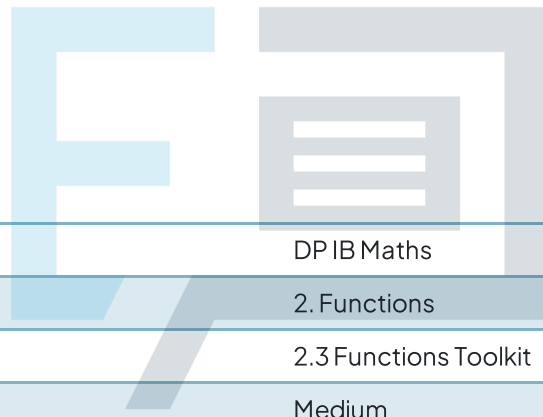




2.3 Functions Toolkit

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Course	DP IB Maths
Section	2. Functions
Topic	2.3 Functions Toolkit
Difficulty	Medium

Exam Papers Practice

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

a) Sub $(4x - 10)$ into $g(x)$.

$$(g \circ f)(x) = \frac{(4x - 10) + 8}{2}$$

$$(g \circ f)(x) = \frac{4x - 2}{2}$$

$$(g \circ f)(x) = 2x - 1$$

b) Set $(g \circ f)(x) = 27$ and solve for x .

$$2x - 1 = 27$$

$$x = 14$$

$$\therefore a = 14$$

c) Sub $\frac{x + 8}{2}$ into $f(x)$.

$$(f \circ g)(x) = 4^2 \left(\frac{x + 8}{2} \right) - 10$$

$$(f \circ g)(x) = 2(x + 8) - 10$$

$$(f \circ g)(x) = 2x + 16 - 10$$

$$(f \circ g)(x) = 2x + 6$$

d) Set $(f \circ g)(x) = 44$ and solve for x .

$$2x + 6 = 44$$

$$x = 19$$

$$\therefore b = 19$$



Question 2

a) $x^2 \geq 0$ ALWAYS POSITIVE

RANGE = OUTPUT VALUES

$$f(x) \geq 0$$

b)

i) $f \circ g(x)$ SUBSTITUTE $g(x)$ INTO $f(x)$
 $(4x-3)^2$

$$f \circ g(x) = 16x^2 - 24x + 9$$

ii) $g \circ f(x)$ SUBSTITUTE $f(x)$ INTO $g(x)$

$$4(x^2) - 3$$

$$g \circ f(x) = 4x^2 - 3$$



c) $x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

FACTORISE OR
USE CALCULATOR

$(x-1)(x-3) = 0$

$x = 1 \quad x = 3$

Question 3

a)

i)

DOMAIN

INPUT VALUES

$x \geq 0 \quad x \in \mathbb{R}$

RANGE

OUTPUT VALUES

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$f(x) \geq 1$

ii)

$y = x$

b)

INVERSE FUNCTION

REFLECTION IN $y = x$

Question 4

a) i) $f \circ g(x)$ SUBSTITUTE $g(x)$ INTO $f(x)$

$$\frac{1}{2} \left(4 \left(\frac{1}{2}x + \frac{3}{4} \right) - 3 \right)$$

$$\frac{1}{2} (2x + \cancel{3} - \cancel{3})$$

$$f \circ g(x) = x$$

ii) $g \circ f(x)$ SUBSTITUTE $f(x)$ INTO $g(x)$

$$\frac{1}{2} \left(\frac{1}{2} (4x - 3) \right) + \frac{3}{4}$$

$$\frac{1}{4} (4x - 3) + \frac{3}{4}$$

$$x - \frac{3}{4} + \frac{3}{4}$$

$$g \circ f(x) = x$$

b) IF $f \circ g(x) = g \circ f(x) = x$

$f(x)$ AND $g(x)$ ARE INVERSE FUNCTIONS
OF EACH OTHER

$$f^{-1}(x) = 0.5x + 0.75$$

DOMAIN $x \in \mathbb{R}$

RANGE $x \in \mathbb{R}$



Question 5

a) Sub $x = \frac{5}{2}$ into $f(x)$.

$$f\left(\frac{5}{2}\right) = 54\left(\frac{5}{2}\right) - 13$$

$$f\left(\frac{5}{2}\right) = 122$$

b) Use the domain of $f(x)$ to find its range.

$$f(-2) = 54(-2) - 13$$

$$f(-2) = -121$$

$$f(20) = 54(20) - 13$$

$$f(20) = 1067$$

$$\text{Range is } \{y \mid -121 < y < 1067\}$$

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c) Set $y = f(x)$.

$$y = 54x - 13$$

Swap x and y .

$$x = 54y - 13$$

Rearrange into $y = mx + c$.

$$54y - 13 = x$$

$$54y = x + 13$$

$$y = \frac{x + 13}{54}$$

$$y = \frac{1}{54}x + \frac{13}{54}$$

$$f^{-1}(x) = \frac{1}{54}x + \frac{13}{54}$$

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d) The domain of $f(x)$ is the range of $f^{-1}(x)$.

$$\text{Range is } \{y \mid -2 < y < 20\}$$

Question 6

a) i) Sub $x = 2$ into $f(x)$.

$$f(2) = -6(2) - 3$$

$$f(2) = -15$$

ii) Set $f(x) = 15$ and rearrange for x .

$$f(x) = 15$$

$$-6x - 3 = 15$$

$$-6x = 18$$

$$x = -3$$

$$\begin{array}{l} +3 \\ \div(-6) \end{array}$$

b) Use the domain of $f(x)$ to find its range.

$$f(-5) = -6(-5) - 3$$

$$f(-5) = 27$$

$$f(3) = -6(3) - 3$$

$$f(3) = -21$$

$$\text{Range is } \{y \mid -21 \leq y \leq 27\}$$

c) The range of $f(x)$ is the domain of $f^{-1}(x)$.

$$\text{Domain is } \{x \mid -21 \leq x \leq 27\}$$



Question 7

a) The graph of f is a parabola.

Axis of symmetry

$$x = -\frac{b}{2a} \quad (\text{in formula booklet})$$

Sub $b = 10$ and $a = 3$ into formula.

$$x = -\frac{10}{2(3)} \quad \therefore x = -\frac{5}{3}$$

Sub $x = -\frac{5}{3}$ into $f(x)$.

$$f\left(-\frac{5}{3}\right) = 3\left(-\frac{5}{3}\right)^2 + 10\left(-\frac{5}{3}\right) + 7$$

$$f\left(-\frac{5}{3}\right) = 3\left(\frac{25}{9}\right) - \frac{50}{3} + 7$$

$$f\left(-\frac{5}{3}\right) = \frac{75}{9} - \frac{150}{9} + \frac{63}{9}$$

$$f\left(-\frac{5}{3}\right) = \frac{-12}{9} = -\frac{4}{3}$$

Range is $\left\{y \mid y \geq -\frac{4}{3}\right\}$

$$b) (g \cdot f)(x) = 3x^2 + 10x + 7 + d$$

$(g \cdot f)(x)$ is a quadratic equation, with

$$a = 3 \quad b = 10 \quad c = 7 + d$$

Discriminant formula.

$$\Delta = b^2 - 4ac \quad (\text{in formula booklet})$$

$$\Delta = (10)^2 - 4(3)(7 + d)$$

$$\Delta = 100 - (84 + 12d)$$

$$\Delta = 16 - 12d$$

$(g \cdot f)(x)$ is positive for all x when $\Delta < 0$.

$$16 - 12d < 0$$

$$d > \frac{4}{3}$$

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Question 8 a) x -intercept is when $g(x) = 0$.

x -intercept is at $(4, 0)$

y -intercept is when $x = 0$.

y -intercept is at $(0, 2)$

Graph $g(x)$ on your GDC to find its shape.

b)i) Sub $x = -5$ into $g(x)$.

$$g(-5) = \sqrt{4 - (-5)}$$

$$g(-5) = \sqrt{9}$$

$$g(-5) = 3$$

ii) Set $g(x) = \frac{1}{2}$ and rearrange for x .

$$g(x) = \frac{1}{2}$$

$$\sqrt{4 - x} = \frac{1}{2}$$

$$4 - x = \frac{1}{4}$$

$$x = 3.75$$

c)i) $g(x)$ is undefined for $x > 4$.

$$\text{Domain is } \{x \mid x \leq 4\}$$

ii) $g(x) = 0$ when $x = 4$.

$$\text{Range is } \{y \mid y \geq 0\}$$

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