Exam Papers Practice

2.2 Quadratic Functions & Graphs

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Course	DP IB Maths
Section	2. Functions
Topic	2.2 Quadratic Functions & Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

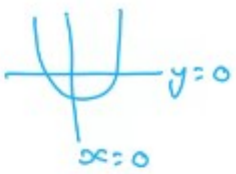
Question 1

a) $x = 0$ y intercept

$y = x^2 - 3x + 2$

$y = 2$

$(0, 2)$



x intercept $y = 0$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

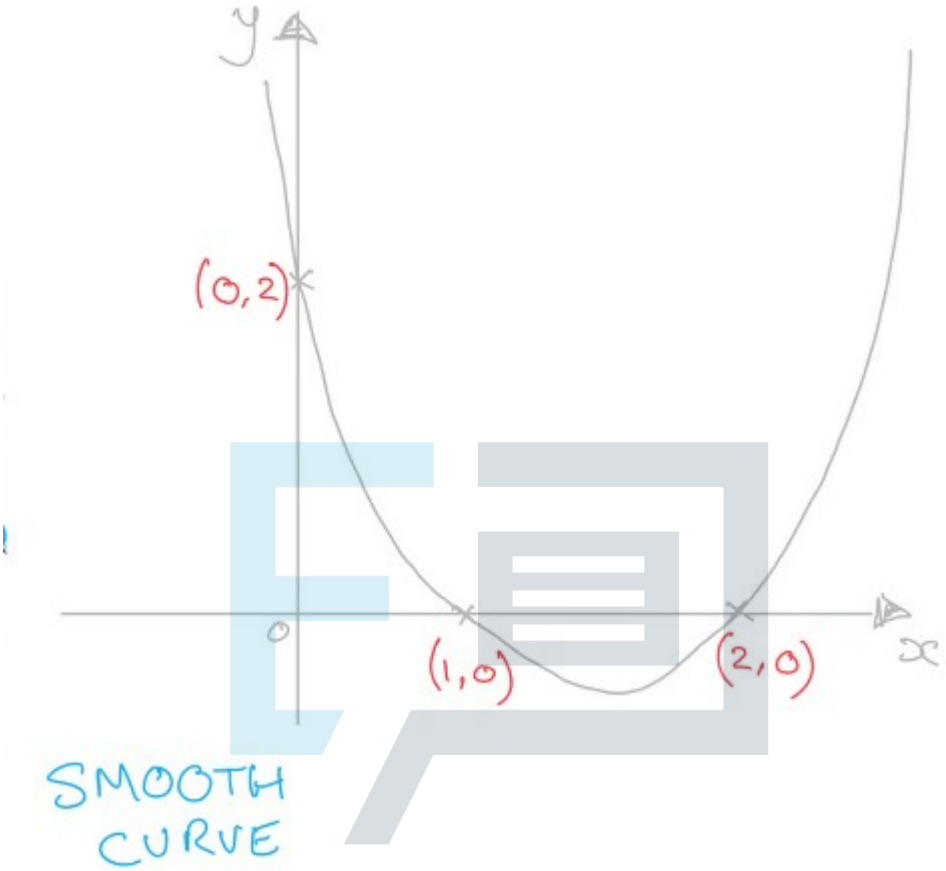
FACTORISE & SOLVE

$x - 1 = 0$ $x - 2 = 0$

$x = 1$ $x = 2$

$(1, 0)$ $(2, 0)$ $(0, 2)$

b) (1, 0) (2, 0) (0, 2)



SMOOTH CURVE

Question 2

a) $x^2 + 8x - 9$ $(x+b)^2 + c$

$(x+4)^2 - 4^2 - 9$

$(x+4)^2 - 16 - 9$

$(x+4)^2 - 25$

b) $(x+4)^2 - 25$ $(x+b)^2+c$

$(-4, -25)$ $(-b, c)$

c) MINIMUM $(-4, -25)$

Y INTERCEPT $x = 0$

$y = x^2 + 8x - 9 = -9$ $(0, -9)$

X INTERCEPT $y = 0$

$x^2 + 8x - 9 = 0$ FACTORISE + SOLVE

$(x-1)(x+9) = 0$

$x = 1$ $x = -9$

$(1, 0)$ $(-9, 0)$

Question 3

a) $2x^2 + x - 6 = 0$ FACTORISE

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ $\quad x + 2 = 0$ SOLVE

$x = \frac{3}{2}$ $\quad x = -2$

b) $2x^2 + x - 6$ COMPLETING THE SQUARE $a(x+b)^2 + c$

$2\left[x^2 + \frac{1}{2}x\right] - 6$

$2\left[\left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] - 6$

$2\left[\left(x + \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6$

$2\left(x + \frac{1}{4}\right)^2 - \frac{1}{8} - 6$

$2\left(x + \frac{1}{4}\right)^2 - \frac{49}{8}$ TURNING POINT $(-b, c)$

$\left(-\frac{1}{4}, -\frac{49}{8}\right)$

c) TURNING POINT $\left(-\frac{1}{4}, -\frac{49}{8}\right)$

Y INTERCEPT $x = 0$

$y = 2x^2 + x - 6 = -6$ $(0, -6)$

X INTERCEPT $y = 0$

$x = \frac{3}{2}$ $x = -2$

$\left(\frac{3}{2}, 0\right)$ $(-2, 0)$

Question 4

a) $x^2 + 4x + 5$ COMPLETING THE SQUARE

$(x+2)^2 - 2^2 + 5$ $a(x+b)^2 + c$

$(x+2)^2 + 1$ MINIMUM POINT

$(-2, 1)$ $(-b, c)$

b) x^2+4x+5

$a=1 \quad b=4 \quad c=5$

$b^2-4ac < 0$

$4^2-4\times1\times5 = 16-20$

$-4 < 0$

$\therefore$ NO REAL ROOTS

OR

COMPLETED SQUARE

$(x+2)^2+1$

$(x+2)^2 \geq 0$ FOR ALL VALUES OF x

$(x+2)^2+1 \geq 0$

$\therefore$ NO REAL ROOTS

Question 5

$kx^2+2kx-3$

$a=k \quad b=2k \quad c=-3$

$b^2-4ac \geq 0$

2 DISTINCT REAL ROOTS

$(2k)^2-4(k)(-3) \geq 0$

$4k^2+12k \geq 0$

$4k(k+3) \geqslant 0$

$k=0 \quad k=-3$

$k<-3 \quad k>0$

Question 6

$2x^2 - 4x + 3 - 2k = 0$ DISCRIMINANT

$a = 2 \quad b = -4 \quad c = 3 - 2k$ $b^2 - 4ac \geqslant 0$

$(-4)^2 - 4(2)(3-2k) \geqslant 0$

EXPAND

$16 - 4(6 - 4k) \geqslant 0$

$16 - 24 + 16k \geqslant 0$

SOLVE

$-8 + 16k \geqslant 0$

$16k \geqslant 8$

$k \geqslant \frac{8}{16} = \frac{1}{2}$

$k \geqslant \frac{1}{2}$

Question 7

$x^2 + px + q$

$a = 1 \quad b = p \quad c = q$

DISCRIMINANT

$b^2 - 4ac < 0$

NO REAL ROOTS

$p^2 - 4q < 0$

$p^2 < 4q$

Question 8

4 m

b) FIND WIDTH BY FINDING SOLUTIONS FOR x

x AXIS SOLUTIONS WHERE $y = 0$

SOLVE $4 - \frac{x^2}{8} = 0$

$4 = \frac{x^2}{8}$

OR USE CALCULATOR

$\frac{x^2}{8} = 4$

$\times 8$ $\quad x^2 = 32$

$\sqrt{}$ $\quad x = \pm\sqrt{32} = \pm 4\sqrt{2}$

WIDTH $= 2 \times 4\sqrt{2} = 8\sqrt{2} = 11.3137...$

$8\sqrt{2} > 11$

BRIDGE SPANS RIVER

Question 9

a) AXIS OF SYMMETRY AT x-COORDINATE OF TURNING POINT $(1.5, 6.25)$

$x = 1.5$

b) USING $a(x-h)^2+k$ WHERE (h,k) = TURNING POINT

$h = 1.5 \qquad k = 6.25$

$a(x-1.5)^2 + 6.25 = 0$

USE $x = -1$ OR $x = 4$

$a(4-1.5)^2 = -6.25$

$6.25a = -6.25$

$a = -1$

$a = -1 \quad , h = 1.5, \quad k = 6.25$

Question 10

$f(x) = x^2$

$x^4 - 13x^2 + 36$

$(x^2)^2 - 13(x^2) + 36 = 0$

let $y = x^2$

$y^2 - 13y + 36 = 0$

FACTORISE

$(y - 9)(y - 4) = 0$

SOLVE

$y = 9 \quad y = 4$

$x^2 = 9 \quad x^2 = 4$

$\surd$

$x = \pm 3 \quad x = \pm 2$

$x = -3, -2, 2, 3$

Question 11

$$x^{\frac{2}{5}} + x^{\frac{1}{5}} - 6 = 0$$

$$f(x) = x^{1/5}$$

$$\left(x^{1/5}\right)^2 + \left(x^{1/5}\right) - 6 = 0$$

let $y = x^{1/5}$

$$y^2 + y - 6 = 0$$

FACTORISE

$$(y-2)(y+3) = 0$$

$$y = 2 \qquad y = -3$$

$$x^{\frac{1}{5}} = 2 \qquad x^{1/5} = -3$$

$$x = 2^5 = 32 \qquad x = (-3)^5 = -243$$

$$x = 32, -243$$

Question 12

a) DISCRIMINANT $b^2 - 4ac$

$a = 2p \quad b = (2p - 5) \quad c = p - \frac{5}{2}$

SUB IN AND SIMPLIFY

$(2p - 5)^2 - 4(2p)(p - \frac{5}{2})$

$(2p - 5)(2p - 5) - 4(2p^2 - 5p)$

$4p^2 - 20p + 25 - 8p^2 + 20p$

$-4p^2 + 25$ AS REQUIRED

DISCRIMINANT $= -4p^2 + 25$

b) TWO DISTINCT ROOTS WHEN DISCRIMINANT

$b^2 - 4ac > 0$

$-4p^2 + 25 > 0$

$+4p^2$ TO AVOID NEED TO SWITCH SIGN IF $\div(-p)$

$25 > 4p^2$

$\div 4$

$\frac{25}{4} > p^2$

$\sqrt{}$

$\pm\frac{5}{2} > p$

$-\frac{5}{2} < p < \frac{5}{2}$