Exam Papers Practice

2.5 Transformations of Graphs

Mark Schemes

Course	DP IB Maths
Section	2. Functions
Topic	2.5 Transformations of Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

i) VERTICAL TRANSLATION y CHANGES $+3$

$(-1, 7)$

ii) HORIZONTAL TRANSLATION x CHANGES -3

$(-4, 4)$

iii) VERTICAL STRETCH y CHANGES $\times 3$

$(-1, 12)$

iv) HORIZONTAL STRETCH x CHANGES $\times \frac{1}{3}$ $\div 3$

$\left(-\frac{1}{3}, 4\right)$

Question 2

i) HORIZONTAL REFLECTION y AXIS x CHANGES

$(3, -4)$

ii) VERTICAL REFLECTION x AXIS y CHANGES

$(-3, 4)$

Question 3

i) VERTICAL TRANSLATION y CHANGED

$(3,2) \Rightarrow (3,-5)$ $+a$

$a = -7$

ii) HORIZONTAL TRANSLATION x CHANGED

$(3,2) \Rightarrow (-1,2)$ $-b$

$b = 4$

iii) VERTICAL STRETCH y CHANGED

$(3,2) \Rightarrow (3,1)$ $\times c$

$c = \frac{1}{2}$

iv) HORIZONTAL STRETCH x CHANGED

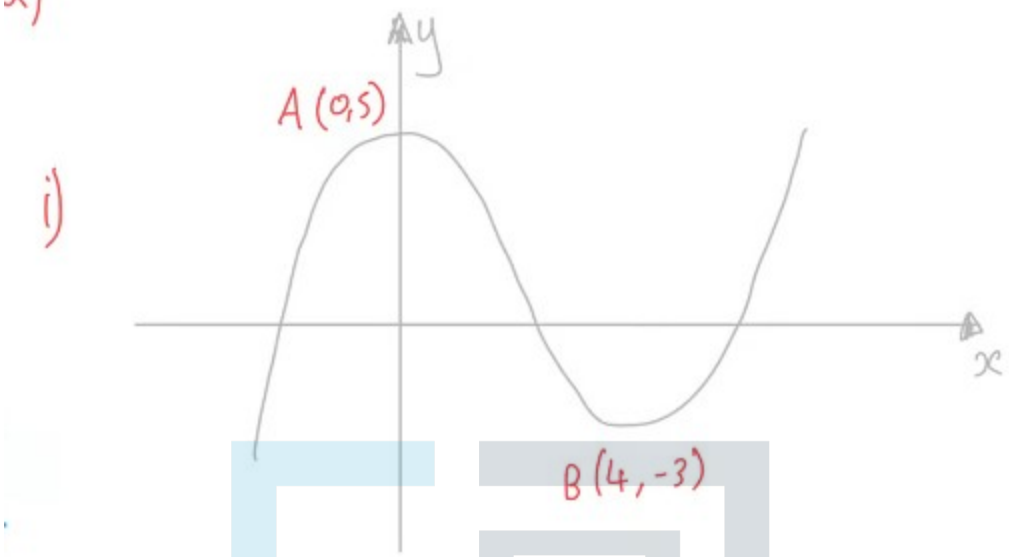
$(3,2) \Rightarrow (1,2)$ $\times \frac{1}{d}$

$d = 3$

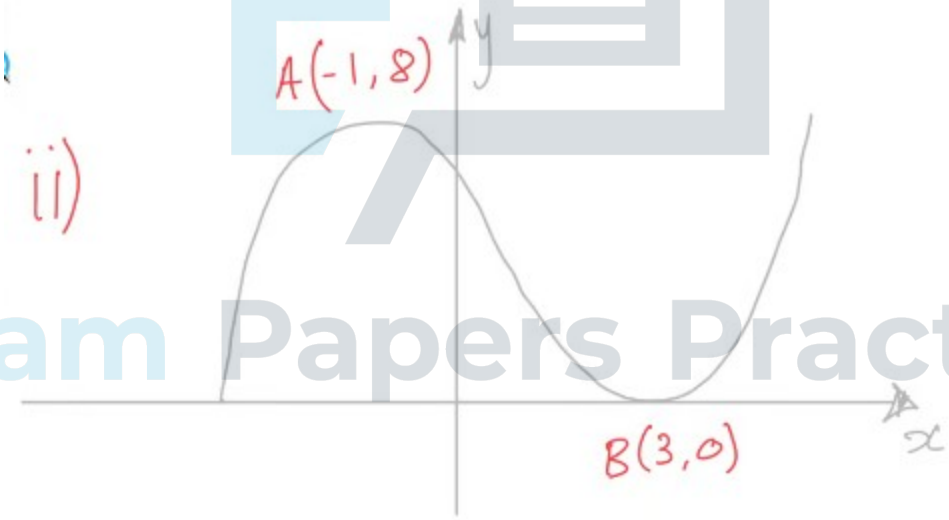
Question 4

a)

i)



ii)



b) HORIZONTAL TRANLATION x CHANGE $-a$

$A = (-1, 5) \Rightarrow (2, 5)$ $a = -3$

$+3$

$B = (3, -3) \Rightarrow (2, -3)$ $a = 1$

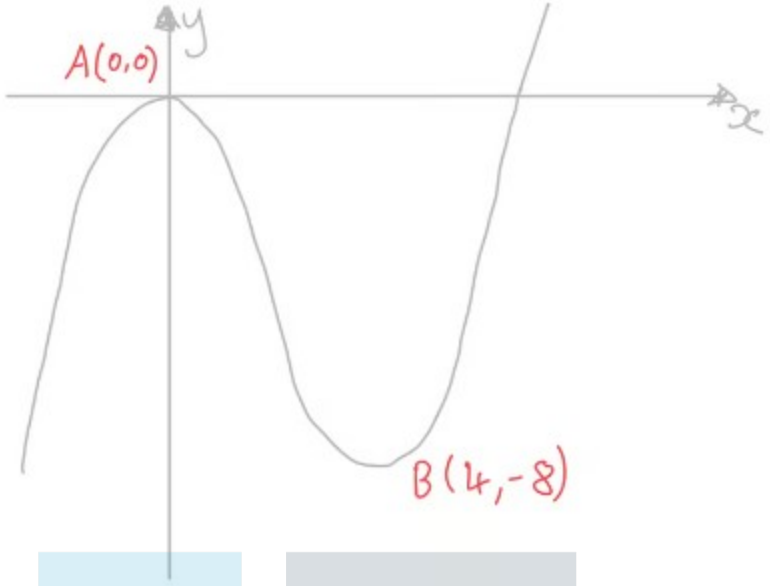
-1

$a = -3$ OR $a = 1$

Question 5

a)

i)



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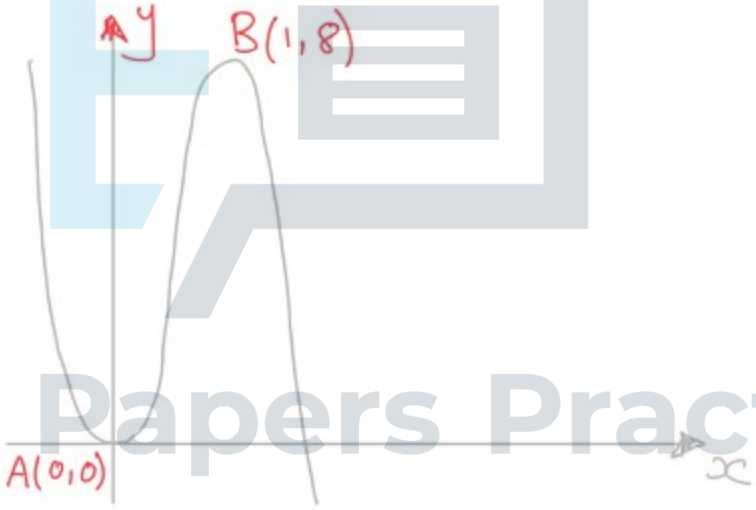


b) VERTICAL STRETCH y CHANGES $\times a$

$A = (0,0)$ WONT CHANGE

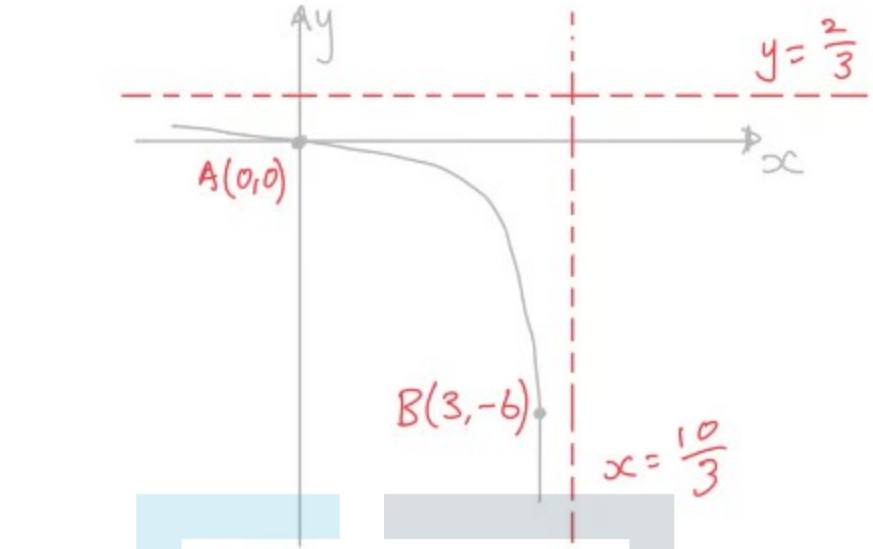
$B = (4,8) \Rightarrow (4,4)$ $\times a$

$a = \frac{1}{2}$

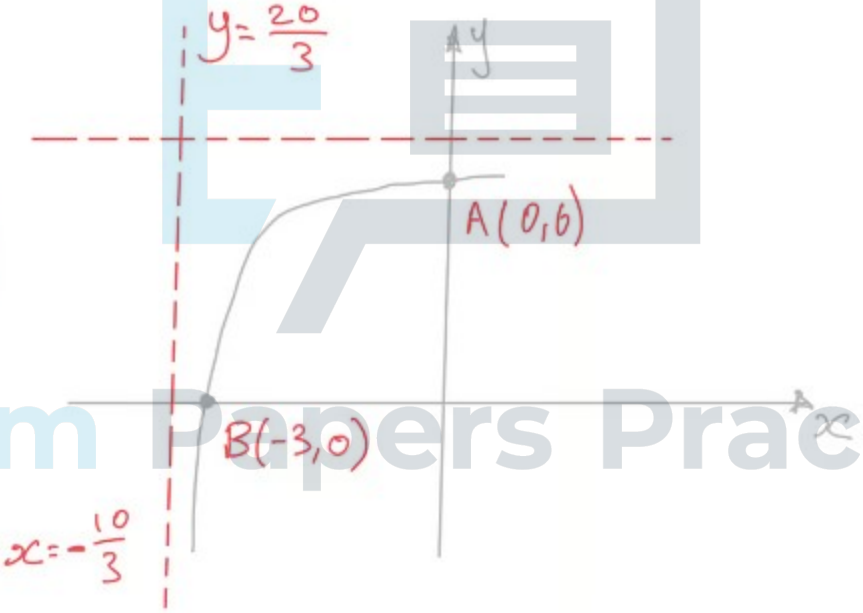
Question 6

a)

i)



ii)



b) HORIZONTAL TRANSLATION

CHANGES

$-a$

$x=\frac{10}{3} \Rightarrow x=0$

$-\frac{10}{3}$

$a=\frac{10}{3}$

Question 7

DEAL WITH INSIDE BRACKETS FIRST

INSIDE = HORIZONTAL

OUTSIDE = VERTICAL

i) TRANSLATION BY $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

VERTICAL STRETCH SF. 3

ii) REFLECTION IN y AXIS (OR $x = 0$)

TRANSLATION BY $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Question 8

1. $f(x) \Rightarrow f(x-2)$

$3x^2 - 2x \Rightarrow 3(x-2)^2 - 2(x-2)$

$3(x^2 - 4x + 4) - 2x + 4$

$3x^2 - 14x + 16$

2. $f(x-2) \Rightarrow 4f(x-2)$

$4(3x^2 - 14x + 16)$

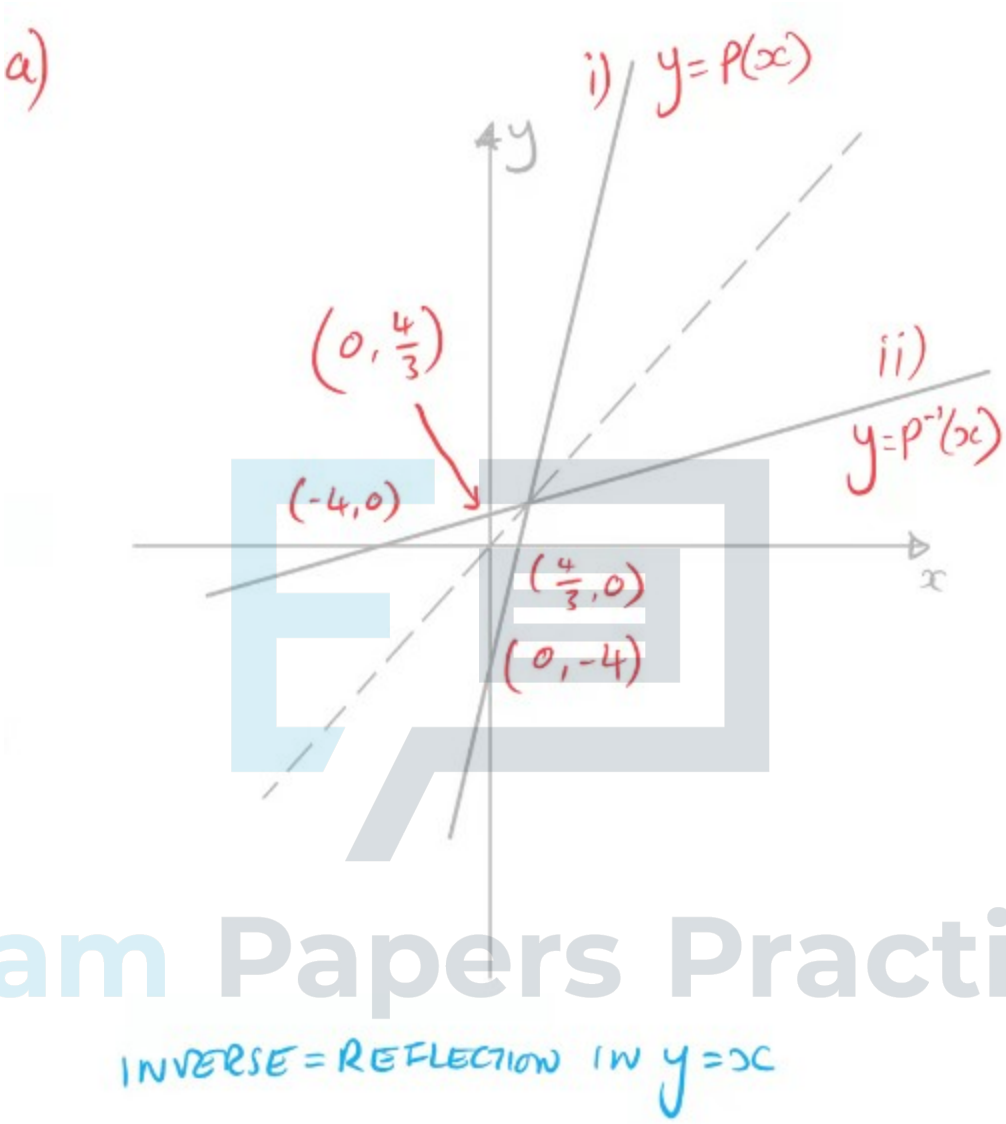
$12x^2 - 56x + 64$

3. $4f(x-2) \Rightarrow 4f(x-2) - 3$

$12x^2 - 56x + 64 - 3$

$g(x) = 12x^2 - 56x + 61$

Question 9



INVERSE = REFLECTION IN y = x

b) i) LET $y = 3x - 4$ REARRANGE

$y + 4 = 3x$

$\frac{y+4}{3} = x$

$P^{-1}(x) = \frac{x+4}{3}$ OR $P^{-1}(x) = \frac{1}{3}(x+4)$

ii) $\frac{1}{9}\left[(3x - 4) + 16\right]$

$\frac{1}{9}(3x + 12)$

$\frac{1}{3}x + \frac{4}{3} = \frac{1}{3}(x+4)$

iii) $\frac{1}{9}\left[P(x) + 16\right] = P^{-1}(x)$

Question 10

a) BOTH POINTS ON AXES SO

$y = 0$ OR $x = 0$

A $x = 0$ $\quad f(x) = (0 - a)^2$

$(-a)^2$

$y = a^2$

$A = (0, a^2)$

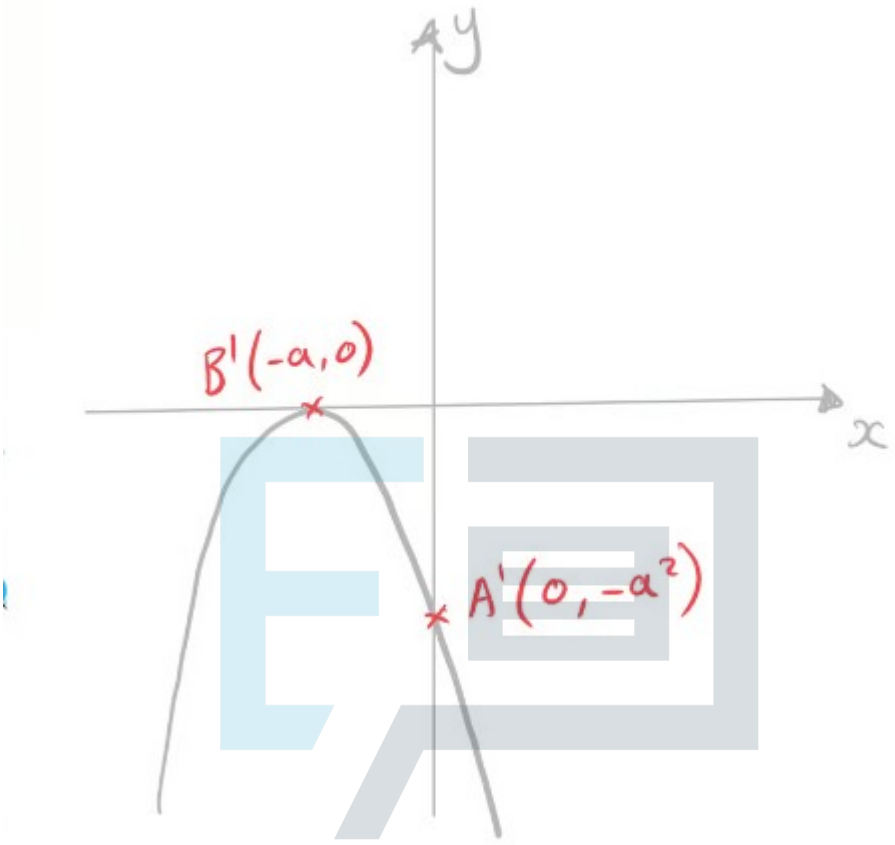
B $y = 0$ $\quad 0 = (x - a)^2$

$x - a = 0$

$x = a$

$B = (a, 0)$

b) $f(-x)$ REFLECTING $-f(x)$ REFLECT IN x



c) BOTH ON AXES SO HORIZONTAL AND VERTICAL DISTANCES FROM ORIGIN

$$A = 3B$$

$$(0, a^2) = (a, 0)$$

$$a^2 = 3a$$

$$\boxed{a = 3}$$

Question 11

1. HORIZONTAL = INSIDE BRACKETS (SCALE FACTOR = $\frac{1}{a}$)

STRETCH = MULTIPLY

$$f\left(\frac{1}{2}x\right)$$

2. VERTICAL REFLECTION = OUTSIDE (NEGATIVE)

$$-f\left(\frac{1}{2}x\right)$$

3. VERTICAL TRANSLATION = OUTSIDE

$$-f\left(\frac{1}{2}x\right)+2$$

$$\boxed{2-f\left(\frac{1}{2}x\right)}$$

Question 12

a) i) VERTICAL STRETCH SF 3

MAX OF SIN 2t = 1

MAX 3(SIN2t) = 3

$y = 3$

ii) VERTICAL STRETCH WON'T AFFECT HORIZONTAL VALUE

$t = 45°$

b) TRANSLATION $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$ ONLY AFFECTS t

VERTICAL STRETCH SF 5 ONLY AFFECTS y

i) MIN $\sin 2t = -1$

MIN $5(\sin 2t + 30) = -5$

$$y = -5$$

ii) TRANSLATION $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$

$t = 135 - 30$

$$t = 105°$$

c) $f(t) = \sin 2t$

$f\left(\frac{1}{2}t\right) = \sin t$

$f\left(\frac{1}{2}t\right) + 2 = 2 + \sin t$

$$f\left(\frac{1}{2}t\right) + 2$$

Question 13

a) SUB IN $x = -3$

$$3(-3)^2 + 18(-3) + 27$$

$$27 - 54 + 27 = 0$$

$$\boxed{f(-3) = 0}$$

b) FACTORISE 3 FROM x TERMS

$$f(x) = 3(x^2 + 6x) + 27$$

COMPLETE THE SQUARE ON $x^2 + 6x$

$$f(x) = 3[(x+3)^2 - 3^2] + 27$$

$$f(x) = 3[(x+3)^2 - 9] + 27$$

EXPAND AND SIMPLIFY INTO FORM $a(x-h)^2 + k$

$$f(x) = 3(x+3)^2 - 27 + 27$$

$$f(x) = 3(x+3)^2$$

$$\boxed{a = 3 \qquad h = -3 \qquad k = 0}$$

ALTERNATE METHOD EXPAND $a(x-h)^2 + k$

$$a(x-h)^2 + k = ax^2 - 2ahx + ah^2 + k$$

COMPARE COEFFICIENTS TO $3x^2 + 18x + 27$

$$ax^2 = 3x^2 \Rightarrow a = 3$$

$$-2ahx = 18x \Rightarrow -2(3)h = 18 \Rightarrow h = -3$$

$$ah^2 + k = 27 \Rightarrow 3(-3)^2 + k = 27 \Rightarrow k = 0$$

c) REFLECTION OF $f(x)$ IN x-AXIS : $-f(x)$

TRANSLATION BY $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$: $-f(x)+1$

$$g(x) = -f(x) + 1$$

$$g(x) = -\left[3x^2 + 18x + 27\right) + 1$$

$$= -3x^2 - 18x - 27 + 1$$

$$g(x) = -3x^2 - 18x - 26$$