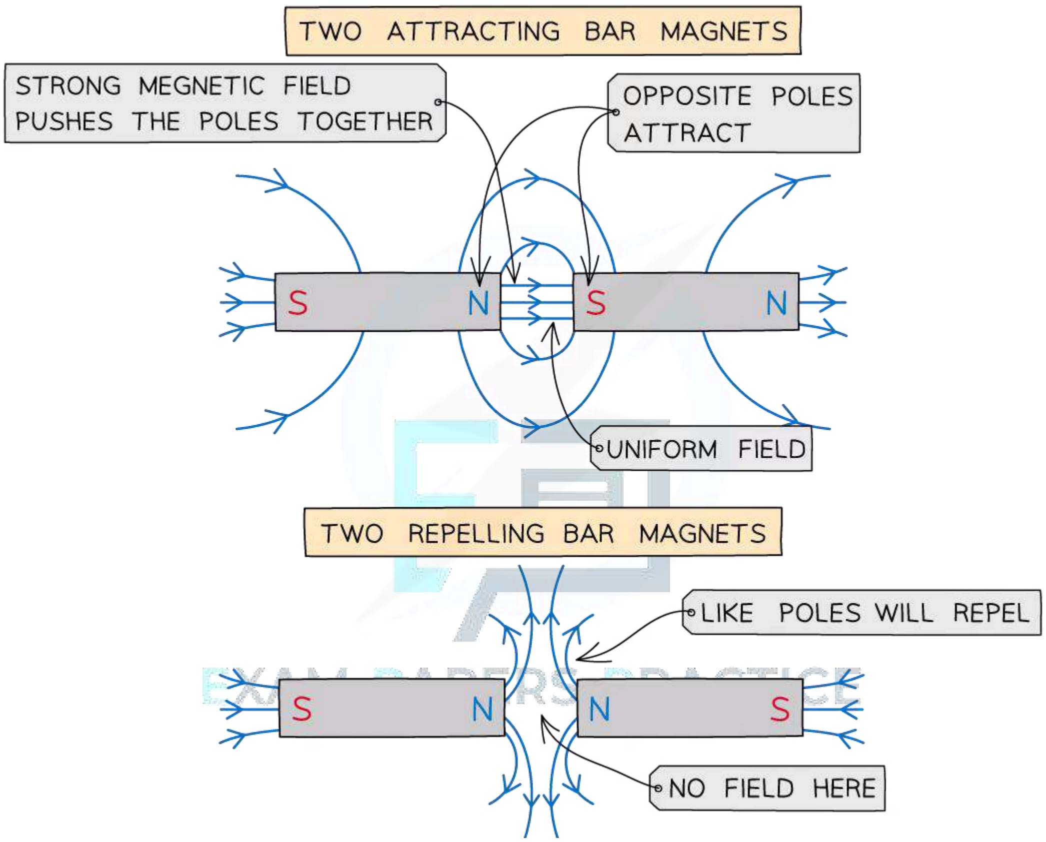
20.1.1 Representing Magnetic Fields

Magnetic Field Definition

- A magnetic field is a **field of force** that is created either by:
  - **Moving** electric charge
  - **Permanent** magnets
- Permanent magnets are materials that produce a magnetic field
- A stationary charge will **not** produce a magnetic field
- A magnetic field is sometimes referred to as a **B-field**
- A magnetic field is created around a **current carrying wire** due to the movement of electrons
- Although magnetic fields are invisible, they can be observed by the force that pulls on magnetic materials, such as iron or the movement of a needle in a plotting compass

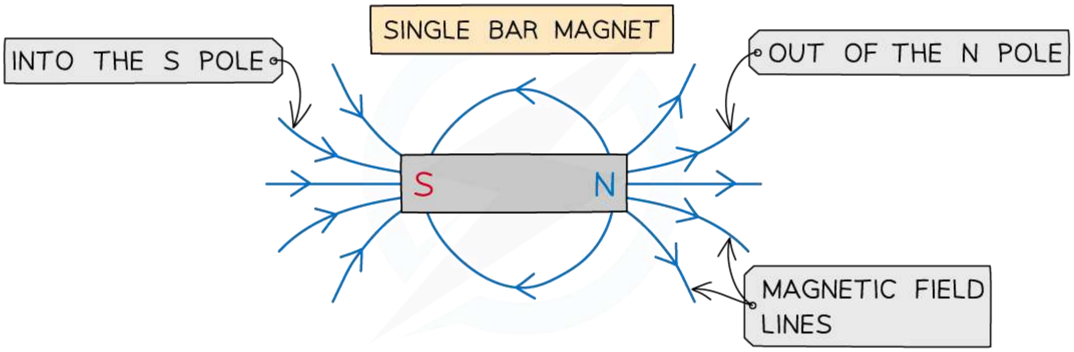
Representing Magnetic Fields

- Magnetic fields are represented by magnetic field lines
  - These can be shown using iron filings or plotting compasses
- Field lines are best represented on **bar magnets**, which consist of a north pole on one end and south pole on the other
- The magnetic field is produced on a bar magnet by the movement of electrons within the atoms of the magnet
- This is a result of the electrons circulating around the atoms, representing a tiny current and hence setting up a magnetic field
- The direction of a magnetic field on a bar magnet is always from **north to south**



Magnetic field lines are directed from the north pole to the south pole

- When two bar magnets are pushed together, they either attract or repel each other:
  - Two **like** poles (north and north or south and south) **repel** each other
  - Two **opposite** poles (north and south) **attract** each other

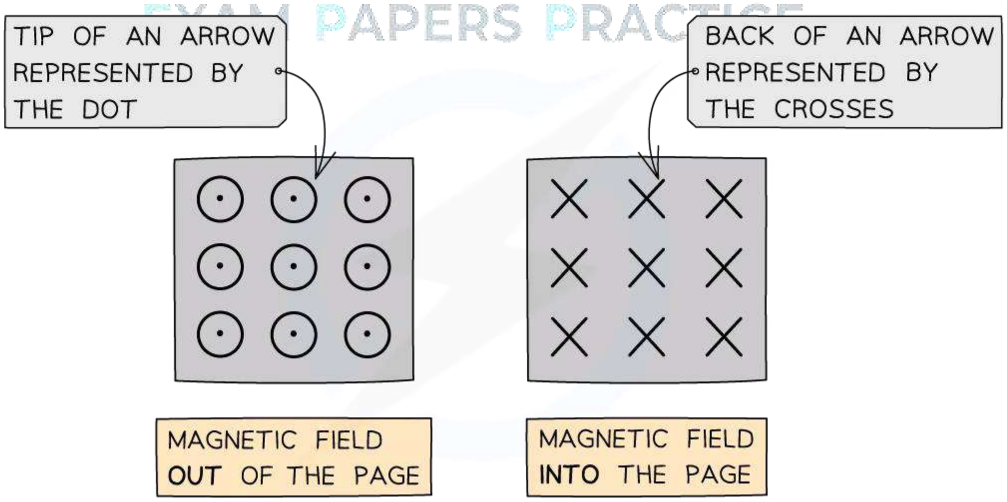


Two opposite poles attract each other and two like poles repel each other

- **The key aspects of drawing magnetic field lines:**
  - The lines come **out** from the north poles and **into** the south poles
  - The direction of the field line shows the direction of the force that a free magnetic north pole would experience at that point
  - The field lines are **stronger** the **closer** the lines are together
  - The field lines are **weaker** the **further apart** the lines are
  - Magnetic field lines **never** cross since the magnetic field is unique at any point
  - Magnetic field lines are **continuous**

- A uniform magnetic field is where the magnetic field strength is the same at all points
  - This is represented by equally spaced parallel lines, just like electric fields

- The direction of the magnetic field into or out of the page in 3D is represented by the following symbols:
  - Dots (sometimes with a circle around them) represent the magnetic field directed **out** of the plane of the page
  - Crosses represent the magnetic field directed **into** the plane of the page



The magnetic field into or out of the page is represented by circles with dots or crosses

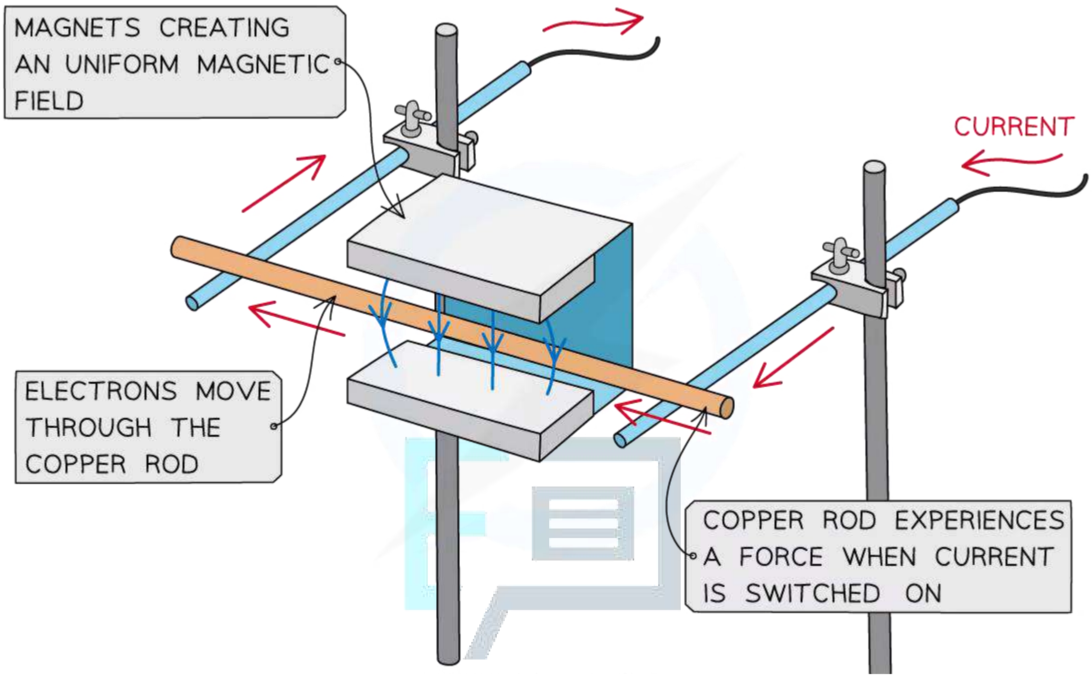
Exam Tip

The best way to remember which way around to draw magnetic fields in 3D is by imagining an arrow coming towards or away from you

- When the head of an arrow is coming towards you, you see the tip as a dot representing the arrow coming **'out'** of the page
- When an arrow is travelling away from you, you see the cross at the back of the arrow representing the arrow going **'into'** the page

Force on a Current-Carrying Conductor

- A current-carrying conductor produces its own magnetic field
  - When interacting with an external magnetic field, it will experience a **force**
- A current-carrying conductor will only experience a force if the current through it is **perpendicular** to the direction of the magnetic field lines
- A simple situation would be a copper rod placed within a uniform magnetic field
- When current is passed through the copper rod, it experiences a force which makes it move



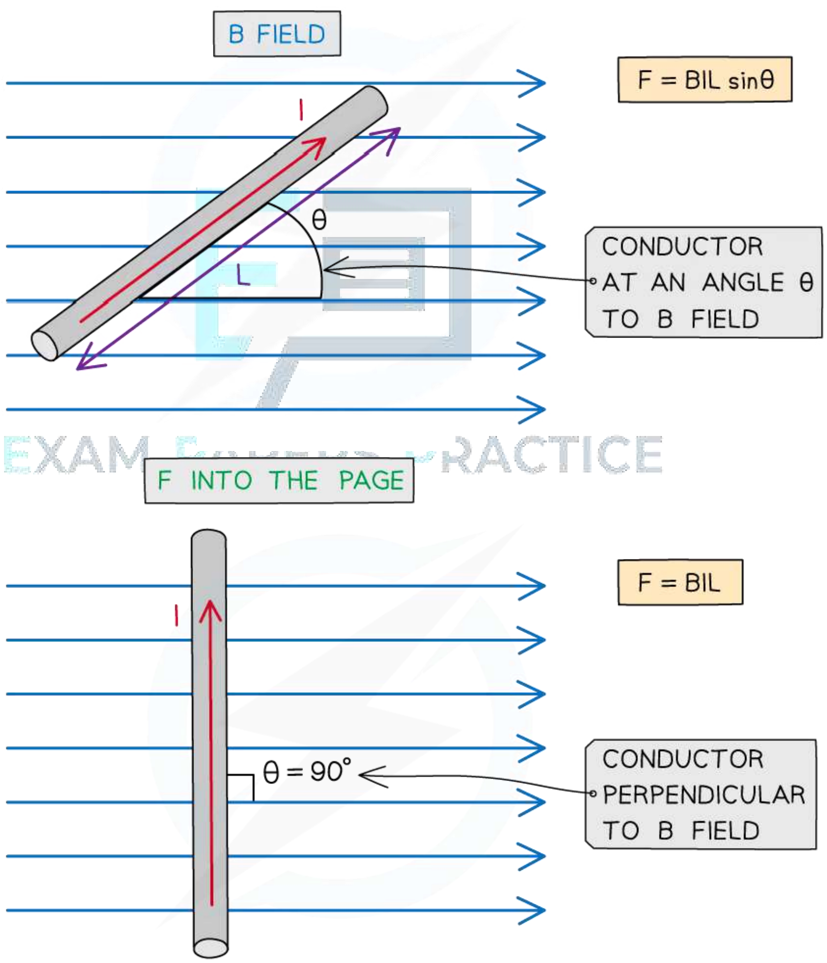
A copper rod moves within a magnetic field when current is passed through it

Calculating Magnetic Force on a Current-Carrying Conductor

- The strength of a magnetic field is known as the **magnetic flux density, B**
  - This is also known as the magnetic field strength
  - It is measured in units of **Tesla (T)**
- The force F on a conductor carrying current I at right angles to a magnetic field with flux density B is defined by the equation

$$F = BIL \sin\theta$$

- Where:
  - F = force on a current carrying conductor in a B field (N)
  - B = magnetic flux density of external B field (T)
  - I = current in the conductor (A)
  - L = length of the conductor (m)
  - θ = angle between the conductor and external B field (degrees)
- This equation shows that the greater the current or the magnetic field strength, the greater the force on the conductor



Magnitude of the force on a current carrying conductor depends on the angle of the conductor to the external B field

- The **maximum** force occurs when sin θ = 1
  - This means θ = 90° and the conductor is **perpendicular** to the B field
  - This equation for the magnetic force now becomes:

$$F = BIL$$

- The **minimum** force (0) is when sin θ = 0
  - This means θ = 0° and the conductor is **parallel** to the B field
- It is important to note that a current-carrying conductor will experience **no** force if the current in the conductor is parallel to the field

Worked Example

A current of 0.87 A flows in a wire of length 1.4 m placed at 30° to a magnetic field of flux density 80 mT.Calculate the force on the wire.

Step 1: Write down the known quantities

Magnetic flux density, B = 80 mT = 80 × 10^{-3} T

Current, I = 0.87 A

Length of wire, L = 1.4 m

Angle between the wire and the magnetic field, θ = 30°

Step 2: Write down the equation for force on a current-carrying conductor

$$F = BIL \sin\theta$$

Step 3: Substitute in values and calculate

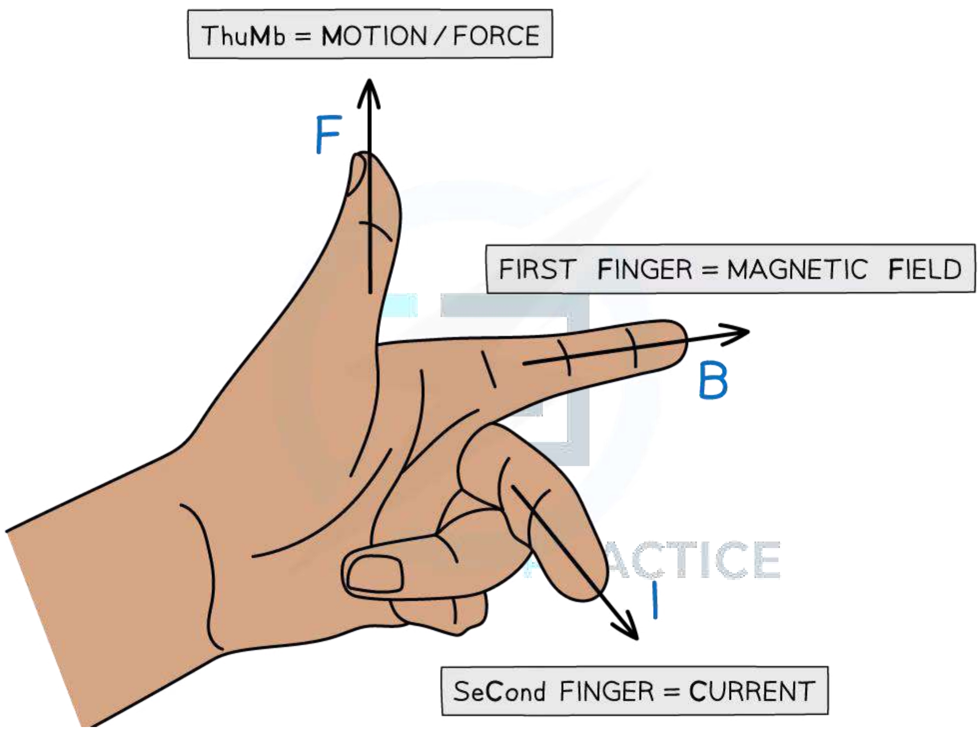
$$F = (80 \times 10^{-3}) \times (0.87) \times (1.4) \times \sin(30) = 0.04872 = 0.049 \text{ N (2 s.f)}$$

Exam Tip

Remember that the direction of current flow is the flow of **positive** charge (positive to negative), and this is in the **opposite direction** to the flow of electrons

Fleming's Left-Hand Rule

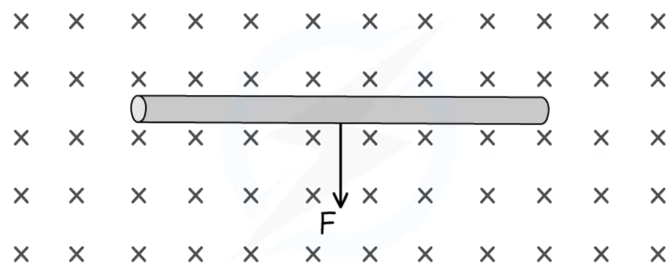
- The **direction** of the force on a charge moving in a magnetic field is determined by the direction of the magnetic field and the current
- Recall that the direction of the current is the direction of **conventional** current flow (positive to negative)
- When the force, magnetic field and current are all mutually perpendicular to each other, the directions of each can be interpreted by **Fleming's left-hand rule:**
  - On the left hand, with the thumb pointed upwards, first finger forwards and second finger to the right ie. all three are perpendicular to each other
  - The **thumb** points in the direction of **motion** of the rod (or the direction of the force) (F)
  - The **first** finger points in the direction of the external **magnetic field** (B)
  - The **second** finger points in the direction of conventional **current** flow (I)



Fleming's left hand rule

ked Example

the direction of the current flowing in the wire in the diagram below.

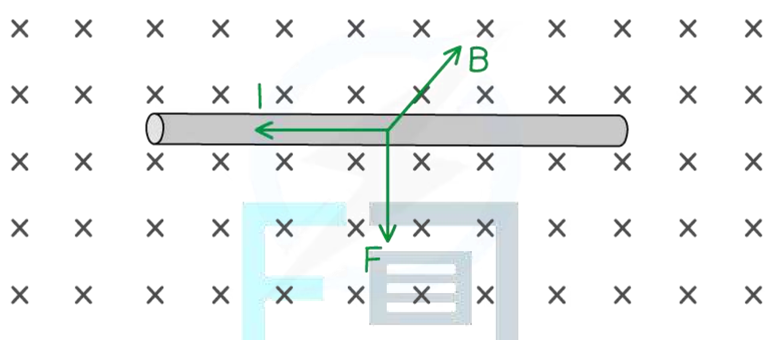


Using Fleming's left-hand rule:

B = into the page

F = vertically downwards

I = from right to left



Exam Tip

Don't be afraid to use Fleming's left-hand rule during an exam. Although, it is best to do it subtly in order not to give the answer away to other students!

Magnetic Flux Density Definition

- The magnetic flux density B is defined as:

  The force acting per unit current per unit length on a current-carrying conductor placed perpendicular to the magnetic field

- Rearranging the equation for magnetic force on a wire, the magnetic flux density is defined by the equation:

$$B = \frac{F}{IL}$$

- **Note:** this equation is only relevant when the B field is perpendicular to the current
- Magnetic flux density is measured in units of **tesla**, which is defined as:

  A straight conductor carrying a current of 1A normal to a magnetic field of flux density of 1 T with force per unit length of the conductor of 1 N m⁻¹

- To put this into perspective, the Earth's magnetic flux density is around 0.032 mT and an ordinary fridge magnet is around 5 mT

Worked Example

A 15 cm length of wire is placed vertically and at right angels to a magnetic field.When a current of 3.0 A flows in the wire vertically upwards, a force of 0.04 N acts on it to the left.Determine the flux density of the field and its direction.

Step 1: Write out the known quantities

Force on wire, F = 0.04 N

Current, I = 3.0 A

Length of wire = 15 cm = 15×10^{-2} m

Step 2: Magnetic flux density *B* equation

$$B = \frac{F}{IL}$$

Step 3: Substitute in values

$$B = \frac{0.04}{3 \times 15 \times 10^{-2}} = 0.089 \text{ T (2 s.f)}$$

Step 4: Determine the direction of the B field

Using Fleming's left-hand rule :

F = to the left

I = vertically upwards

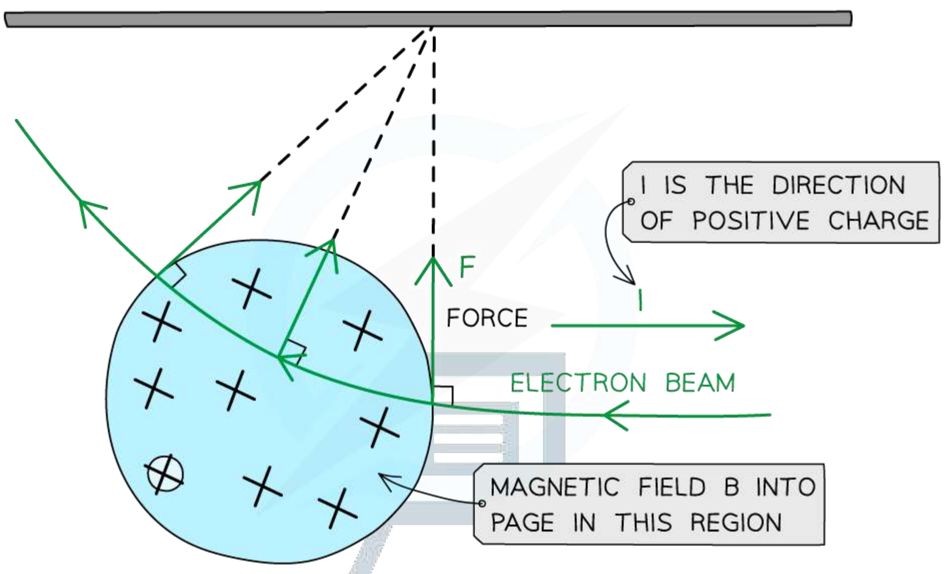
therefore, B = into the page

Calculating Magnetic Force on a Moving Charge

- The magnetic force on an isolating moving charge, such an electron, is given by the equation:

$$F = BQv \sin\theta$$

- Where:
  - F = force on the charge (N)
  - B = magnetic flux density (T)
  - Q = charge of the particle (C)
  - v = speed of the charge (m s^{-1})
  - θ = angle between charge's velocity and magnetic field (degrees)



The force on an isolated moving charge is perpendicular to its motion and the magnetic field B

- Equivalent to the force on a wire, if the magnetic field *B* is perpendicular to the direction of the charge's velocity, the equation simplifies to:

$$F = BQv$$

- According to Fleming's left hand rule:
  - When an electron enters a magnetic field from the **left**, and if the magnetic field is directed **into the page**, then the force on it will be directed **upwards**

- The equation shows:
  - If the direction of the electron changes, the magnitude of the force will change too

- The force due to the magnetic field is always perpendicular to the velocity of the electron

- **Note:** this is equivalent to circular motion

- Fleming's left-hand rule can be used again to find the direction of the force, magnetic field and velocity
  - The key difference is that the second finger representing current I (direction of positive charge) is now the **direction of velocity *v*** of the positive charge

Worked Example

An electron is moving at 5.3×10^7 m s^{-1} in a uniform magnetic field of flux density 0.2 T.Calculate the force on the electron when it is moving at 30° to the field, and state the factor it increases by compared to when it travels perpendicular to the field.

Step 1: Write out the known quantities

Speed of the electron, $v = 5.3 \times 10^7$ m s^{-1}

Charge of an electron, $Q = 1.60 \times 10^{-19}$ C

Magnetic flux density, $B = 0.2$ T

Angle between electron and magnetic field, $\theta = 30^\circ$

Step 2: Write down the equation for the magnetic force on an isolated particle

$$F = BQv \sin\theta$$

Step 3: Substitute in values, and calculate the force on the electron at 30°

$$F = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) \times \sin(30) = 8.5 \times 10^{-13} \text{ N}$$

Step 4: Calculate the electron force when travelling perpendicular to the field

$$F = BQv = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) = 1.696 \times 10^{-12} \text{ N}$$

Step 5: Calculate the ratio of the perpendicular force to the force at 30°

$$\frac{1.696 \times 10^{-12}}{8.5 \times 10^{-13}} = 1.995 = \mathbf{2}$$

Therefore, the force on the electron is twice as strong when it is moving perpendicular to the field than when it is moving at 30° to the field

Exam Tip

Remember not to mix this up with F = BIL!

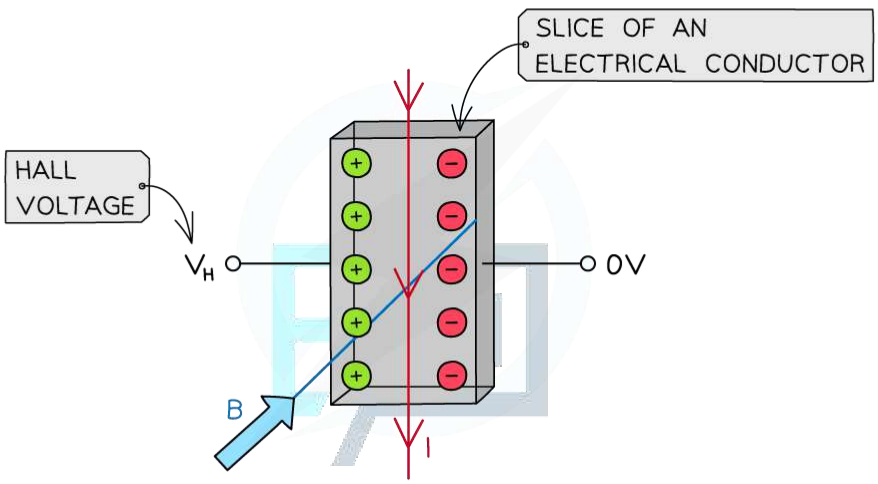
- **F = BIL** is for a current carrying conductor
- **F = Bqv** is for an isolated moving charge (which may be inside a conductor)

Hall Voltage

- The Hall voltage is a product of the Hall effect
- Hall voltage is defined as:

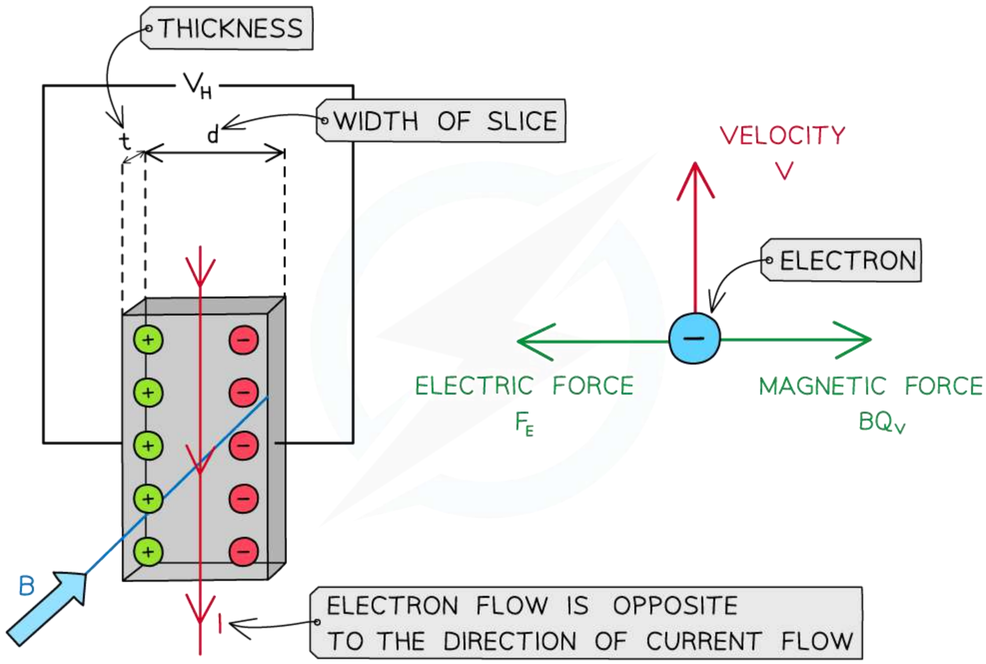
  The potential difference produced across an electrical conductor when an external magnetic field is applied perpendicular to the current through the conductor

- When an external magnetic field is applied perpendicular to the direction of current through a conductor, the electrons experience a magnetic force
- This makes them drift to one side of the conductor, where they all gather and becomes more negatively charged
- This leaves the opposite side deficient of electrons, or positively charged
- There is now a potential difference across the conductor
  - This is called the Hall Voltage, V_H



The positive and negative charges drift to opposite ends of the conductor producing a hall voltage when a magnetic field is applied

- An equation for the Hall voltage V_H is derived from the electric and magnetic forces on the charges



The electric and magnetic forces on the electrons are equal and opposite

- The voltage arises from the electrons accumulating on one side of the conductor slice
- As a result, an electric field is set up between the two opposite sides
- The two sides can be treated like oppositely charged parallel plates, where the electric field strength E is equal to:

$$E = \frac{V_H}{d}$$

- Where:
  - V_H = Hall voltage (V)
  - d = width of the conductor slice (m)

- A single electron has a drift velocity of v within the conductor. The magnetic field is into the plane of the page, therefore the electron has a magnetic force F_B to the right:

$$F_B = Bqv$$

- This is equal to the electric force F_E to the left:

$$F_E = qE$$

$$qE = Bqv$$

- Substituting E and cancelling the charge q

$$\frac{V_H}{d} = Bv$$

- Recall that current I is related to the drift velocity v by the equation:

$$I = nAvq$$

- Where:
  - A = cross-sectional area of the conductor (m^2)
  - n = number density of electrons (m^{-3})
- Rearranging this for v and substituting it into the equation gives:

$$\frac{V_H}{d} = B\frac{I}{nAq}$$

- The cross-sectional area A of the slice is the product of the width d and thickness t:

$$A = dt$$

- Substituting A and rearranging for the Hall voltage V_H leads to the equation:

$$\frac{V_H}{d} = B\frac{I}{n(dt)q}$$

$$V_H = B\frac{I}{ntq}$$

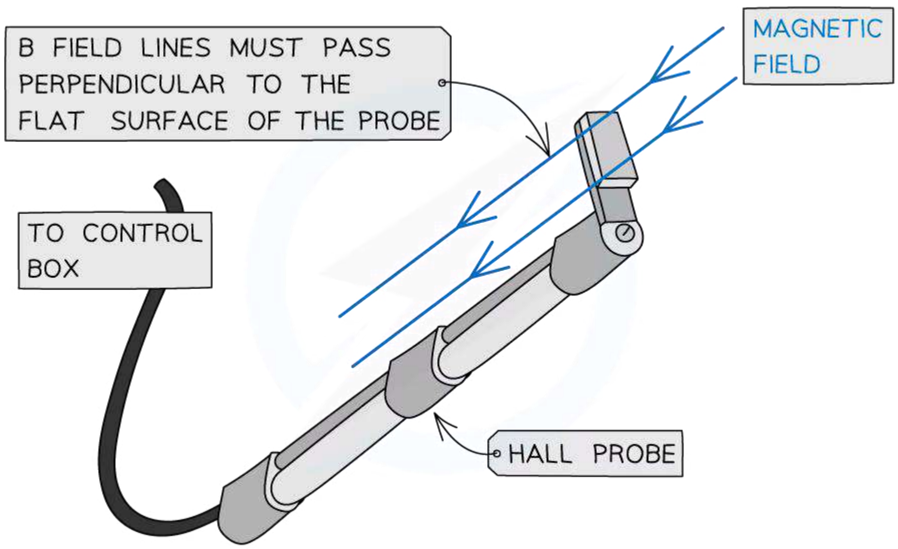
- Where:
  - B = magnetic flux density (T)
  - q = charge of the electron (C)
  - I = current (A)
  - n = number density of electrons (m^{-3})
  - t = thickness of the conductor (m)
- This equation shows that the smaller the electron density n of a material, the larger the magnitude of the Hall voltage
  - This is why a semiconducting material is often used for a Hall probe
- **Note:** if the electrons were placed by positive charge carriers, the negative and positive charges would still deflect in opposite directions
  - This means there would be no change in the polarity (direction) of the Hall voltage

Exam Tip

Remember to use Fleming's left-hand rule to obtain the direction the electrons move due to the magnetic force created by the magnetic field.

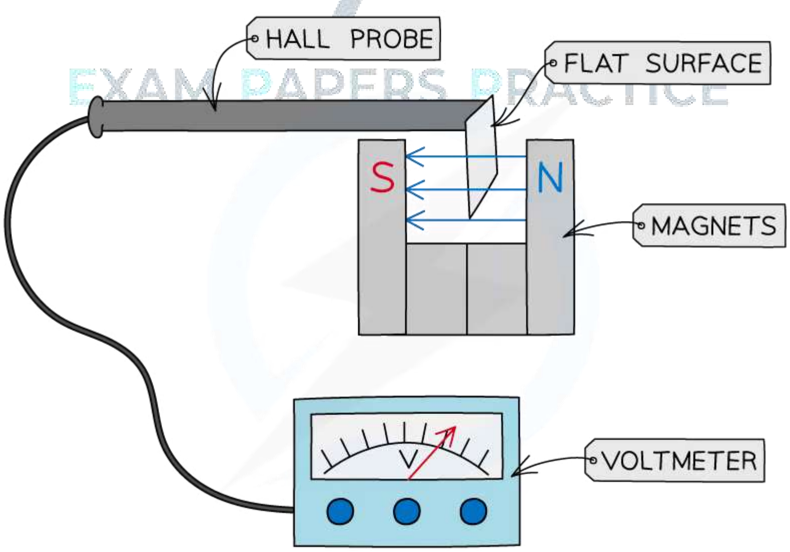
Measuring Magnetic Flux Density using a Hall Probe

- A Hall probe can be used to measure the magnetic flux density between two magnets based on the Hall effect
- It consists of a cylinder with a flat surface at the end



A Hall probe consists of a flat surface and is held so the magnetic field lines are perpendicular to it

- To measure the magnetic flux density between two magnets, the flat surface of the probe must be directed between the magnets so the magnetic field lines pass completely perpendicular to this surface

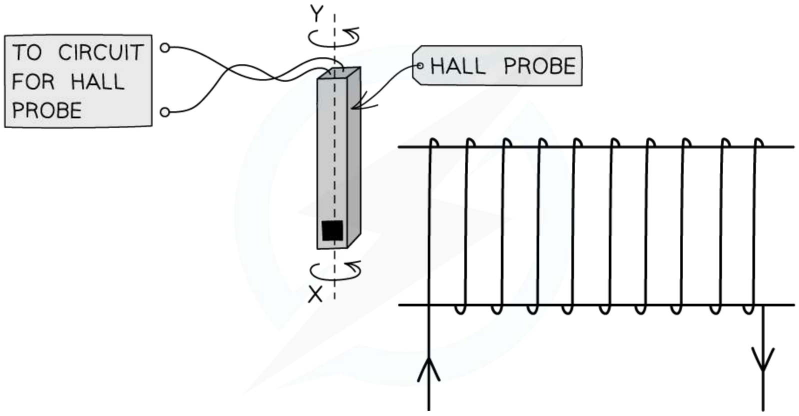


- The probe is connected to a voltmeter to measure the Hall voltage

- If the probe is not held in the correct orientation (perpendicular to the field lines), the voltmeter reading will be reduced
- Since the Hall voltage is directly proportional to the magnetic flux density, the flux density of the magnets can be obtained
- A Hall probe is sensitive enough to measure even the Earth's magnetic flux density

Worked Example

A Hall probe is placed near one end of a solenoid, as shown in the diagram.

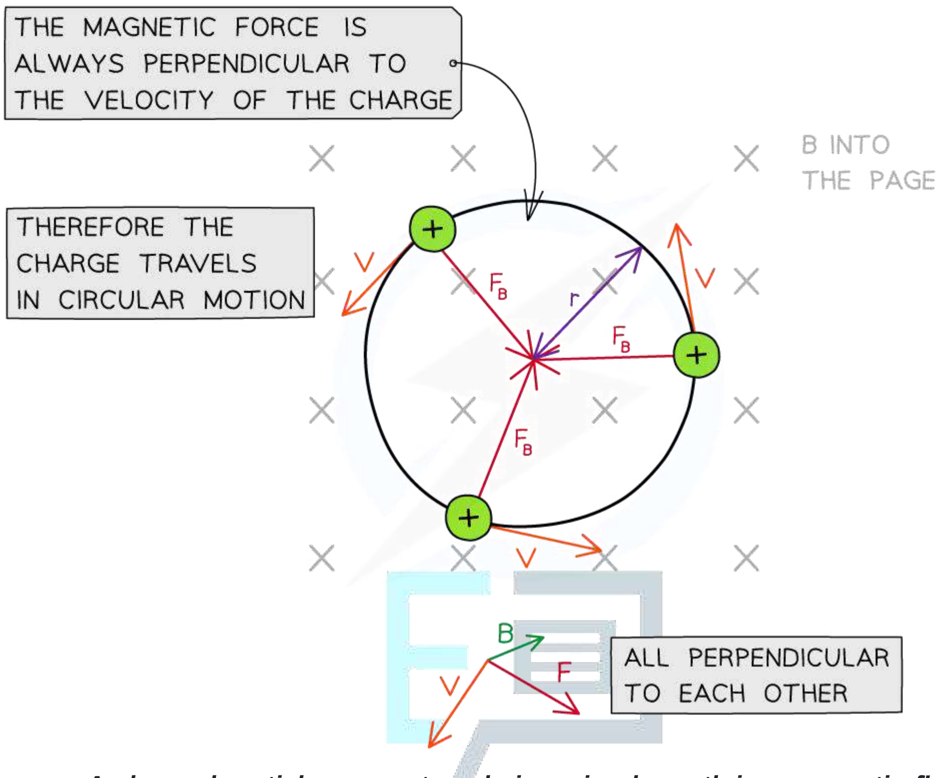


The Hall probe is rotated about the axis XY.State and explain why the magnitude of the Hall voltage varies.

- The Hall voltage depends on angle between the magnetic field and the plane of the probe
- The Hall voltage reaches a **maximum** when the field is **perpendicular** to the probe
- The Hall voltage is **zero** when the field is **parallel** to the probe

Motion of a Charged Particle in a Uniform Magnetic Field

- A charged particle in uniform magnetic field which is perpendicular to its direction of motion travels in a **circular** path
- This is because the magnetic force F_B will always be perpendicular to its velocity v
    - F_B will always be directed towards the centre of the path



A charged particle moves travels in a circular path in a magnetic field

- The magnetic force F_B provides the **centripetal force** on the particle
- Recall the equation for centripetal force:

$$F = \frac{mv^2}{r}$$

- Where:
    - m = mass of the particle (kg)
    - v = linear velocity of the particle (m s^{-1})
    - r = radius of the orbit (m)

- Equating this to the force on a moving charged particle gives the equation:

$$\frac{mv^2}{r} = Bqv$$

- Rearranging for the radius r obtains the equation for the radius of the orbit of a charged particle in a perpendicular magnetic field:

$$r = \frac{mv}{Bq}$$

- This equation shows that:
  - Faster moving particles with speed v move in larger circles (larger r): $\mathbf{r \propto v}$
  - Particles with greater mass m move in larger circles: $\mathbf{r \propto m}$
  - Particles with greater charge q move in smaller circles: $\mathbf{r \propto 1 / q}$
  - Particles moving in a strong magnetic field B move in smaller circles: $\mathbf{r \propto 1 / B}$

Worked Example

An electron with charge-to-mass ratio of 1.8×10^{11} C kg^{-1} is travelling at right angles to a uniform magnetic field of flux density 6.2 mT. The speed of the electron is 3.0×10^6 m s^{-1}.Calculate the radius of the circle path of the electron.

Step 1: Write down the known quantities

$$\text{Charge-to-mass ratio} = \frac{q}{m} = \mathbf{1.8 \times 10^{11}\ C\ kg^{-1}}$$

Magnetic flux density, B = 6.2 mT

Electron speed, v = 3.0×10^6 m s^{-1}

Step 2: Write down the equation for the radius of a charged particle in a perpendicular magnetic field

$$r = \frac{mv}{Bq}$$

Step 3: Substitute in values

$$\frac{m}{q} = \frac{1}{1.8 \times 10^{11}}$$

$$r = \frac{(3.0 \times 10^6)}{(1.8 \times 10^{11})(6.2 \times 10^{-3})} = 2.688 \times 10^{-3}\ \text{m} = \mathbf{2.7\ mm}\ \text{(2 s.f.)}$$

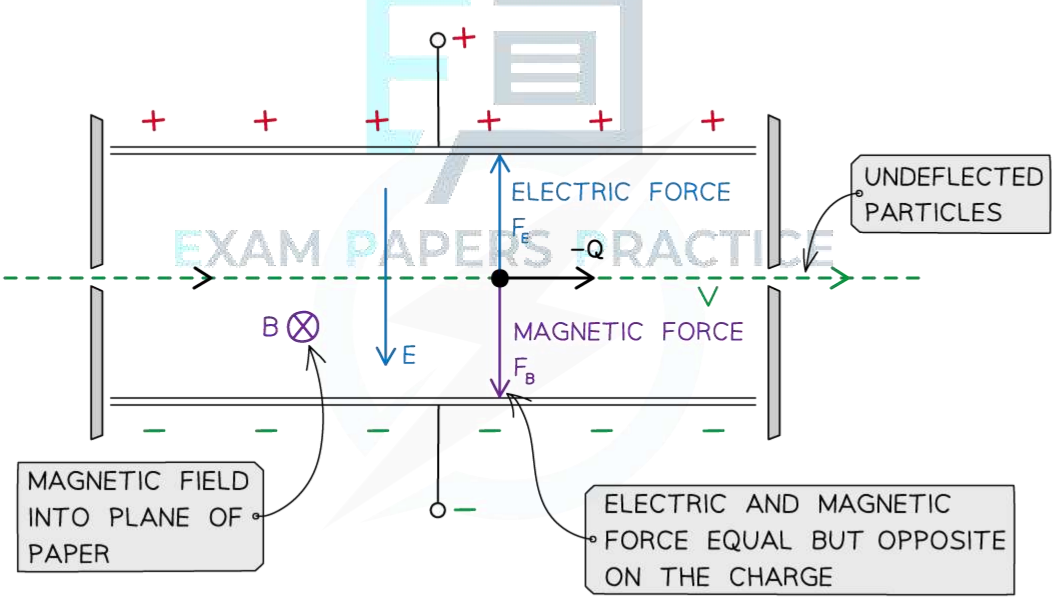
Velocity Selection

- A velocity selector is defined as:

  A device consisting of perpendicular electric and magnetic fields where charged particles with a specific velocity can be filtered

- Velocity selectors are used in devices, such as mass spectrometers, in order to produce a beam of charged particles all travelling at the same velocity
- The construction of a velocity selector consists of two horizontal oppositely charged plates situated in a vacuum chamber
  - The plates provide a uniform electric field with strength E between them
- There is also a uniform magnetic field with flux density B applied perpendicular to the electric field
  - If a beam of charged particles enter between the plates, they may all have the same charge but travel at different speeds v
- The electric force **does not** depend on the velocity: $F_E = EQ$
- However, the magnetic force **does** depend on the velocity: $F_B = BQv$
  - The magnetic force will be greater for particles which are travelling faster
- To select particles travelling at exactly the desired the speed v, the electric and magnetic force must therefore be **equal**, but in **opposite** directions

$$F_E = F_B$$



The particles travelling at the desired speed v will travel through undeflected due to the equal and opposite electric and magnetic forces on them

- The resultant force on the particles at speed *v* will be zero, so they will remain undeflected and pass straight through between the plates
- By equating the electric and magnetic force equations:

$$EQ = BQv$$

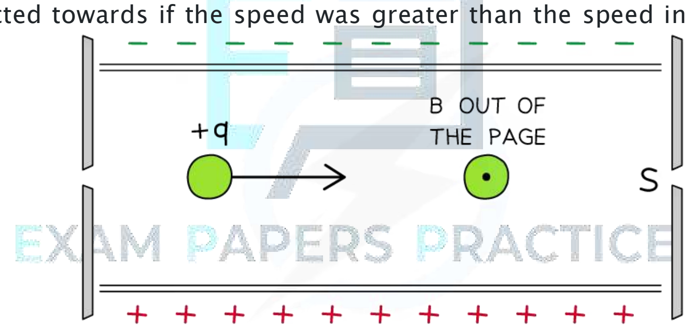
- The charge *Q* will cancel out on both sides to give the selected velocity *v* equation:

$$v = \frac{E}{B}$$

- Therefore, the speed *v* in which a particle will remain undeflected is found by the **ratio of the electric and magnetic field strength**
  - If a particle has a speed greater or less than *v*, the magnetic force will deflect it and collide with one of the charged plates
  - This would remove the particles in the beam that are not exactly at speed *v*
- **Note:** the gravitational force on the charged particles will be negligible compared to the electric and magnetic forces and therefore can be ignored in these calculations

Worked Example

A positive ion travels between two charged plates towards a slit Sa) State the direction of the electric and magnetic fields on the ionb) Calculate the speed of the ion emerging from slit S when the magnetic flux density is 0.50 T and the electric field strength is 2.8 kV m^{-1}c) Which plate will the ion be deflected towards if the speed was greater than the speed in part (b)



Part (a)

Step 1: Direction of E field

- Electric field lines point from the positive to negative to charge
- Therefore, it must be directed **vertically upwards**

Step 2: Direction of B field

- Using Fleming's left-hand rule:
  - The charge or current I is to the right
  - B is out of the page
  - Therefore, the force F is **vertically downwards**

Part (b)

Velocity selector equation

Electric field strength, $E = 2.8\ \text{kV m}^{-1}$ = **$2.8 \times 10^3\ \text{V m}^{-1}$**

Magnetic flux density, B = **0.50 T**

$$v = \frac{E}{B} = \frac{2.8 \times 10^3}{0.50} = \mathbf{5600\ m\ s^{-1}}$$

Part (c)

If the speed increases, the magnetic force must be greater because $F_B \propto v$

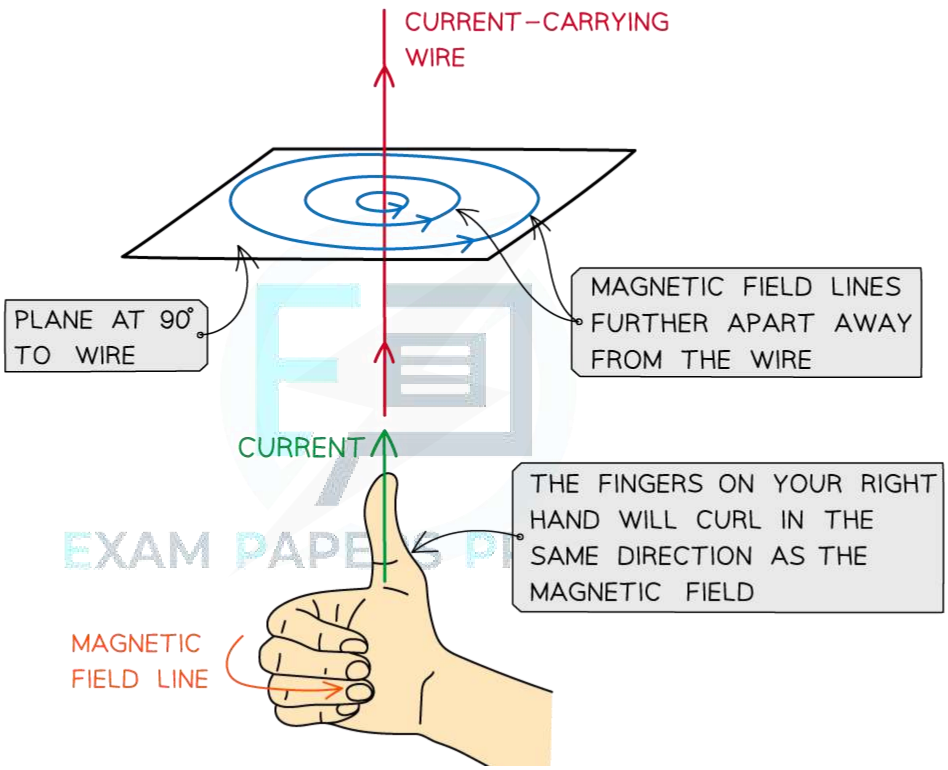
Since the magnetic force would direct the ion downwards in the direction of the field, the ion will be deflected **towards the positive plate**

Magnetic Fields in Wires, Coils & Solenoids

- Magnetic field patterns are not only observed around bar magnets, magnetic fields are formed wherever current is flowing, such as in:
  - Long straight wires
  - Long solenoids
  - Flat circular coils

Field Lines in a Current-Carrying Wire

- Magnetic field lines in a current carrying wire are circular rings, centered on the wire
- The field lines are strongest near the wire and become further part away from the wire
- Reversing the current reverses the direction of the field



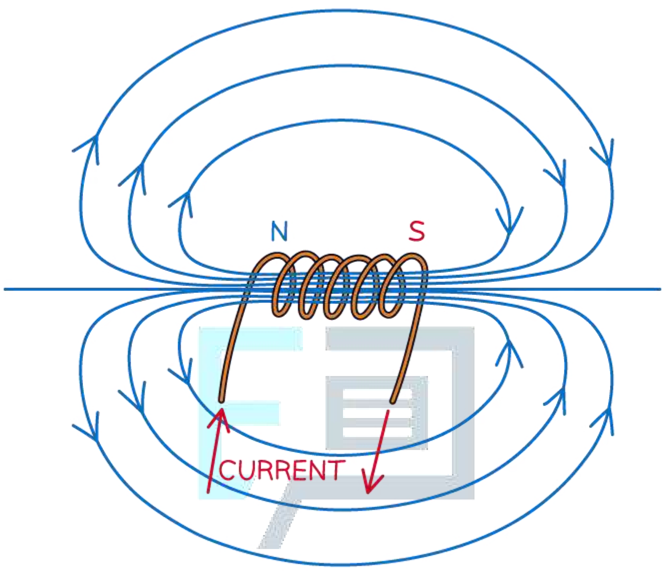
The direction of magnetic field lines on a current carrying wire can be determined by the right hand thumb rule

- The field lines are clockwise or anticlockwise around the wire, depending on the direction of the current
- The direction of the magnetic field is determined by **Maxwell's right hand screw rule**

- This is determined by pointing the **right-hand** thumb in the direction of the current in the wire and curling the fingers onto the palm
- The direction of the curled fingers represents the direction of the magnetic field around the wire
- For example, if the current is travelling vertically upwards, the magnetic field lines will be directed anticlockwise, as seen from directly above the wire
- **Note:** the direction of the current is taken to be the conventional current ie. from **positive** to **negative**, **not** the direction of electron flow

Field Lines in a Solenoid

- As seen from a current carrying wire, an electric current produces a magnetic field
- An electromagnetic makes use of this by using a coil of wire called a **solenoid** which concentrates the magnetic field
- One ends becomes a north pole and the other the south pole



Magnetic field lines around a solenoid are similar to a bar magnet

- Therefore, the magnetic field lines around a solenoid are very similar to a bar magnet
  - The field lines **emerge** from the **north** pole
  - The field lines **return** to the **south** pole

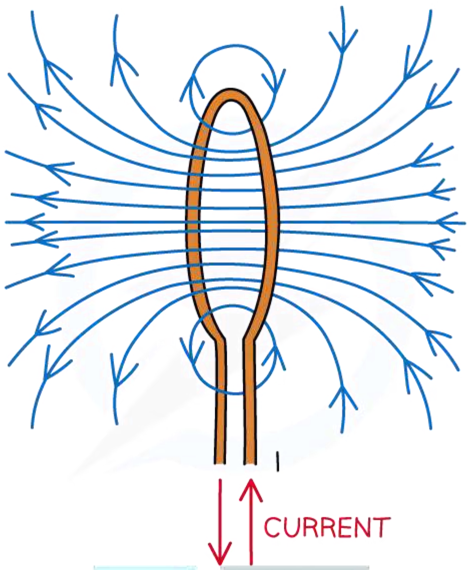
- Which is the north or south pole depends on the direction of the current
  - This is found by the **right hand grip rule**

- This involves gripping the electromagnet so the fingers represent the direction of the current flow of the wire
- The thumb points in the direction of the field lines inside the coil, or in other words, point towards the electromagnet's **north pole**

Field Lines in a Flat Circular Coil

- A flat circular coil is equal to one of the coils of a solenoid

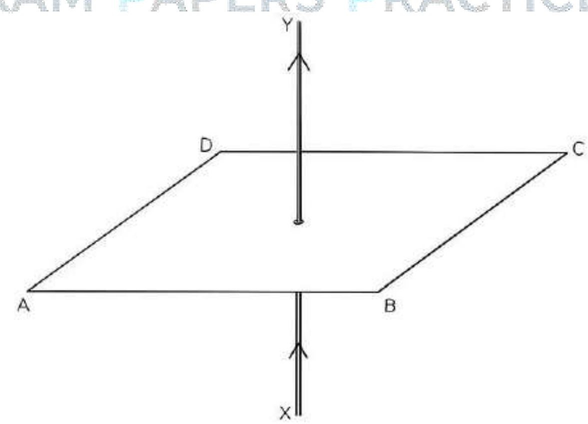
* The field lines will emerge through one side of the circle (north pole) and leave the other (south pole)
* As before, the direction of the north and south pole depend on the direction of the current
    * This can be determined by using the **right hand thumb rule**
    * It easier to find the direction of the magnetic field on the straight part of the circular coil to determine which direction the field lines are passing through



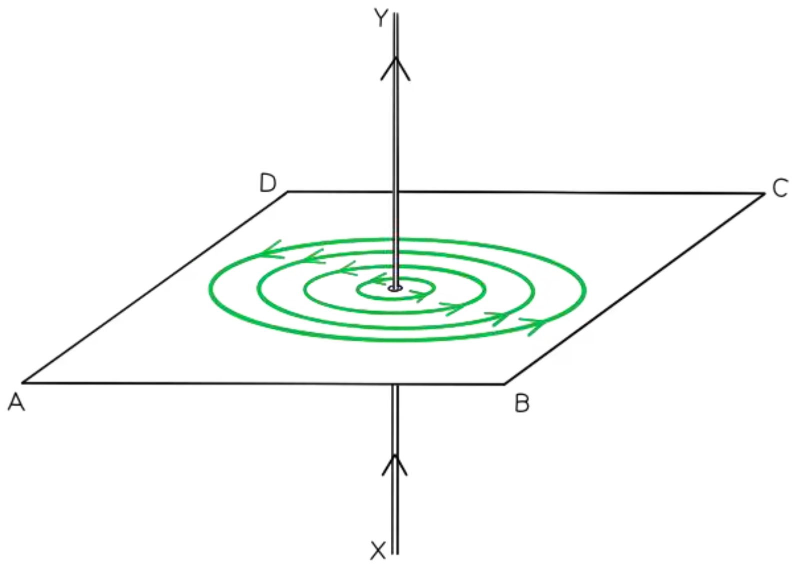
Magnetic field lines of a single circular coil are added up together to make to make the field lines of a solenoid

Worked Example

The current in a long, straight vertical wire is in the direction XY, as shown in the diagram.



Sketch the pattern of the magnetic flux in the horizontal plane ABCD due to the current-carrying wire. Draw at least four flux lines.



- √ **Concentric circles**
- √ **Increasing separation between each circle**
- √ **Arrows drawn in anticlockwise direction**

Exam Tip

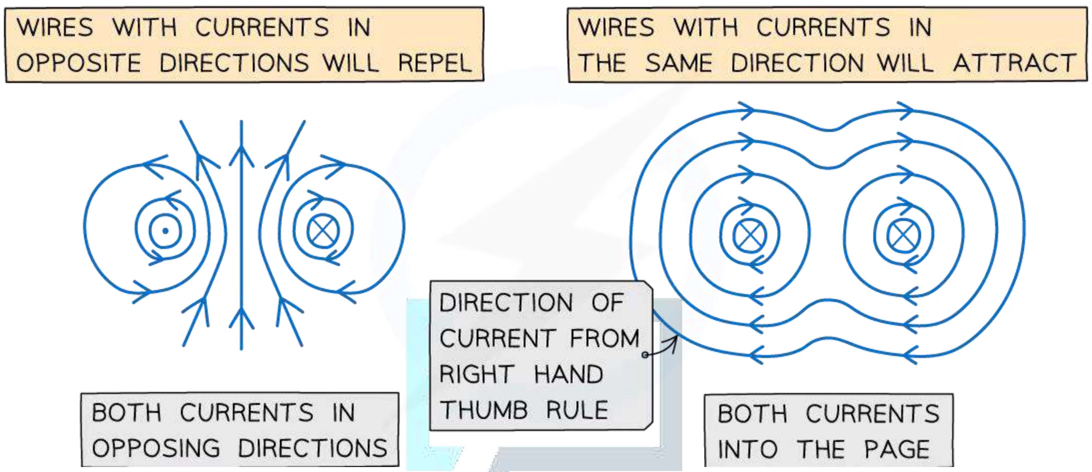
Remember to draw the arrows showing the direction of the field lines on every single field line you draw. Also, ensure that in a uniform magnetic field, the field lines are equally spaced.

Factors Affecting Magnetic Field Strength

- The strength of the magnetic field of a solenoid can be increased by:
  - Adding a core made from a **ferrous** (iron-rich) material eg. an iron rod
  - Adding **more turns** in the coil
- When current flows through the solenoid with an iron core, it becomes magnetised, creating an even stronger field
  - The addition of an iron core can strengthen the magnetic field up to a several hundred times more
- When more turns are added in the coil, this concentrates the magnetic field lines, causing the magnetic field strength to increase

Origin of the Forces Between Current-Carrying Conductors

- A current carrying conductor, such as a wire, produces a magnetic field around it
- The direction of the field depends on the direction of the current through the wire
  - This is determined by the **right hand thumb rule**
- Parallel current-carrying conductors will therefore either attract or repel each other
  - If the currents are in the **same** direction in both conductors, the magnetic field lines between the conductors cancel out – the conductors will **attract** each other
  - If the currents are in the **opposite** direction in both conductors, the magnetic field lines between the conductors push each other apart – the conductors will **repel** each other



Both wires will attract if their currents are in the same direction and repel if in opposite directions

- When the conductors **attract**, the direction of the magnetic forces will be **towards** each other
- When the conductors **repel**, the direction of the magnetic forces will be **away** from each other
- The magnitude of each force depend on the amount of current and length of the wire

Worked Example

Two long, straight, current-carrying conductors, WX and YZ, are held at a close distance, as shown in diagram 1.

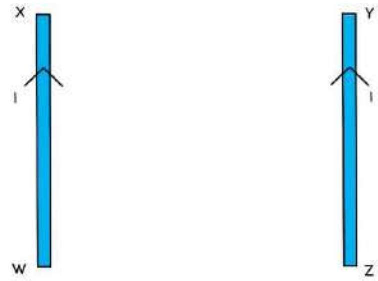


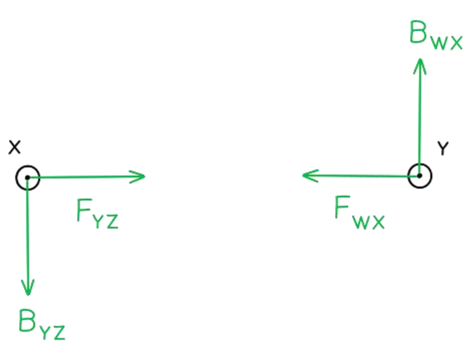
diagram 1

The conductors each carry the same magnitude current in the same direction. A plan view from above the conductors is shown in diagram 2.

diagram 2

On diagram 2, draw arrows, one in each case, to show the direction of:

- The magnetic field at X due to the current in wire YZ (label this arrow B_{YZ})
- The force at X as a result of the magnetic field due to the current in the wire YZ (label this arrow F_{YZ})
- The magnetic field at Y due to the current in wire WX (label this arrow B_{WX})
- The force at Y as a result of the magnetic field due to the current in the wire WX (label this arrow F_{WX})



- Newton's Third Law states:
  - When two bodies interact, the force on one body is **equal but opposite** in direction to the force on the other body
- Therefore, the forces on the wires act in equal but opposite directions

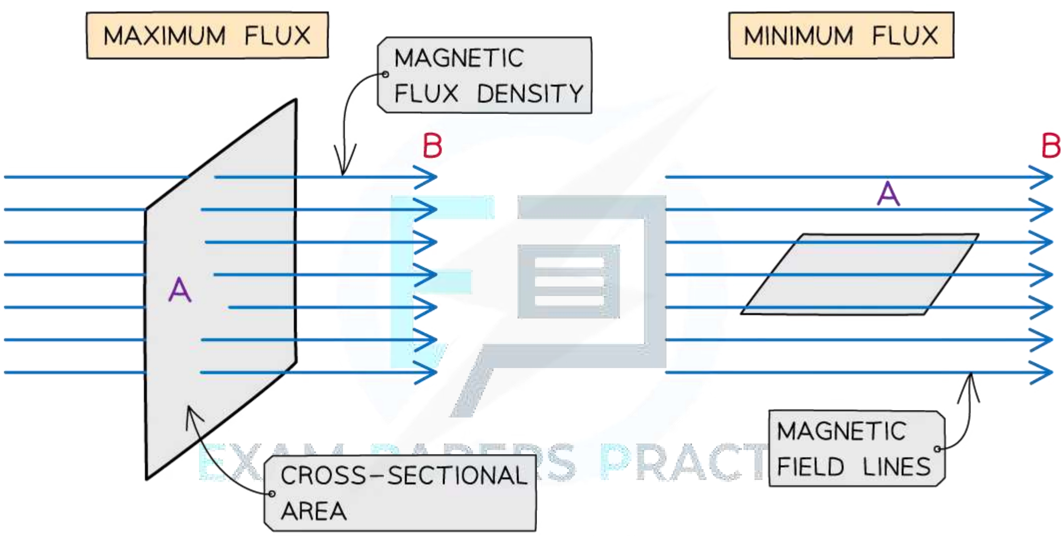
20.2 Electromagnetic Induction

20.2.1 Magnetic Flux

Magnetic Flux Definition

- Electromagnetic induction is when an e.m.f is induced in a closed circuit conductor due to it moving through a magnetic field
- This happens when a conductor **cuts** through magnetic field lines
- The amount of e.m.f induced is determined by the magnetic flux
- The amount of magnetic flux varies as the coil rotates within the field
  - The flux is the total magnetic field that passes through a given area
  - It is a maximum when the magnetic field lines are **perpendicular** to the area
  - It is at a minimum when the magnetic field lines are **parallel** to the area
- The **magnetic flux** is defined as:

The product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density



The magnetic flux is maximum when the magnetic field lines and the area they are travelling through are perpendicular

- In other words, magnetic flux is the **number of magnetic field lines through a given area**

Calculating Magnetic Flux

- Magnetic flux is defined by the symbol Φ (greek letter 'phi')
- It is measured in units of **Webers (Wb)**
- Magnetic flux can be calculated using the equation:

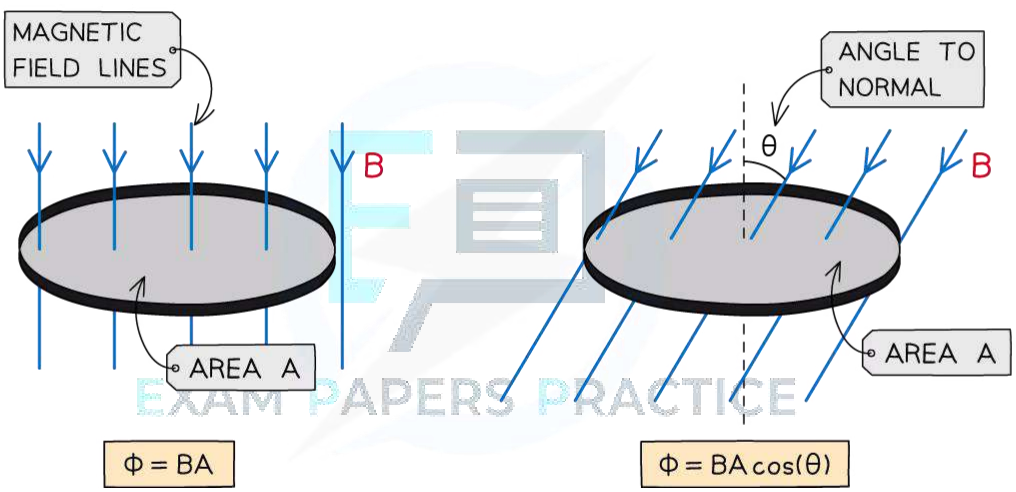
$$\Phi = BA$$

- Where:
  - Φ = magnetic flux (Wb)
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)

- When the magnet field lines are not completely perpendicular to the area *A*, then the component of magnetic flux density *B* perpendicular to the area is taken
- The equation then becomes:

$$\Phi = BA\cos(\theta)$$

- Where:
  - θ = angle between magnetic field lines and the line perpendicular to the plane of the area (often called the normal line) (degrees)



The magnetic flux decreases as the angle between the field lines and plane decrease

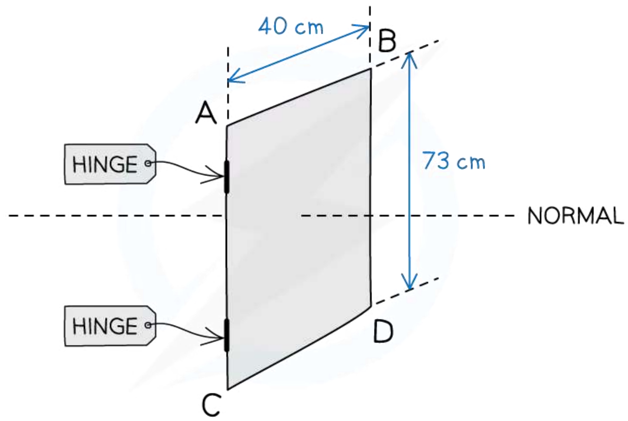
- This means the magnetic flux is:
  - **Maximum** = BA when cos(θ) =1 therefore **θ = 0°.** The magnetic field lines are perpendicular to the plane of the area
  - **Minimum** = 0 when cos(θ) = 0 therefore **θ = 90°.** The magnetic fields lines are parallel to the plane of the area

- An e.m.f is induced in a circuit when the magnetic flux linkage changes with respect to time
- This means an e.m.f is induced when there is:
  - A changing magnetic flux density *B*
  - A changing cross-sectional area *A*

- A change in angle θ

Worked Example

An aluminium window frame has a width of 40 cm and length of 73 cm as shown in the figure below



The frame is hinged along the vertical edge AC.When the window is closed, the frame is normal to the Earth's magnetic field with magnetic flux density 1.8×10^{-5} T

a) Calculate the magnetic flux through the window when it is closed

b) Sketch the graph of the magnetic flux against angle between the field lines and the normal when the window is opened and rotated by 180°

Part (a)

Step 1: Write out the known quantities

Cross-sectional area, A = 40 cm × 73 cm = (40 × 10^{-2}) × (73 × 10^{-2}) = 0.292 m^2

Magnetic flux density, B = 1.8 × 10^{-5} T

Step 2: Write down the equation for magnetic flux

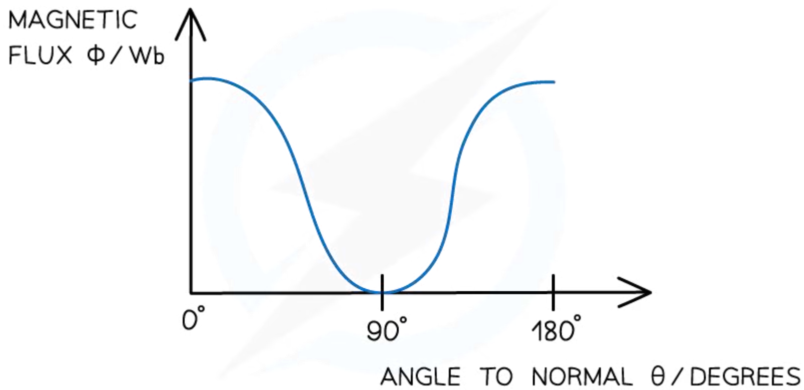
$$\Phi = BA$$

Step 3: Substitute in values

$$\Phi = (1.8 \times 10^{-5}) \times 0.292 = 5.256 \times 10^{-6} = 5.3 \times 10^{-6} \text{ Wb}$$

Part (b)

The magnetic flux will be at a minimum when the window is opened by 90° and a maximum when fully closed or opened to 180°



Exam Tip

Consider carefully the value of θ, it is the angle between the field lines and the line **normal** (perpendicular) to the plane of the area the field lines are passing through. If it helps, drawing the normal on the area provided will help visualise the correct angle.

Magnetic Flux Linkage

- The **magnetic flux linkage** is a quantity commonly used for solenoids which are made of *N* turns of wire
- Magnetic flux linkage is defined as:

The product of the magnetic flux and the number of turns

- It is calculated using the equation:

$$\Phi N = BAN$$

- Where:
  - Φ = magnetic flux (Wb)
  - N = number of turns of the coil
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)
- The flux linkage ΦN has the units of **Weber turns (Wb turns)**
- As with magnetic flux, if the field lines are not completely perpendicular to the plane of the area they are passing through
- Therefore, the component of the flux density which is perpendicular is equal to:

$$\Phi N = BAN\cos(\theta)$$

Worked Example

A solenoid of circular cross-sectional radius of 0.40 m^2 and 300 turns is facing perpendicular to a magnetic field with magnetic flux density of 5.1 mT.Determine the magnetic flux linkage for this solenoid.

Step 1: Write out the known quantities

Cross-sectional area, $A = \pi r^2 = \pi(0.4)^2 = 0.503\ m^2$

Magnetic flux density, B = 5.1 mT

Number of turns of the coil, N = 300 turns

Step 2: Write down the equation for the magnetic flux linkage

$$\Phi N = BAN$$

Step 3: Substitute in values and calculate

$$\Phi N = (5.1 \times 10^{-3}) \times 0.503 \times 300 = 0.7691 = 0.8 \text{ Wb turns (2 s.f)}$$

Principles of Electromagnetic Induction

- Electromagnetic induction is a phenomenon which occurs when an e.m.f is induced when a conductor moves through a magnetic field
- When the conductor cuts through the magnetic field lines:
  - This causes a change in **magnetic flux**
  - Which causes **work to be done**
  - This work is then transformed into **electrical energy**

- Therefore, if attached to a complete circuit, a current will be induced
- This is known as **electromagnetic induction** and is defined as:

The process in which an e.m.f is induced in a closed circuit due to changes in magnetic flux

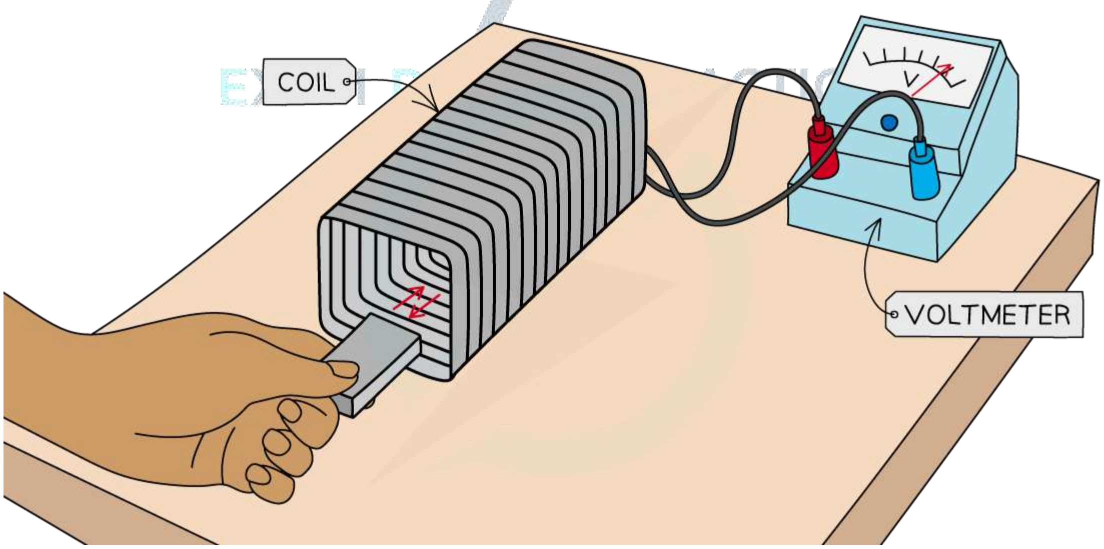
- This can occur either when:
  - A conductor cuts through a magnetic field
  - The direction of a magnetic field through a coil changes

- Electromagnetic induction is used in:
  - Electrical generators which convert mechanical energy to electrical energy
  - Transformers which are used in electrical power transmission

- This phenomenon can easily be demonstrated with a magnet and a coil, or a wire and two magnets

Experiment 1: Moving a magnet through a coil

- When a coil is connected to a sensitive voltmeter, a bar magnet can be moved in and out of the coil to induce an e.m.f



A bar magnet is moved through a coil connected to a voltmeter to induce an e.m.f

The expected results are:

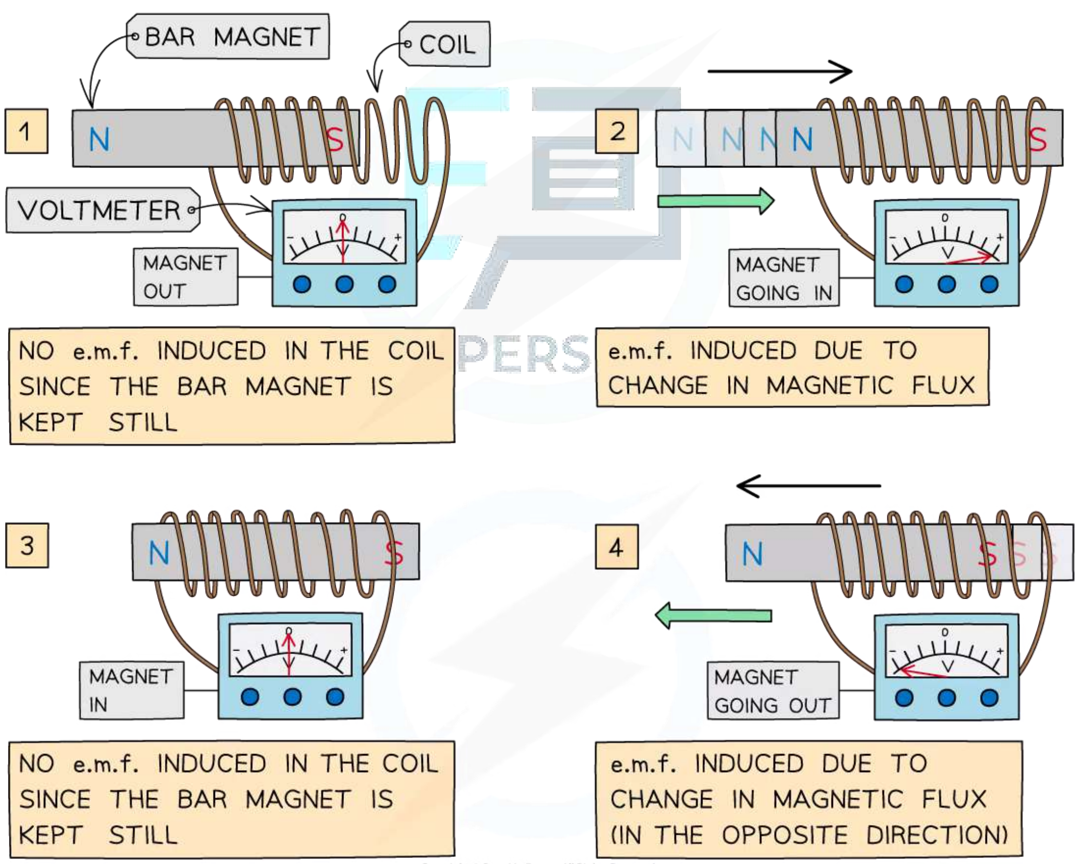
- When the bar magnet is **not moving**, the voltmeter shows a **zero reading**
  - When the bar magnet is held still inside, or outside, the coil, the rate of change of flux is zero, so, there is **no e.m.f induced**

- When the bar magnet begins to move inside the coil, there is a reading on the voltmeter
  - As the bar magnet moves, its magnetic field lines 'cut through' the coil, generating a **change in magnetic flux**
  - This induces an **e.m.f** within the coil, shown momentarily by the reading on the voltmeter

- When the bar magnet is taken back out of the coil, an e.m.f is induced in the **opposite direction**
  - As the magnet changes direction, the direction of the current changes
  - The voltmeter will momentarily show a reading with the opposite sign

- Increasing the **speed** of the magnet induces an e.m.f with a **higher magnitude**
  - As the speed of the magnet increases, the rate of change of flux increases

- The direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it

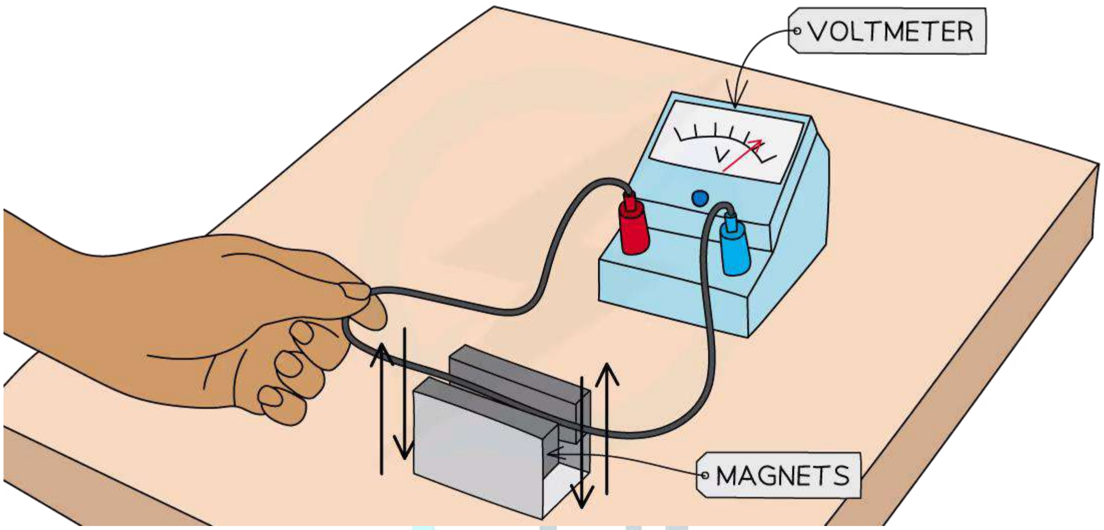


An e.m.f is induced only when the bar magnet is moving through the coil

- Factors that will increase the induced e.m.f are:
  - Moving the magnet faster through the coil
  - Adding more turns to the coil
  - Increasing the strength of the bar magnet

Experiment 2: Moving a wire through a magnetic field

- When a long wire is connected to a voltmeter and moved between two magnets, an e.m.f is induced
- **Note:** there is no current flowing through the wire to start with



A wire is moved between two magnets connected to a voltmeter to induce an e.m.f

The expected results are:

- When the wire is **not moving**, the voltmeter shows a **zero reading**
  - When the wire is held still inside, or outside, the magnets, the rate of change of flux is zero, so, there is **no e.m.f induced**
- As the wire is moved through between the magnets, an **e.m.f** is induced within the wire, shown momentarily by the reading on the voltmeter
  - As the wire moves, it 'cuts through' the magnetic field lines of the magnet, generating a **change in magnetic flux**
- When the wire is taken back out of the magnet, an e.m.f is induced in the **opposite direction**
  - As the wire changes direction, the direction of the current changes
  - The voltmeter will momentarily show a reading with the opposite sign
- As before, the direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it
- Factors that will increase the induced e.m.f are:
  - Increasing the length of the wire

- Moving the wire between the magnets faster
- Increasing the strength of the magnets

Faraday's & Lenz's Laws

- Faraday's law tells us the magnitude of the induced e.m.f in electromagnetic induction and is defined as:

  The magnitude of the induced e.m.f is directly proportional to the rate of change in magnetic flux linkage

$$\varepsilon = N \frac{\Delta\phi}{\Delta t}$$

- Where:
  - ϵ = induced e.m.f (V)
  - N = number of turns of coil
  - $\Delta\phi$ = change in magnetic flux (Wb)
  - Δt = time interval (s)
- Lenz's Law gives the **direction** of the induced e.m.f as defined by Faraday's law:

  The induced e.m.f acts in such a direction to produce effects which oppose the change causing it

- Lenz's law combined with Faraday's law is:

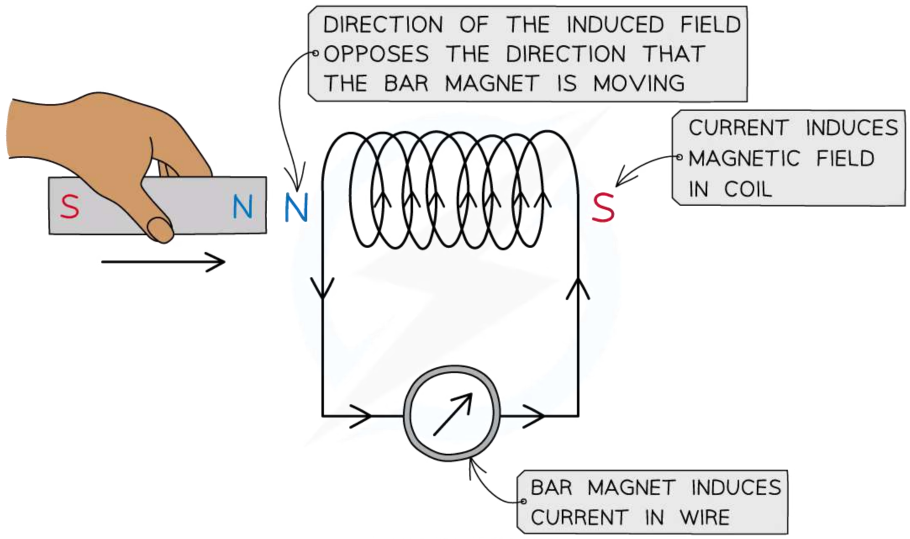
$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

- This equation shows:
  - When a bar magnet goes through a coil, an e.m.f is induced within the coil due to a change in magnetic flux
  - A current is also induced which means the coil now has its own magnetic field
  - The coil's magnetic field acts in the **opposite direction** to the magnetic field of the bar magnet
- If a direct current (d.c) power supply is replaced with an alternating current (a.c) supply, the e.m.f induced will also be alternating with the same frequency as the supply

Experimental Evidence for Lenz's Law

- To verify Lenz's law, the only apparatus needed is:
  - A bar magnet
  - A coil of wire
  - A sensitive ammeter
- **Note:** a cell is **not** required
- A known pole (either north or south) of the bar magnet is pushed into the coil, which induces a magnetic field in the coil
  - Using the right hand grip rule, the curled fingers indicate the direction of the **current** and the thumb indicates the direction of the **induced magnetic field**

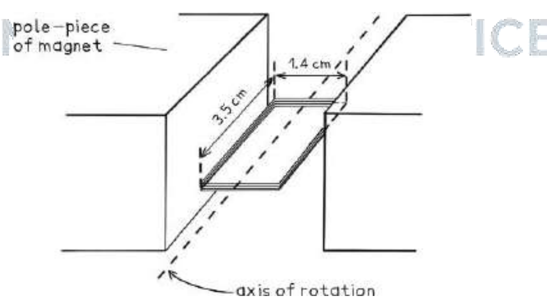
- The direction of the current is observed on the ammeter
  - Reversing the magnet direction would give an opposite deflection on the meter
- The induced field (in the coil) **repels** the bar magnet
- This is because of **Lenz's law**:
  - The direction of the induced field in the coil pushes against the change creating it, ie. the bar magnet



Lenz's law can be verified using a coil connected in series with a sensitive ammeter and a bar magnet

Worked Example

A small rectangular coil contains 350 turns of wire. The longer sides are 3.5 cm and the shorter sides are 1.4 cm.



The coil is held between the poles of a large magnet so that the coil can rotate about an axis through its centre.The magnet produces a uniform magnetic field of flux density 80 mT between its poles. The coil is positioned horizontally and then turned through an angle of 40° in a time of 0.18 s.Calculate the magnitude of the average e.m.f induced in the coil.

Step 1: Write down the known quantities

Magnetic flux density, $B = 80\text{ mT} = 80 \times 10^{-3}\text{ T}$

Area, $A = 3.5 \times 1.4 = (3.5 \times 10^{-2}) \times (1.4 \times 10^{-2}) = 4.9 \times 10^{-4}\text{ m}^2$

Number of turns, $N = 350$

Time interval, $\Delta t = 0.18\text{ s}$

Angle between coil and field lines, $\theta = 40^{\circ}$

Step 2: Write out the equation for Faraday's law:

$$\varepsilon = N\frac{\Delta\phi}{\Delta t}$$

Step 3: Write out the equation for flux linkage:

$$\phi N = BAN\cos(\theta)$$

Step 4: Substitute values into flux linkage equation:

$$\phi N = (80 \times 10^{-3}) \times (4.9 \times 10^{-4}) \times 350 \times \cos(40) = 0.0105\text{ Wb turns}$$

Step 5: Substitute flux linkage and time into Faraday's law equation:

$$\varepsilon = \frac{0.0105}{0.18} = 0.05839 = \mathbf{58\text{ mV}}\ (2\text{ s.f.})$$