A Level Physics CIE

2. Kinematics

CONTENTS

2.1 Equations of Motion

2.1.1 Displacement, Velocity & Acceleration

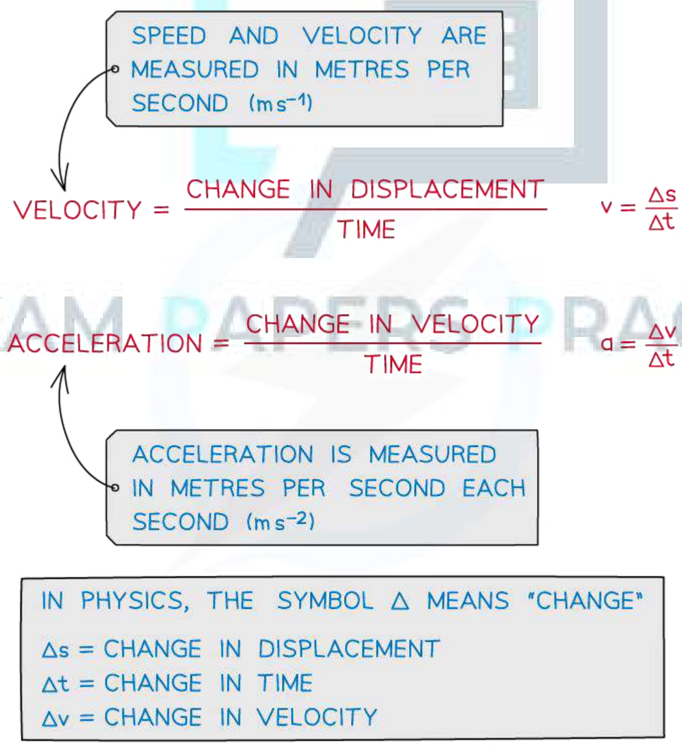
Defining Displacement, Velocity & Acceleration

Scalar quantities

- Remember scalar quantities only have a magnitude (size)
  - **Distance:** the total length between two points
  - **Speed:** the total distance travelled per unit of time

Vector quantities

- Remember vector quantities have both magnitude and direction
  - **Displacement:** the distance of an object from a fixed point in a specified direction
  - **Velocity:** the rate of change of displacement of an object
  - **Acceleration:** the rate of change of velocity of an object

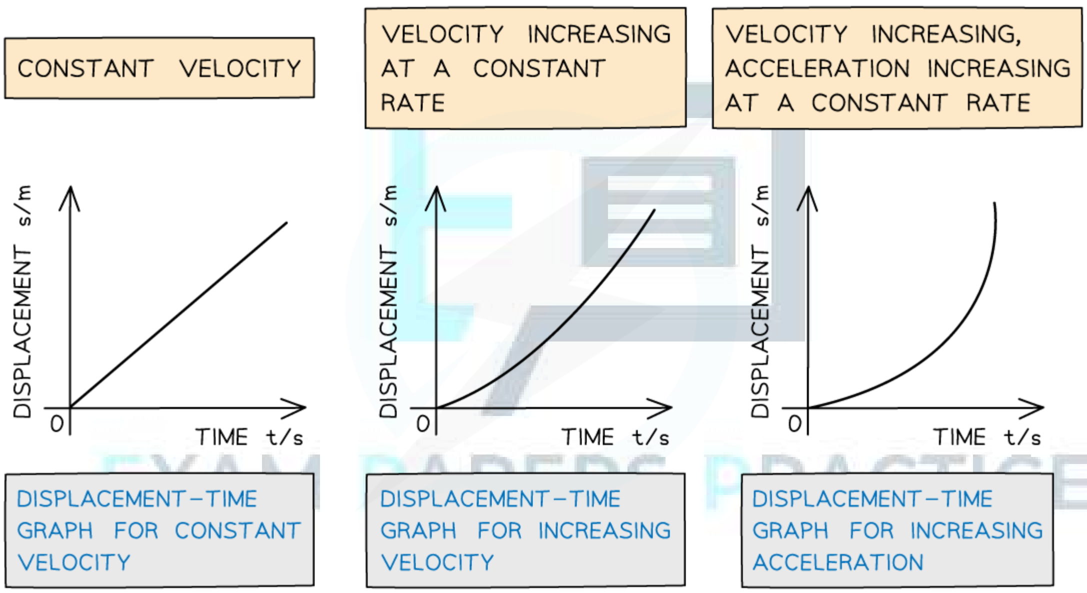


Equations

Equations linking displacement, velocity and acceleration

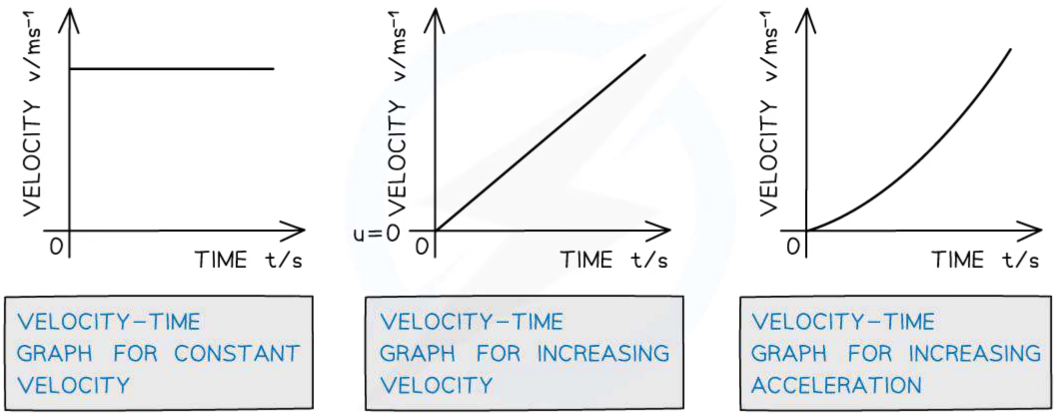
Motion Graphs

- Three types of graph that can represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- On a **displacement-time graph...**
  - **slope** equals **velocity**
  - the **y-intercept** equals the **initial displacement**
  - a **straight**(diagonal) line represents a **constant** velocity
  - a **curved** line represents an **acceleration**
  - a **positive slope** represents motion in the **positive direction**
  - a **negative slope** represents motion in the **negative direction**
  - a **zero** slope (horizontal line) represents a state of **rest**
  - the area under the curve is meaningless



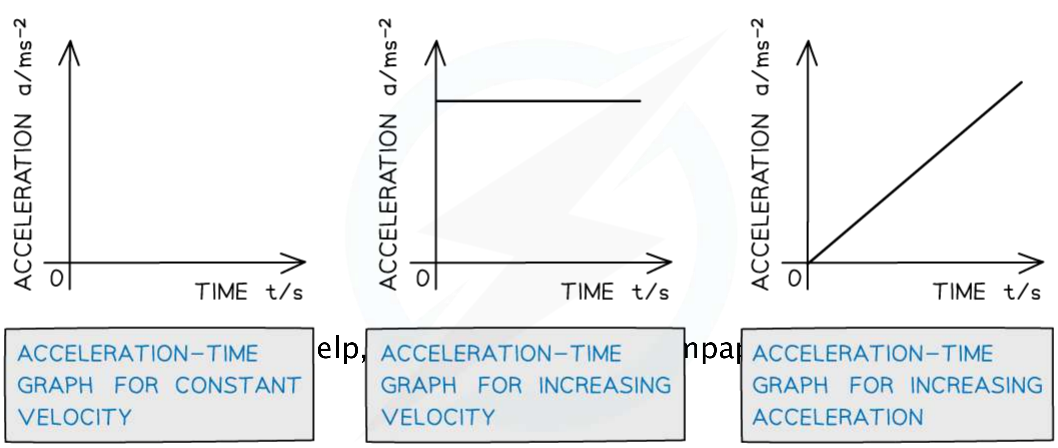
On a velocity-ti

- **me graph... slope** equals **acceleration**
  - the **y-intercept** equals the **initial velocity**
  - a **straight** line represents **uniform** acceleration
  - a **curved** line represents **non-uniform acceleration**
  - a **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - a **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - a **zero** slope (horizontal line) represents motion with **constant velocity**
  - the **area** under the curve equals the **displacement** or **distance** travelled



- On an **acceleration-time graph...**
  - slope is meaningless
  - the y-intercept equals the initial acceleration
  - a zero slope (horizontal line) represents an object undergoing constant acceleration
  - the area under the curve equals the change in velocity

How displacement, velocity and acceleration graphs relate to each other



2.1.3 Area under a Velocity-Time Graph

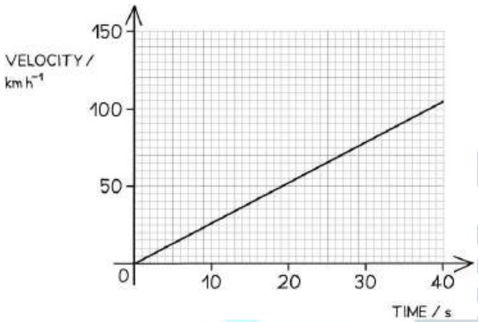
Area under a Velocity-Time Graph

- Velocity-time graphs show the speed and direction of an object in motion over a specific period of **time**
- The area under a velocity-time graph is equal to the **displacement** of a moving object

displacement = area under a velocity-time graph

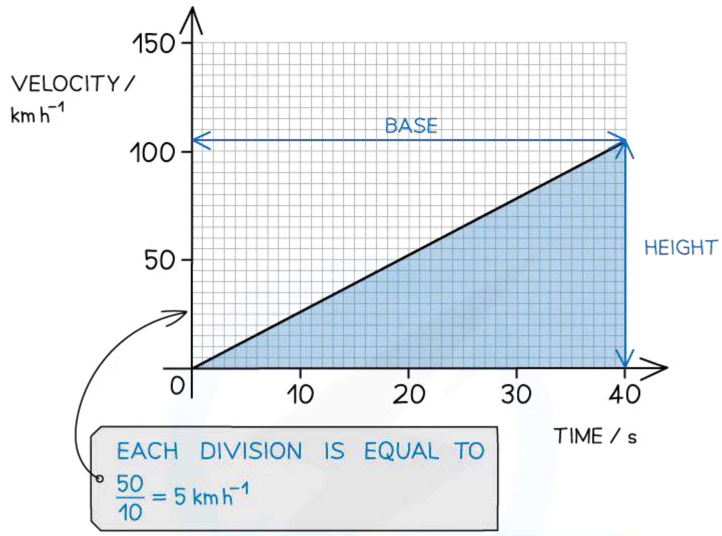
Worked Example

The velocity-time graph of a vehicle travelling with uniform acceleration is shown in the diagram below.



Calculate the displacement of the vehicle at 40 s.

THE DISPLACEMENT IS EQUAL TO THE AREA UNDER A VELOCITY–TIME GRAPH



CONVERT $km\,h^{-1}$ TO $km\,s^{-1}$

BASE = TIME = 40s

HEIGHT = VELOCITY = 105 $km\,h^{-1}$

$\frac{105}{60 \times 60} = 0.0292\ km\,s^{-1}$

WORK OUT THE DISPLACEMENT

AREA OF A TRIANGLE = $\frac{1}{2} \times$ BASE $\times$ HEIGHT

DISPLACEMENT = VELOCITY $\times$ TIME = $\frac{1}{2} \times 40 \times 0.0292 = 0.6\ km$ OR 600 m

Exam Tip

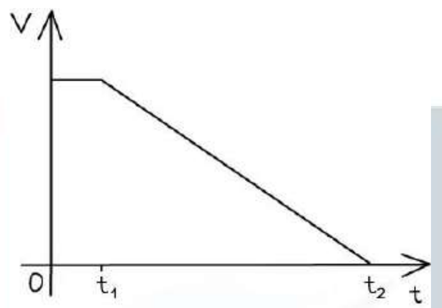
Always check the values given on the y-axis of a motion graph – students often confuse displacement-time graphs and velocity-time graphs.The area under the graph can often be broken down into triangles, squares and rectangles, so make sure you are comfortable with calculating area!

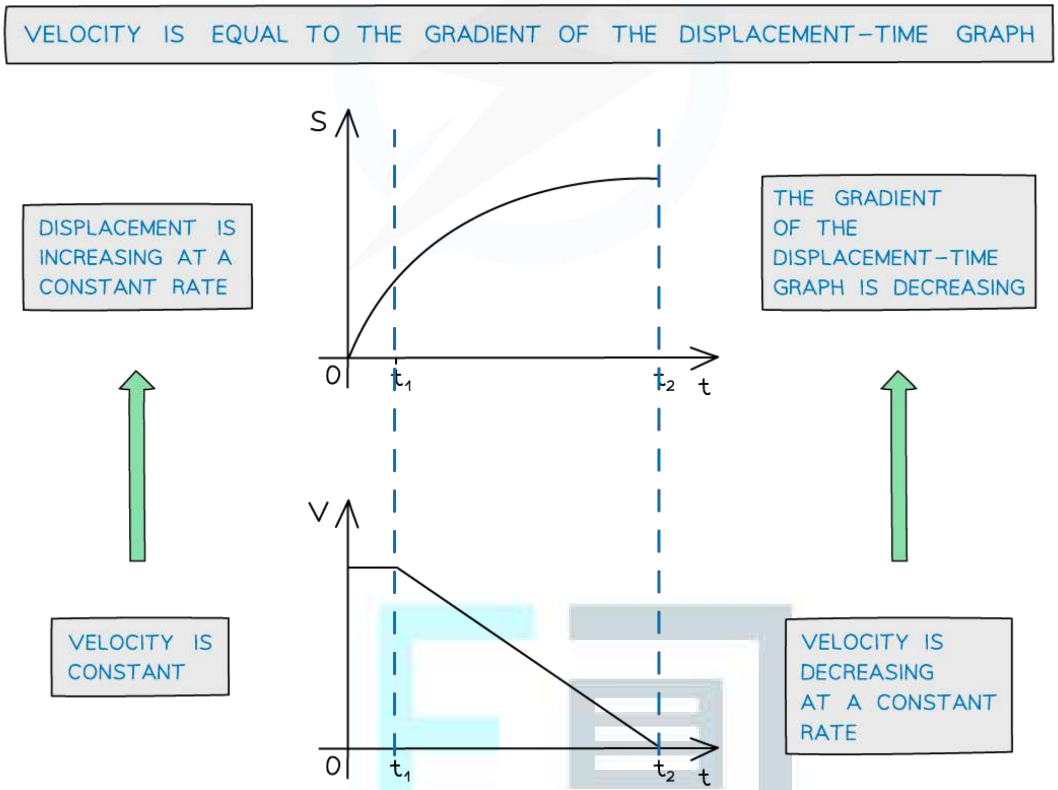
Gradient of a Displacement-Time Graph

- Displacement-time graphs show the changing position of an object in motion
- They also show whether an object is moving forwards (positive displacement) or backwards (negative displacement)
    - A negative gradient = a negative velocity (the object is moving backwards)
- The gradient (slope) of a displacement-time graph is equal to **velocity**
    - The greater the slope, the greater the velocity

Worked Example

A car driver sees a hazard ahead and applies the brakes to bring the car to rest.What does the displacement-time graph look like?





Exam Tip

Don't forget that velocity is a vector quantity; it has a size and a direction. If velocity is initially positive and then becomes negative, then the object has changed direction.

Gradient of a Velocity-Time Graph

- **Acceleration** is any change in the velocity of an object in a given time

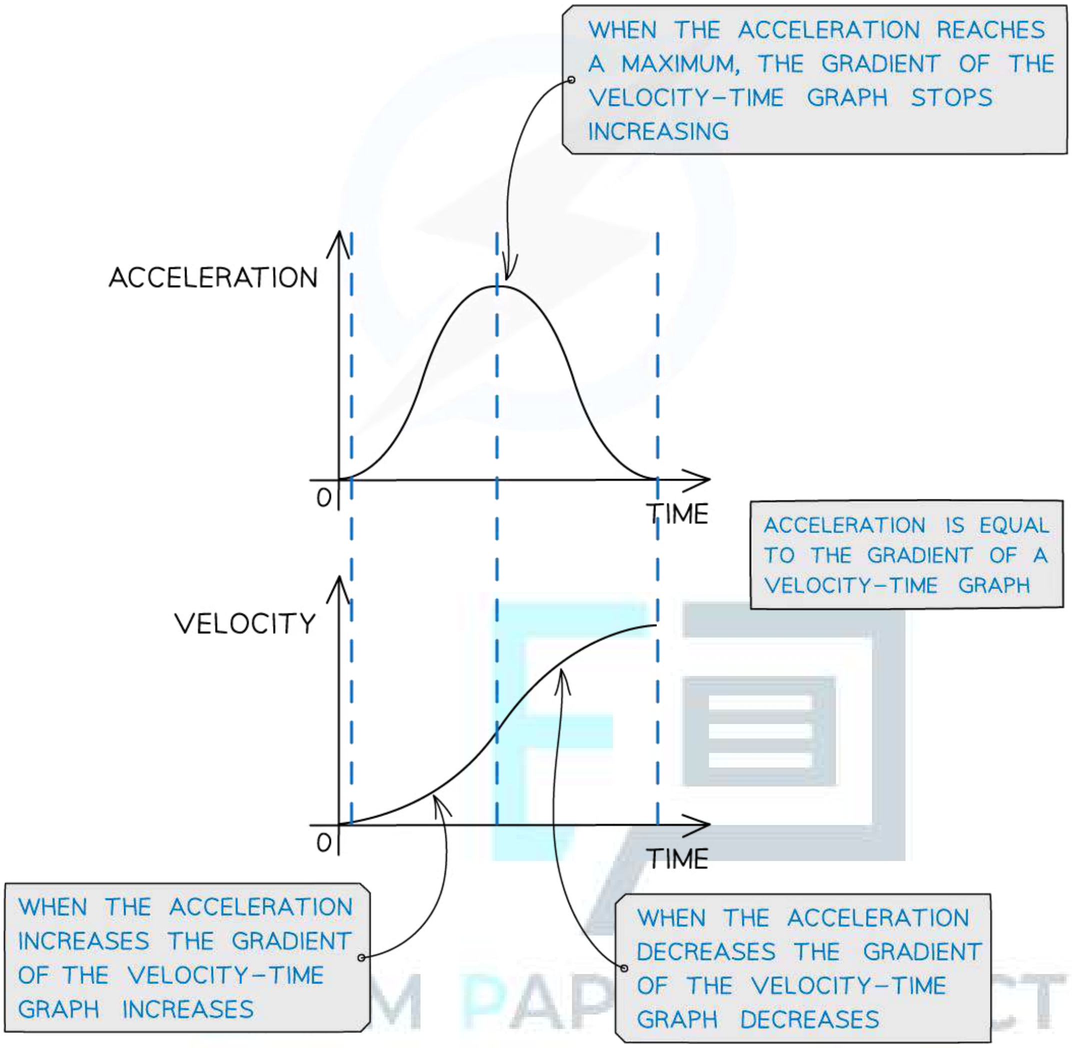
$$acceleration = \frac{change\ in\ velocity}{time} = \frac{(v - u)}{t}$$

- As velocity is a vector quantity, this means that if the **speed** of an object **changes**, or its **direction changes**, then it is accelerating
  - An object that slows down tends to be described as 'decelerating'
- The gradient of a velocity-time graph is equal to **acceleration**

Worked Exan

What does the
graph?

ACCE

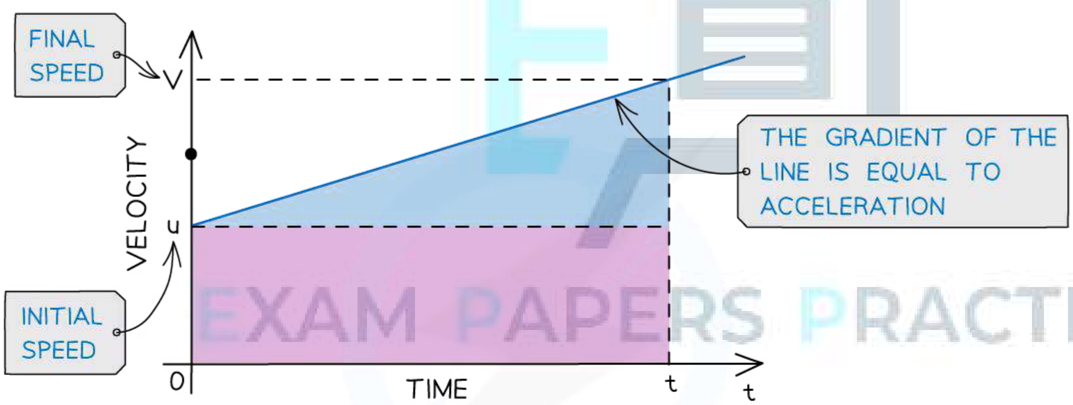

WHEN THE ACCELERATION REACHES A MAXIMUM, THE GRADIENT OF THE VELOCITY−TIME GRAPH STOPS INCREASING
ACCELERATION
0
TIME
ACCELERATION IS EQUAL TO THE GRADIENT OF A VELOCITY−TIME GRAPH
VELOCITY
0
TIME
WHEN THE ACCELERATION INCREASES THE GRADIENT OF THE VELOCITY−TIME GRAPH INCREASES
WHEN THE ACCELERATION DECREASES THE GRADIENT OF THE VELOCITY−TIME GRAPH DECREASES

Deriving Kinematic Equations of Motion

- The kinematic equations of motion are a set of four equations which can describe any object moving with **constant** acceleration
- They relate the five variables:
  - s = **displacement**
  - u = **initial velocity**
  - v = **final velocity**
  - a = **acceleration**
  - t = **time interval**

- It's important to know where these equations come from and how they are derived:

Deriving $v = u + at$



THE VELOCITY–TIME GRAPH SHOWS A STRAIGHT LINE, THEREFORE, THE OBJECT'S ACCELERATION IS CONSTANT

FROM THE GRADIENT WE CAN DEDUCE ACCELERATION IS EQUAL TO

- $\frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{(v - u)}{t}$
- $a = \frac{(v - u)}{t}$

MULTIPLY BOTH SIDES BY t

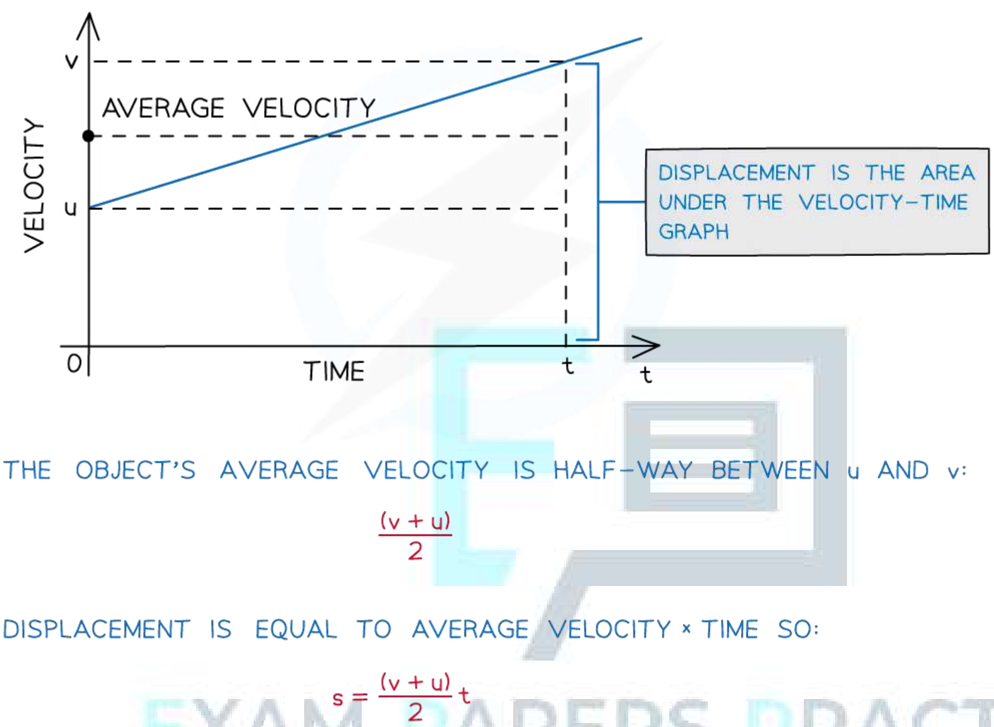
- $at = (v - u)$

REARRANGING LEADS TO

- $v = u + at$

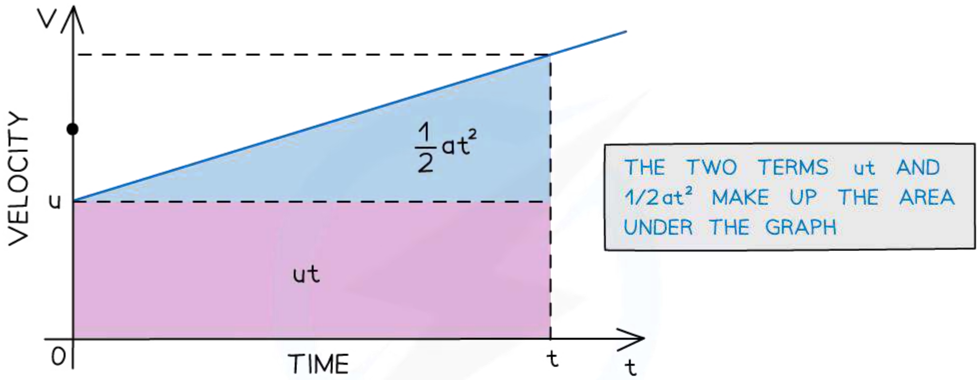
A graph showing how the velocity of an object varies with time

Deriving $s = \frac{(u + v)}{2}t$



The average velocity is halfway between u and v

Deriving $s = ut + \frac{1}{2}at^2$



TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at$ (1)
- $s = \frac{(v + u)}{2}t$ (2)

SUBSTITUTING EQUATION (1) AND (2)

MULTIPLY EVERYTHING IN THE BRACKET BY t

- $s = \frac{(u + u + at)}{2}t$

SEPARATE THE t AND t² TERMS

- $s = \frac{2ut}{2} + \frac{at^2}{2}$
- $s = ut + \frac{1}{2}at^2$

The two terms ut and ½at² make up the area under the graph

Deriving $v^2 = u^2 + 2as$

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at \longrightarrow t = \frac{v - u}{a}$ (1)

$s = \frac{(v + u)}{2} t$ (2)

SUBSTITUTING (1) INTO (2)

- $s = \frac{(v + u)}{2} \times \frac{(v - u)}{a}$
- $s = \frac{v^2 - u^2}{2a}$ MULTIPLY BOTH SIDES BY 2a
- $v^2 = u^2 + 2as$

This final equation can be derived from two of the others

SUMMARY

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{(v + u)}{2} t$

$v^2 = u^2 + 2as$

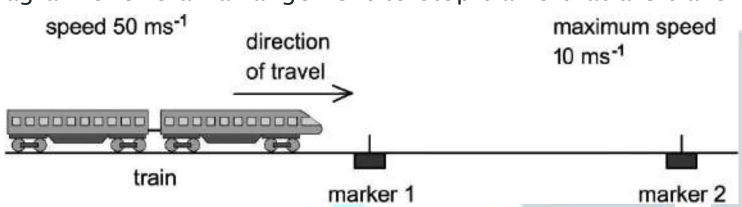
Summary of the four equations of uniformly accelerated motion

Solving Problems with Kinematic Equations

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm$ 9.81 m s^{-2}, an object which starts or finishes at rest will have u = 0 or v = 0
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links s, u, a and t is $s = ut + \frac{1}{2}at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

Worked Example

The diagram shows an arrangement to stop trains that are travelling too fast.



Trains coming from the left travel at a speed of 50 ms^{-1}. At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 ms^{-1}.The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s.How far from marker 2 should marker 1 be placed?

STEP 1

OUR KNOWN VARIABLES ARE

- $u = 50\ ms^{-1}$
- $v = 10\ ms^{-1}$
- $t = 20\ s$

AND WE ARE ASKED TO FIND DISTANCE, s.

STEP 2

SO THE EQUATION THAT LINKS u, v, t AND s IS

$$s = \frac{(u+v)}{2}t$$

STEP 3

NO REARRANGING IS REQUIRED SO WE SIMPLY PLUG IN THE VARIABLES:

$$s = \frac{(50+10)}{2} \times 20 = 30 \times 20 = 600\ m$$

Exam Tip

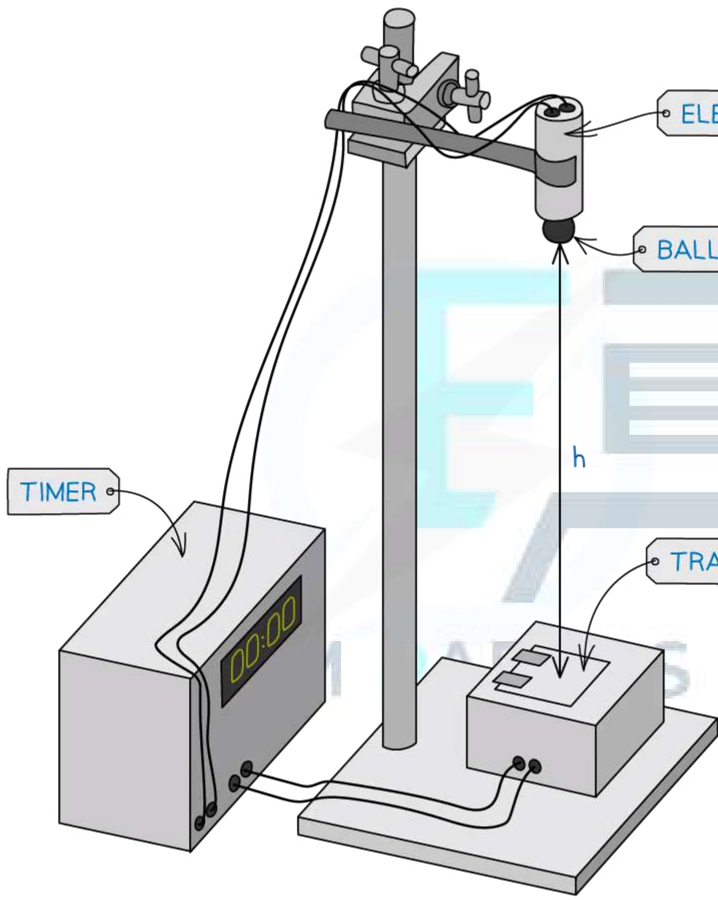
- This is arguably the most important section of this topic, you can always be sure there will be one, or more, questions in the exam about solving problems with the kinematic equations
- The best way to master this section is to practice as many questions as possible

Acceleration of Free Fall Experiment

- A common experiment to determine acceleration of a falling object which can be carried out in the lab

Apparatus

- Metre rule, ball bearing, electromagnet, electronic timer, trapdoor



Apparatus used to measure g

Method

- When the current to the magnet switches off, the ball drops and the timer starts
- When the ball hits the trapdoor, the timer stops
- The reading on the timer indicates the time it takes for the ball to fall a distance, h
- This procedure is repeated several times for different values of h, in order to reduce random error
- The distance, h, can be measured using a metre rule as it would be preferable to use for distances between 20 cm – 1 m

Analysing data

- To find g , use the same steps as in the problem solving section
- The known quantities are
  - Displacement $s = h$
  - Time taken $= t$
  - Initial velocity $u = 0$
  - Acceleration $a = g$
- The equation that links these quantities is
  - $s = ut + ½ at^2$
  - $h = ½ gt^2$
- Using this equation, deduce g from the gradient of the graph of h against t^2

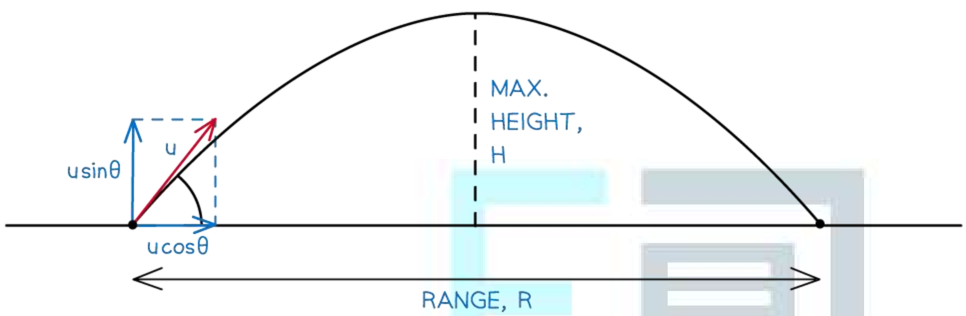
Sources of error

- **Systematic error:** residue magnetism after the electromagnet is switched off may cause the time to be recorded as longer than it should be
- **Random error:** large uncertainty in distance from using a metre rule with a precision of 1mm, or from parallax error

2.1.9 Projectile Motion

Projectile Motion

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance travelled by the projectile



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ \text{ms}^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \dfrac{u\sin\theta}{g}$

$2t = \dfrac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

$2gH = (u\sin\theta)^2$

$H = \frac{(u\sin\theta)}{2g}$

HORIZONTAL MOTION (→)

INITIAL SPEED, $u = u\cos\theta$

ACCELERATION, $a = 0$

DISPLACEMENT = R

RANGE (R)

$u = u\cos\theta \quad t = \frac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

DISTANCE = SPEED × TIME

$R = (u\cos\theta)t$

$R = \frac{2u^2\sin\theta\cos\theta}{g}$

USING THE TRIG IDENTITY:

$2\sin\theta\cos\theta = \sin 2\theta$

$R = \frac{u^2\sin 2\theta}{g}$

How to find the time of flight, maximum height and range

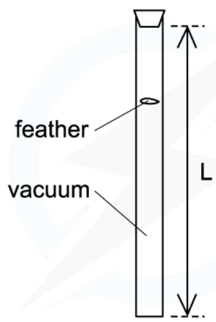
- **Remember:** the only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**
- Let's consider each in turn:

Worked Example

To calculate vertical projection(free fall)

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



What it the length of the tube, L?

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ ms^{-2}$ $\quad u = 0$ $\quad t = 0.5\ s$ $\quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

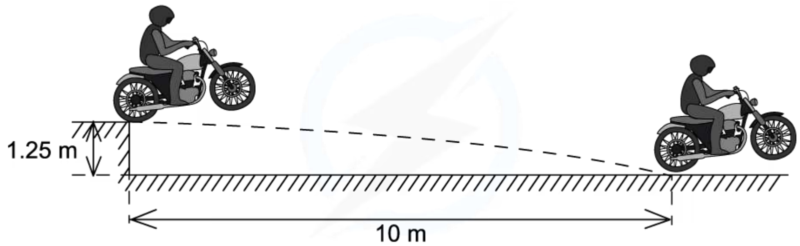
$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ m$$

Worked Example

To calculate horizontal projection

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off?



IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $\quad a = 9.81$ ms^{-2} $\quad u = 0$ $\quad t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $\quad a = 0$ $\quad t = 0.5$ s $\quad u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

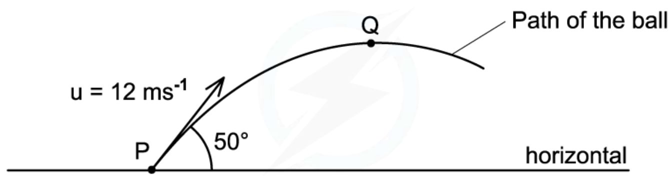
$$v = \frac{10}{0.5} = 20 \text{ ms}^{-1}$$

Worked Example

To calculate projection at an angle

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q?



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $\quad$ $a = -9.81\ \text{ms}^{-2}$ $\quad$ $v = 0$ $\quad$ $H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ \text{m}$$

Exam Tip

Make sure you don't make these common mistakes:

- Forgetting that deceleration is negative as the object rises
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres