2.8 Inequalities

IB Maths - Revision Notes

AA HL

2.8.1 Solving Inequalities Graphically

Solving Inequalities Graphically

How can I solve inequalities graphically?

- Consider the inequality ***f*(*x*) ≤ *g*(*x*)**, where *f*(*x*) and *g*(*x*) are functions of *x*
  - if we **move *g*(*x*) to the LHS** we get
    - ***f*(*x*) – *g*(*x*) ≤ 0**
- Solve ***f*(*x*) – *g*(*x*) = 0 to** find the **zeros** of *f*(*x*) – *g*(*x*)
  - These correspond to the *x*-coordinates of the points of intersection of the graphs ***y* = *f*(*x*)** and ***y* = *g*(*x*)**
- To solve the inequality we can use a **graph**
  - **Graph *y* = *f*(*x*) – *g*(*x*)** and label its zeros
  - Hence find the intervals of *x* that satisfy the inequality *f*(*x*) – *g*(*x*) ≤ 0
    - These are the **intervals which satisfies the original inequality** *f*(*x*) ≤ *g*(*x*)
  - This method is particularly useful when finding the intersections between the functions is difficult due to needing large *x* and *y* windows on your GDC

Be careful when rearranging inequalities!

- Remember to **flip the sign** of the inequality when you **multiply or divide** both sides by a **negative** number
  - e. **1 < 2** → [times both sides by (–1)] → **–1 > –2** (sign flips)
- **Never multiply or divide** by a **variable** as this could be **positive or negative**
  - You can only multiply by a term if you are certain it is always positive (or always negative)
    - Such as x^2, $|x|$, e^x
- Some **functions reverse the inequality**
  - Taking reciprocals of positive values
    - $0 < x < y \Rightarrow \frac{1}{x} > \frac{1}{y}$
  - Taking logarithms when the base is 0 < *a* < 1
    - $0 < x < y \Rightarrow \log_a(x) > \log_a(y)$
- The **safest way** to rearrange is simply to add & subtract to move all the terms onto one side

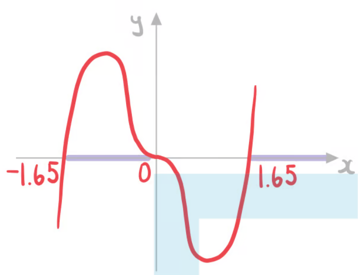
Worked example

Use a GDC to solve the inequality $2x^3 < x^5 - 2x$.

Rearrange to get one side as zero

$x^5 - 2x^3 - 2x > 0$

On GDC sketch $y = x^5 - 2x^3 - 2x$ and find zeros



Identify the sections where the graph is above the x-axis

$-1.65 < x < 0$ or $x > 1.65$

2.8.2 Polynomial Inequalities

Polynomial Inequalities

How do I solve polynomial inequalities?

- **STEP 1**: **Rearrange the inequality** so that **one of the sides is equal to zero**
  - For example: $P(x) \le 0$
- **STEP 2**: Find the **roots** of the polynomial
  - You can do this by factorising or using GDC to solve $P(x) = 0$
- **STEP 3**: Choose one of the following methods:
- **Graph method**
  - Sketch a graph of the polynomial (with or without a GDC)
  - Choose the intervals for x corresponding to the sections of the graph that satisfy the inequality
    - For example: for $P(x) \le 0$ you would want the sections below the x-axis
- **Sign table method**
  - If you are unsure how to sketch a polynomial graph then this method is best
  - **Split the real numbers** into the possible **intervals** using the roots
    - If the roots are a and b then the intervals would be $x<a$, $a<x<b$, $x>b$
  - **Test a value** from each interval using the inequality
    - Choose a value within an interval and substitute into $P(x)$ to determine if it is positive or negative
  - Alternatively if the polynomial is factorised you can **determine the sign of each factor** in each interval
    - An odd number of negative factors in an interval will mean the polynomial is negative on that interval
  - If the value satisfies the inequality then that interval is part of the solution

Exam Tip

- In exams most solutions will be intervals but some could be a single point
  - For example: Solution to $(x-3)^2 \le 0$ is $x = 3$

Worked example

Solve the inequality $x^3 + 2x^2 > x + 2$ using an algebraic method.

Rearrange $x^3 + 2x^2 - x - 2 > 0$

Let $P(x) = x^3 + 2x^2 - x - 2$

Find a factor $P(1) = 0 \quad \therefore \quad (x-1)$ is a factor

Factorise $(x-1)(x^2+3x+2) > 0$ Compare coefficients or use division

$(x-1)(x+1)(x+2) > 0$

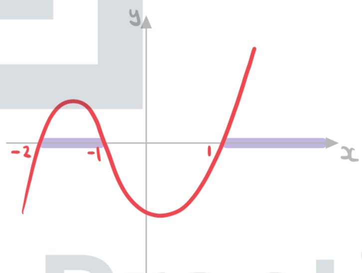
Find the roots $1, -1, -2$

Construct a sign table

For $x < -2$:	For $-2 < x < -1$:	For $-1 < x < 1$:	For $x > 1$:
$(x+2) < 0$	$(x+2) > 0$	$(x+2) > 0$	$(x+2) > 0$
$(x+1) < 0$	$(x+1) < 0$	$(x+1) > 0$	$(x+1) > 0$
$(x-1) < 0$	$(x-1) < 0$	$(x-1) < 0$	$(x-1) > 0$
$\therefore P(x) < 0$	$\therefore P(x) > 0$	$\therefore P(x) < 0$	$\therefore P(x) > 0$
✗	✓	✗	✓

-2, -1, 1

Or sketch



Choose the regions that satisfy the inequality

$-2 < x < -1$ or $x > 1$