2.5 Transformations of Graphs

Question Paper
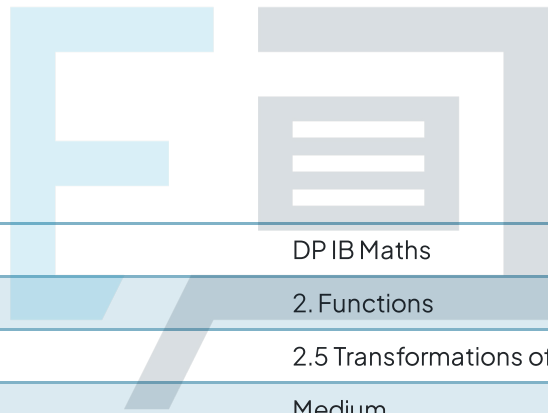
Course	DP IB Maths
Section	2. Functions
Topic	2.5 Transformations of Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

The point $P(-1,4)$ lies on the curve with equation $y = f(x)$. .

State the coordinates of the image of point P on the curves with the following equations:

(i)
$y = f(x) + 3$

(ii)
$y = f(x + 3)$

(iii)
$y = 3f(x)$

(iv)
$y = f(3x)$

[4 marks]

Question 2

The point $P(-3, -4)$ lies on the curve with equation $y = f(x)$. .

State the coordinates of the image of point P on the curves with the following equations:

(i)
$y = f(-x)$
(ii)
$y = -f(x)$

[2 marks]

Question 3

The point $P(3,2)$ lies on the curve with equation $y = f(x).$

(i)

On the graph of $y = f(x) + a$, where a is a constant, the point P is mapped to the point $(3, -5)$. Determine the value of a.

(ii)

On the graph of $y = f(x + b)$, where b is a constant, the point P is mapped to the point $(-1, 2).$ Determine the value of b.

(iii)

On the graph of $y = cf(x),$, where c is a constant, the point P is mapped to the point $(3, 1)$. Determine the value of c.

(iv)

On the graph of $y = f(dx),$, where d is a constant, the point P is mapped to the point $(1, 2)$. Determine the value of d.
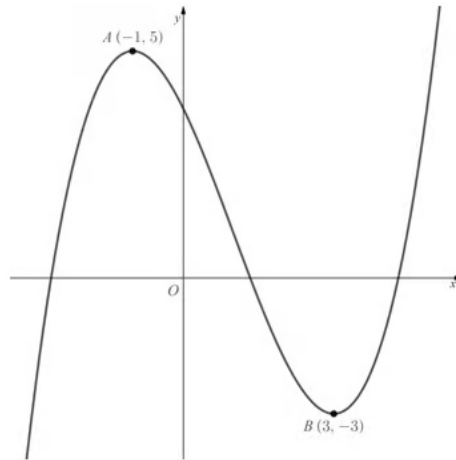
[4 marks]

Question 4a

The diagram below shows the graph of $y = \mathrm{f}(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.

In separate diagrams, sketch the curves with equation

(i)
$$y = \mathrm{f}(x - 1)$$

(ii)
$$y = \mathrm{f}(x) + 3$$

On each diagram, give the coordinates of the images of points A and B under the given transformation.
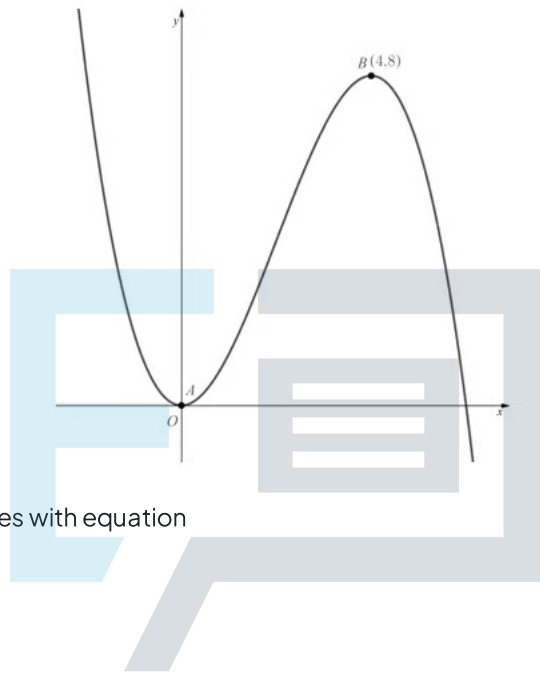
[4 marks]

Question 4b

On the graph of $y = \mathrm{f}(x + a)$ the image of one of the two marked points has an x coordinate of 2. Find the two possible values of a.

[2 marks]

Question 5a

The diagram below shows the graph of $y = \mathrm{f}(x)$.. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A.

In separate diagrams, sketch the curves with equation

(i)
$$y = -\mathrm{f}(x)$$

(ii)
$$y = \mathrm{f}(4x)$$

On each diagram, give the coordinates of the images of points A and B under the given transformation.
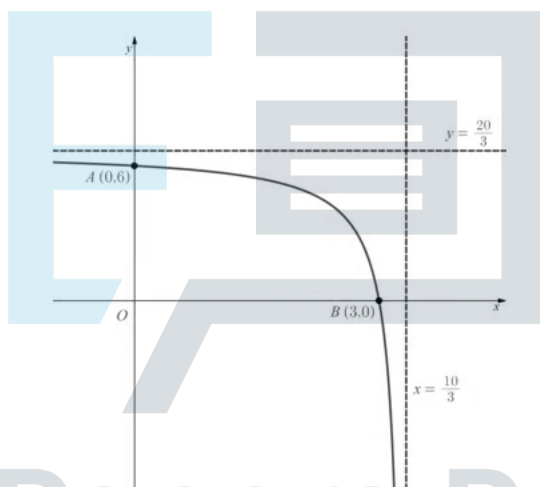
[4 marks]

Question 5b

On the graph of $y = af(x)$ the image of one of the two marked points has a y coordinate of 4. Find the value of a.

[2 marks]

Question 6a

The diagram below shows the graph of $y = f(x)$. The graph intersects the coordinate axes at the two marked points $A(0,6)$ and $B(3,0)$. The graph has two asymptotes as shown, with equations $y = \dfrac{20}{3}$ and $x = \dfrac{10}{3}$

In separate diagrams, sketch the curves with equation

(i) $y = f(x) - 6$

(ii) $y = f(-x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

[6 marks]

Question 6b

The graph of $y = f(x + a)$ has an asymptote at one of the coordinate axes. Find the value of a.

[2 marks]

Question 7

Describe, in order, a sequence of transformations that maps the graph of $y = f(x)$ onto the following graphs:

(i)
$y = 3f(x + 2)$

(ii)
$y = f(-x) - 1$

[3 marks]

Question 8

Given that $f(x) = 3x^2 - 2x$ find an expression for $g(x)$, where $g(x)$ is obtained by applying the following sequence of transformations to $f(x)$.

Translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Vertical stretch of scale factor 4

Translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

[4 marks]

Question 9a

(i)
Sketch the graph of $y = p(x)$, where $p(x) = 3x - 4$.

(ii)
On the same set of axes, sketch the graph of $y = p^{-1}(x)$.
Label the coordinates of the points where each graph crosses the coordinate axes

[4 marks]

Question 9b

(i)
Find an expression for $p^{-1}(x)$.

(ii)
Find an expression for $\dfrac{1}{9}\,[p(x) + 16]$.

(iii)
What can you deduce about the sequence of transformations given
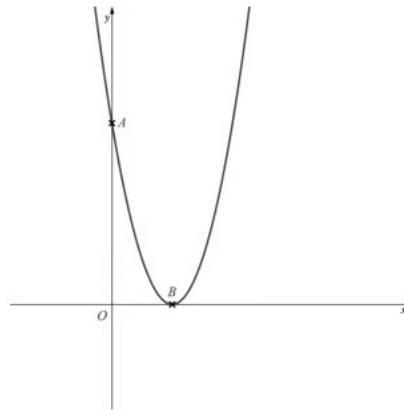by $\dfrac{1}{9}[p(x) + 16]$?

[4 marks]

Question 10a

The equation $y = f(x)$, where $f(x) = (x - a)^2$, with $a > 1$, is shown below.

The points A and B are the points where the graph intercepts the coordinate axes.

Write down, in terms of a, the coordinates of A and B.

[2 marks]

Question 10b

Sketch the graph of $y = -f(-x)$, labelling the images of the points A and B stating their coordinates in terms of a.

[3 marks]

Question 10c

Write down the value of a such that the point A is three times as far from the origin as the point B.

[1 mark]

Question 11

The function $f(x)$ is to be transformed by a sequence of functions, in the order detailed below:

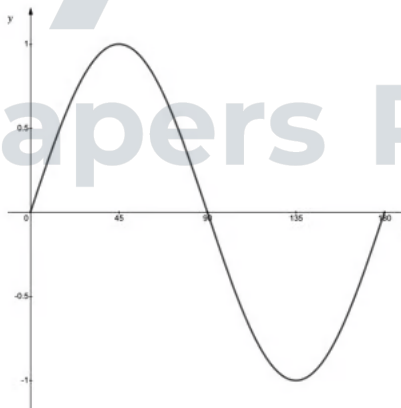A horizontal stretch by scale factor 2

A reflection in the x-axis

A translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Write down an expression for the combined transformation in terms of $f(x)$.

[3 marks]

Question 12a

The diagram shows the graph of $y = f(t)$, where $f(t) = \sin 2t, 0° \le x \le 180°$.

(i)
Write down the maximum value of y when $y = 3f(t)$.

(ii)
Write down the first value of t for which this maximum occurs.

[2 marks]

Question 12b

(i)
Write down the minimum value of when $y = 5f(t + 30°)$.

(ii)
Write down the first value of t for which this minimum occurs.

[2 marks]

Question 12c

Find, in terms of $f(t)$, the combination of transformations that would map the graph of $y = f(t)$ onto the graph of $y = 2 + \sin t$, $0° \leq x \leq 180°$.

[2 marks]

Question 13a

Let $f(x) = 3x^2 + 18x + 27$.

Write down the value of $f(-3)$.

[1 mark]

Question 13b

The function f can be written in the form of $f(x) = a(x - h)^2 + k$.

Find the values of a, h and k.

[3 marks]

Question 13c

The graph of g is obtained from the graph of f by a reflection in the x-axis followed by a translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Find $g(x)$, giving your answer in the form of $g(x) = rx^2 + sx + t$.

[4 marks]