2.3 Modelling with Functions

Mark Schemes

Course	DP IB Maths
Section	2. Functions
Topic	2.3 Modelling with Functions
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) 52 weeks in a year.

Sub $t = 52$ into C.

$C = 16.99(52) + 49$

$\boxed{C = 932.48 \text{ NZD } (2dp)}$

b) Set $C = 2000$ and rearrange for t.

$16.99t + 49 = 2000$ $\quad -49$

$16.99t = 1951$ $\quad \div 16.99$

$t \approx 114.8$

$\boxed{\therefore 115 \text{ weeks}}$

c) Initial payment is 20 NZD less.

Initial payment $= 49 - 20$
$= 29$

Weekly cost is 8.51 NZD more.

Weekly cost $= 16.99 + 8.51$
$= 25.50$

$\boxed{C = 25.50t + 29}$

d) Les Mills > Cityfitness

$25.50t + 29 > 16.99t + 49$) -29

$25.50t > 16.99t + 20$) $-16.99t$

$8.51t > 20$) $\div 8.51$

$t > 2.35$

$\boxed{\therefore 3 \text{ weeks}}$

Question 2

a) i) c represents the y-intercept.

$\boxed{c = 0}$

ii) Sub point Q* into y.

$4 = a(4)^2$) expand

$4 = 16a$) $\div 16$

$\boxed{a = \frac{1}{4}}$

*N.B you could also use point P.

iii) $\boxed{y = \frac{1}{4}x^2}$

b) y represents the height of the water tank.
x represents half the width of the water tank.
Set $y = 2.25$ and rearrange for x.

$$2.25 = \frac{1}{4}x^2 \quad \times 4$$
$$9 = x^2 \quad \sqrt{}$$
$$3 = x \qquad \text{(reject } x = -3\text{)}$$

The width of the water tank is 6m.

Question 3

a) When $t = 0$, $W(t) = 2400$.

$$W(0) = 2400$$
$$a \times \underbrace{b^{-(0)}}_{=1} + 320 = 2400 \qquad b^{-(0)} = 1$$
$$a + 320 = 2400 \qquad -320$$
$$\boxed{a = 2080}$$

b) When $t = 2$, $W(t) = 1020$.

$W(2) = 1020$

$2080 \times b^{-(2)} + 320 = 1020$

-320

$2080 \times b^{-(2)} = 700$

$\div 2080$

$b^{-(2)} = \frac{700}{2080}$

reciprocate

$b^{2} = \frac{2080}{700}$

$\sqrt{}$

$\boxed{b \approx 1.72}$

c) As t tends towards infinity (∞),

$2080 \times 1.72^{-(\infty)}$ tends towards zero

$W(t) = 2080 \times 1.72^{-t} + 320$

$\lim_{x \to \infty} W(t) = 0 + 320$
$= 320$

$\boxed{\therefore c = 320}$

Question 4 a) Sub $t = 0$ into $D(t)$.

$D(0) = 1.4 \times 0.77^{(0)}$

$\boxed{D(0) = 1.4 \text{ mg } L^{-1}}$

b) Sub $t=4$ into $D(t)$.

$D(4) = 1.4 \times 0.77^{(4)}$

$\boxed{D(4) = 0.492 \text{ mg L}^{-1}}$

c) Set $D(t) = 0.22$ and solve for t

on your GDC.

$1.4 \times 0.77^{t} = 0.22$

$\boxed{t = 7.08 \text{ hours}}$

d) 45 minutes is 0.75 hours.

Sub $t=0.75$ into $D(t)$.

$D(0.75) = 1.4 \times 0.77^{(0.75)}$

$\boxed{D(0.75) = 1.15 \text{ mg L}^{-1}}$

Question 5

a) Sub $t=0$ into $N(t)$.

$N(0) = 75 \times 2^{0.5(0)}$

$\boxed{N(0) = 75 \text{ bacteria}}$

b) Sub $t=10$ into $N(t)$.

$N(10) = 75 \times 2^{0.5(10)}$

$\boxed{N(10) = 2400 \text{ bacteria}}$

c) Set $N(t) = 10\,000$ and solve for t on your GDC.

$75 \times 2^{0.5t} = 10\,000$

$\boxed{t = 14.1 \text{ hours}}$

Question 6 a) Sub $w = 20$ into $V(w)$.

$V(20) = 0.0025(20)(2-20)(20-35)$

$\boxed{V(20) = 13.5 \text{ km h}^{-1}}$

b) Set $V(w) = 5.94$ and solve for t on your GDC.

$0.0025w(2-w)(20-w) = 5.94$

$\boxed{w = 11 \text{ km h}^{-1}}$

c) Expand $V(w)$.

$V(w) = 0.0025w(2-w)(w-35)$

Expand first bracket.

$V(w) = (0.005w - 0.0025w^2)(w-35)$

Expand second bracket.

$V(w) = 0.005w^2 - 0.175w - 0.0025w^3 + 0.0875w^2$

Rearrange and collect like terms.

$$\boxed{V(w) = -0.0025w^3 + 0.0925w^2 - 0.175w}$$

d) Graph $V(w)$ on your GDC and find its maximum.

maximum = (23.7, 14.5)

maximum velocity = 14.5 km h^{-1} when the windspeed = 23.7 km h^{-1}

Question 7

a) Graph $H(t)$ on your GDC and find its minimum and maximum.

i) $\boxed{H_{min} = 2\text{ m}}$

ii) $\boxed{H_{max} = 30\text{ m}}$

c) $H(t)$ is in the form $a\cos bx + c$.

one full rotation = period of $H(t)$.

Period formula

Period $= \dfrac{360°}{b}$ (not in formula booklet)

Period $= \dfrac{360°}{10°}$

Period $= 36$

$\boxed{\text{one full rotation} = 36\text{ s}}$

Question 8 a) $t = 0$ at midnight.

Sub $t = 0$ into $D(t)$.

$D(0) = 5\sin(30° \times (0)) + 15$

$\boxed{D(0) = 15\text{ m}}$

b) $D(t)$ is in the form $a\sin bx + c$.

P is equal to the period of $D(t)$.

Period formula

Period $= \dfrac{360°}{b}$ (not in formula booklet)

$P = \dfrac{360°}{30°}$

$\boxed{P = 12\text{ hours}}$

c) Graph $D(t)$ on your GDC and find its maximums and minimums within $0 \le t \le 24$.

maximums = $(3, 20)$ and $(15, 20)$

minimums = $(9, 10)$ and $(21, 10)$

i) $\boxed{\text{maximum depth} = 20\text{ m}}$
$\boxed{\text{minimum depth} = 10\text{ m}}$

ii) $\boxed{\text{maximum depth at } 3{:}00 \text{ and } 15{:}00}$
$\boxed{\text{minimum depth at } 9{:}00 \text{ and } 21{:}00}$

Question 9

a) The shape of the box is a cuboid.

Volume of a cuboid formula

$V = lwh$ $\quad l$ = length, w = width, h = height

$l = 100 - 2x \quad w = 60 - 2x \quad h = x$

Sub l, w and h into formula.

$V = (100 - 2x)(60 - 2x)x$

Expand brackets.

$V = (6000 - 320x + 4x^2)x$

Expand fully.

$V = 6000x - 320x^2 + 4x^3$

Rearrange into the form given.

$$\boxed{V = 4x^3 - 320x^2 + 6000x}$$

Question 1

b) Dimensions of the box are

$l = 100 - 2x \quad w = 60 - 2x \quad h = x$

$\therefore$ the volume of the box can be given by

$V = (100 - 2x)(60 - 2x)x$

If $x = 0$

then $h = 0$ and so $V = 0$.

If $x = 30$

then $w = 60 - 2(30) = 0$ and so $V = 0$.

$\boxed{\text{Domain is } \{x \mid 0 < x < 30\}}$

c) Graph V on your GDC and find its maximum.

maximum = (12.1, 32 800) (3sf)

$\boxed{\therefore V_{max} = 32\,800\ \text{cm}^3 \text{ when } x = 12.1\ \text{cm}}$

Question 10

a) i) During the first minute.

$88 - 58 = 30$

$\boxed{30°C}$

ii) During the second minute.

$58 - 43 = 15$

$\boxed{15°C}$

iii) During the third minute.

$43 - 35.5 = 7.5$

$\boxed{7.5°C}$

b) The decrease in the temperature of the tea in any given minute is half the decrease from the previous minute.

$\frac{15}{30} = \frac{7.5}{15} = \frac{1}{2}$

∴ the decrease during the fourth minute is

$7.5 \times \frac{1}{2} = 3.75$

$\therefore k = 35.5 - 3.75$

$\boxed{k = 31.75°C}$

c) $a+b$ is the initial temperature $(t=0)$.

$$\boxed{88 = a + b}$$

When $t=1$, $y=58$.

$$58 = a(2^{-(1)}) + b$$

$$\boxed{58 = \frac{1}{2}a + b}$$

d) Simultaneous equations

① $88 = a + b$

② $58 = \frac{1}{2}a + b$

① − ②

$$\begin{array}{rl} 88 = & a + b \\ -\ 58 = & \frac{1}{2}a + b \\ \hline 30 = & \frac{1}{2}a \end{array}$$

$$\boxed{\therefore a = 60}$$

Sub a into ①.

$$88 = 60 + b$$

$$\boxed{\therefore b = 28}$$

Alternatively you could solve the simultaneous equations on your GDC.