2.4 Further Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.4 Further Functions & Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1a

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

Find the coordinates of the y-intercepts for the graph of

(i)
f

(ii)
g.

[2 marks]

Question 1b

Find the coordinates of the x-intercepts for the graph of

(i)
f

(ii)
g.

[3 marks]

Question 1c

For the graph of g, find the equation of

(i)
the vertical asymptote

(ii)
the horizontal asymptote.

[2 marks]

Question 2a

Let $f(x) = \frac{2x + 1}{x - 4}$, $x \neq 4$.

For the graph of f, find the equation of:

(i)
the vertical asymptote

(ii)
the horizontal asymptote.

[3 marks]

Question 2b

Find $f^{-1}(x)$.

[2 marks]

Question 2c

Write down the equation of the vertical asymptote to the graph of $f^{-1}(x)$.

[1 mark]

Question 3a

Let $f(x) = \ln(x + 2)$, $x > -2$.

Find the coordinates of:

(i)
the x-intercept

(ii)
the y-intercept.

[2 marks]

Question 3b

State the equation of the vertical asymptote to the graph of f.

[2 marks]

Question 3c

The graph of $y = f(x)$ intersects with its inverse, twice.

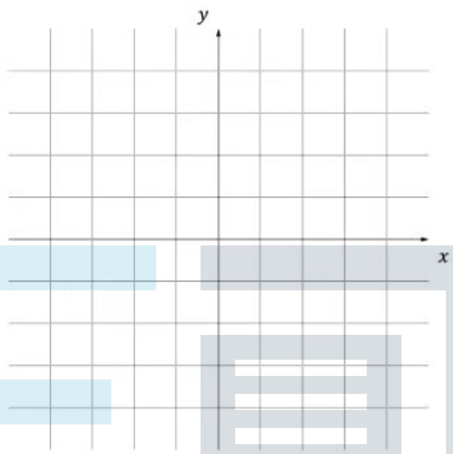
Find the two coordinates where $f(x) = f^{-1}(x)$.

[2 marks]

Question 4a

Let $f(x) = 0.5e^{2x} + 1$, for $-1 \le x \le 2$.

On the following grid, sketch the graph of $y = f(x)$.



[3 marks]

Question 4b

The inverse of f can be written in the form of $f^{-1}(x) = A \ln b(x - c)$.
Find the values of A, b and of c.

[4 marks]

Question 5a

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.

The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_0 and k are constants.

For an object initially containing 100g of carbon-14, write down the value of m_0.

[1 mark]

Question 5b

Briefly explain why, if $m_0 = 100, m$ will equal 50g when $t = 5700$ years.

[2 marks]

Question 5c

Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2 marks]

Question 5d

A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

[2 marks]

Question 6a

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.

£ P is the profit in the y^{th} year of business.

P_0 and k are constants.

Write down two equations connecting P_0 and k.

[2 marks]

Question 6b

Find the values of P_0 and k.

[2 marks]

Question 6c

Find the predicted profit for years 3 and 4.

[2 marks]

Question 6d

Show that

$$P = P_0\, y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y$$

[2 marks]

Question 7a

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.

The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

Write down the number of birds the scientists released into the nature reserve.

[1 mark]

Question 7b

According to this model, how many birds will be in the reserve after 3 years?

[2 marks]

Question 7c

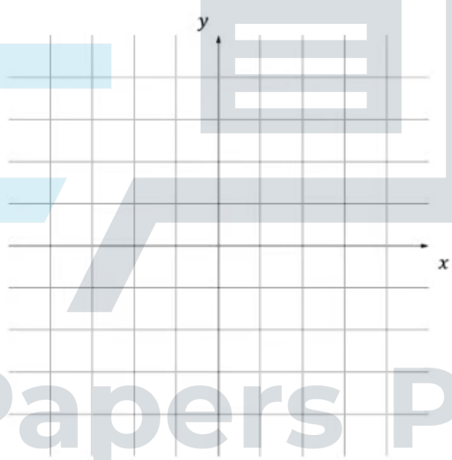
How long will it take for the population of birds within the reserve to reach 500?

[2 marks]

Question 8a

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

On the following grid, sketch the graph of $y = V_1(t)$.



[3 marks]

Question 8b

Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2 marks]

Question 8c

Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3 marks]

Question 8d

The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2 marks]

Question 8e

Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$. Calculate, to the nearest minute, how much faster V_2. took to reach a maximum volume compared to V_1.

[1 mark]

Question 9a

Let $f(x) = \frac{-3}{x - 3}$, for $x \neq 3$.

For the graph of f, find:

(i)
the x- intercept

(ii)
the y- intercept

(iii)
the range of f.

[4 marks]

Question 9b

Find the value of $f^{-1}(-1)$.

[2 marks]

Question 9c

Given that $g(x) = f(x+3) + 1$, find the domain and range of g.

[2 marks]