Exam Papers Practice

2.2 Further Functions & Graphs

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Course	DP IB Maths
Section	2. Functions
Topic	2.2 Further Functions & Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) Sub $x = \frac{5}{2}$ into $f(x)$.

$f\left(\frac{5}{2}\right) = 54\left(\frac{5}{2}\right) - 13$

$$\boxed{f\left(\frac{5}{2}\right) = 122}$$

b) Use the domain of $f(x)$ to find its range.

$f(-2) = 54(-2) - 13$

$f(-2) = -121$

$f(20) = 54(20) - 13$

$f(20) = 1067$

$$\boxed{\text{Range is } \{y \mid -121 < y < 1067\}}$$

c) The inverse of a function reverses the effect of the function.

$\therefore$ if $f\left(\frac{5}{2}\right) = 122$

$$\boxed{f^{-1}(122) = \frac{5}{2}}$$

d) The domain of $f(x)$ is the range of $f^{-1}(x)$.

$$\boxed{\text{Range is } \{y \mid -2 < y < 20\}}$$

Question 2

a)i) Sub $x = 2$ into $f(x)$.

$f(2) = -6(2) - 3$

$\boxed{f(2) = -15}$

ii) Set $f(x) = 15$ and rearrange for x.

$f(x) = 15$

$-6x - 3 = 15$

$-6x = 18$

$\boxed{x = -3}$

$+3$

$\div(-6)$

b) Use the domain of $f(x)$ to find its range.

$f(-5) = -6(-5) - 3$

$f(-5) = 27$

$f(3) = -6(3) - 3$

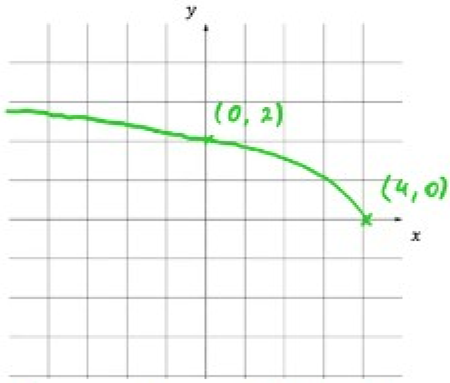
$f(3) = -21$

$\boxed{\text{Range is } \{y \mid -21 \le y \le 27\}}$

c) The range of $f(x)$ is the domain of $f^{-1}(x)$.

$\boxed{\text{Domain is } \{x \mid -21 \le x \le 27\}}$

Question 3



a) x - intercept is when $g(x) = 0$.

$\boxed{x\text{-intercept is at } (4, 0)}$

y - intercept is when $x = 0$.

$\boxed{y\text{-intercept is at } (0, 2)}$

Graph $g(x)$ on your GDC to find its shape.

b)i) Sub $x = -5$ into $g(x)$.

$g(-5) = \sqrt{4-(-5)}$

$g(-5) = \sqrt{9}$

$\boxed{g(-5) = 3}$

ii) Set $g(x) = \frac{1}{2}$ and rearrange for x.

$g(x) = \frac{1}{2}$

$\sqrt{4-x} = \frac{1}{2}$

$4 - x = \frac{1}{4}$

$\boxed{x = 3.75}$

c)i) $g(x)$ is undefined for $x > 4$.

$\boxed{\text{Domain is } \{x \mid x \leq 4\}}$

ii) $g(x) = 0$ when $x = 4$.

$\boxed{\text{Range is } \{y \mid y \geq 0\}}$

Question 4

a)i) y-intercepts occur when $x = 0$.

Sub $x = 0$ into $f(x)$.

$f(0) = -(0)^5 + 2020$

$f(0) = 2020$

$\boxed{\text{Hence the y-intercept for } f \text{ is } (0, 2020).}$

ii) Sub $x = 0$ into $g(x)$.

$g(0) = \frac{1}{\sqrt{(1-(0))^3}} - 2$

$g(0) = -1$

$\boxed{\text{Hence the y-intercept for } g \text{ is } (0, -1).}$

b) i) x-intercepts occur when the function equals zero.

Set $f(x) = 0$ and solve for x on your GDC.

$-x^5 + 2020 = 0$

$x \approx 4.58$

Hence the x-intercept for f is $(4.58, 0)$.

ii) Set $g(x) = 0$ and solve for x on your GDC.

$\frac{1}{\sqrt{(1-x)^3}} - 2 = 0$

$x \approx 0.370$

Hence the x-intercept for g is $(0.37, 0)$.

c)i) The vertical asymptote is when the denominator of $g(x)$ equals zero.

[denominator of g] = 0

$\sqrt{(1-x)^3} = 0$

$x = 1$

Hence the equation of the vertical asymptote is $x = 1$

ii) As x tends towards negative infinity $(-\infty)$, $\frac{1}{\sqrt{(1-x)^3}}$ tends towards zero.

$g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$

$\lim_{x \to -\infty} g(x) = 0 - 2$
$= -2$

Hence the equation of the horizontal asymptote is $y = -2$.

Question 5

a) For $g(x)$ to be defined $x - 1 \geq 0$.

Domain is $\{x \mid x \geq 1\}$

Sub $x = 1$ into $g(x)$.

$g(1) = 2 - \sqrt{(1) - 1}$

$g(1) = 2$

Range is $\{y \mid y \leq 2\}$

b) i) $f(x)$ is undefined when $x = 0$.

Hence the equation of the vertical asymptote is $x = 0$.

ii) As x tends towards $\pm\infty$,

x^{-4} tends towards zero.

$f(x) = x^{-4} - 2021$

$\lim_{x \to \pm\infty} f(x) = 0 - 2021$

Hence the equation of the horizontal asymptote is $y = -2021$.

c) i) x-intercepts occur when the function equals zero.

Set $f(x) = 0$ and solve for x on your GDC.

$x^{-4} - 2021 = 0$

$x = \pm 2021^{-\frac{1}{4}}$

$x \approx \pm 0.149$

x-intercepts at $(0.149, 0)$ and $(-0.149, 0)$.

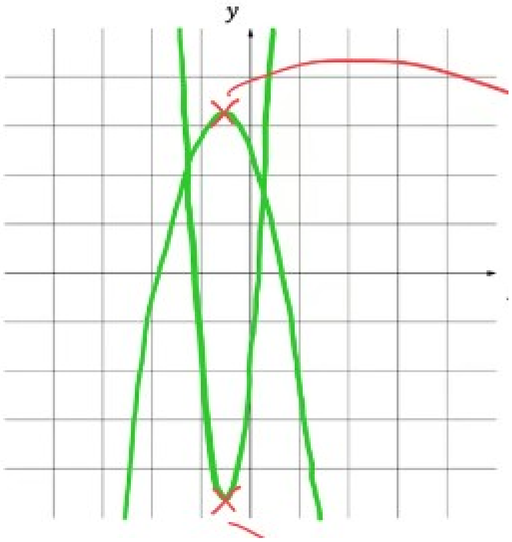
ii) Set $g(x) = 0$ and solve for x on your GDC.

$2 - \sqrt{x - 1} = 0$

$x = 5$

x-intercept at $(5, 0)$.

Question 6



a) Graph $f(x)$ and $g(x)$ on your GDC.

$f(x)$ is a negative quadratic.

$\therefore$ the vertex of $f(x)$ is a maximum.

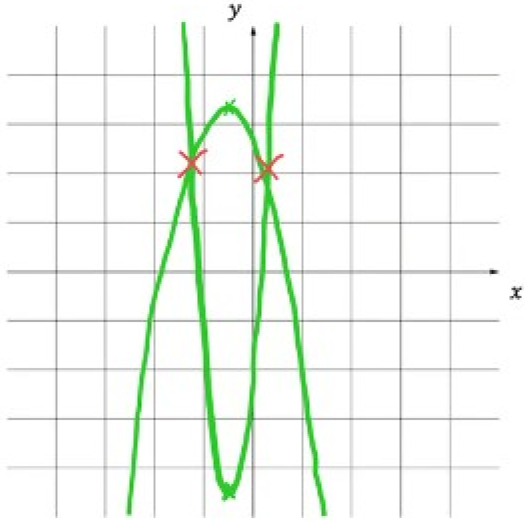
$g(x)$ is a positive quadratic.

$\therefore$ the vertex of $g(x)$ is a minimum.

$\boxed{\text{Vertex of } f(x) \text{ at } (-0.5, 6.25)}$

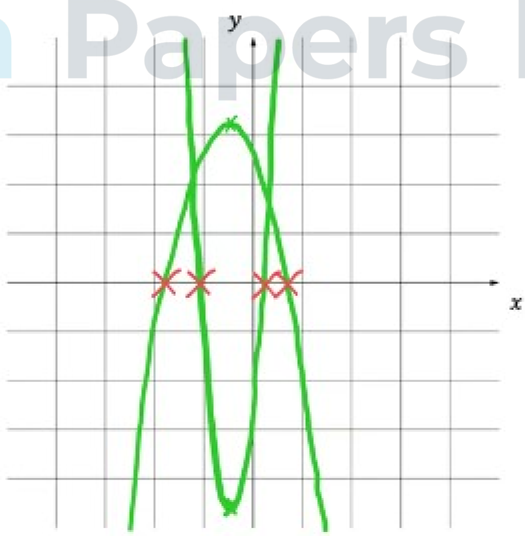
$\boxed{\text{Vertex of } g(x) \text{ at } (-0.5, -9)}$

Question 1



b) Graph f(x) and g(x) on your GDC and find their intersection.

Intersection points are (-2.25, 3.2) and (1.25, 3.2)



c) Graph $f(x)$ and $g(x)$ on your GDC and find the x-intercepts.*

$f(x)$ x-intercepts at $(-3, 0)$ and $(2, 0)$
$g(x)$ x-intercepts at $(-2, 0)$ and $(1, 0)$.

* N.B x-intercepts are also known as "zeros".

Question 7

a) Point $Q(0, 12)$ is the y-intercept.

$g(0) = 12$

$-(0)^2 + b(0) + c = 12$

$\boxed{c = 12}$

b) Sub *$P(-2, 0)$ into $f(x)$.

$f(-2) = 0$

$-(-2)^2 + b(-2) + 12 = 0$

$-4 - 2b + 12 = 0$

$2b = 8$

$b = 4$

$\therefore f(x) = -x^2 + 4x + 12$

*N.B you can also use point R.

c) $f(x)$ is a negative quadratic.

$\therefore$ the vertex of $f(x)$ is a maximum.

Graph $f(x)$ on your GDC and find its maximum.

$\boxed{V(2, 16)}$

Question 8

a) i) $g(x)$ can be obtained by an appropriate translation of the graph $y = 2x^2$

$\boxed{\therefore a = 2}$

$g(x)$ intersects the y-axis at $(0, -16)$

$g(0) = -16$

$2(0)^2 + b(0) + c = -16$

$\boxed{\therefore c = -16}$

$g(x)$ intersects the x-axis at $(-4, 0)$

$g(-4) = 0$

$2(-4)^2 + b(-4) - 16 = 0$

algebraic solver on GDC

$\boxed{\therefore b = 4}$

ii) $\boxed{\therefore g(x) = 2x^2 + 4x - 16}$

b) x-intercepts occur when the function equals zero.

$g(x) = 0$

$2x^2 + 4x - 16 = 0$ } factorise

$(2x - 4)(x + 4) = 0$ } null factor law

$\therefore x = \underline{2}$ and -4

$\boxed{x\text{-intercept at } (2,0)}$

Alternatively you could graph $g(x)$ and find its x-intercepts ("zeros").

c) x-coordinate of the vertex is between the x-intercepts.

$$x = \frac{(-4) + 2}{2}$$

$x = -1$

Sub $x = -1$ into $g(x)$.

$g(-1) = 2(-1)^2 + 4(-1) - 16$

$g(-1) = -18$

$\boxed{\text{Vertex at } (-1, -18)}$

Alternatively you could graph $g(x)$ and find its vertex.

Question 9

a) Amplitude formula

$$\text{Amplitude} = \frac{f_{max} - f_{min}}{2}$$ (not in formula booklet)

$$\text{Amplitude} = \frac{2 - (-2)}{2}$$

$$\boxed{\text{Amplitude} = 2}$$

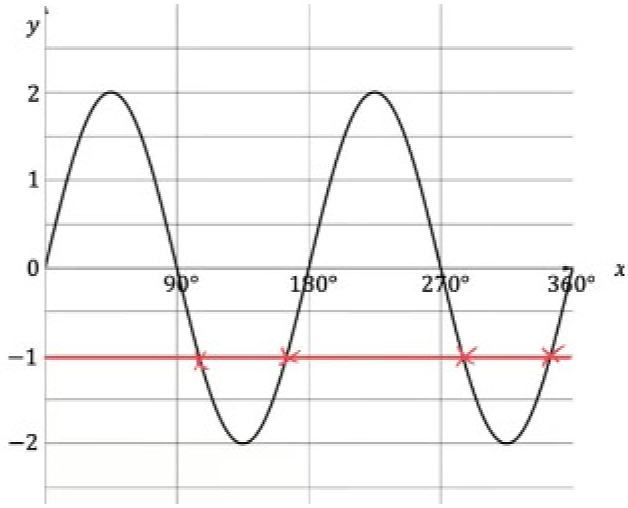
b) $f(x)$ is in the form $a \sin bx$

Period formula

$$\text{Period} = \frac{360°}{b}$$ (not in formula booklet)

$$\text{Period} = \frac{360°}{2}$$

$$\boxed{\text{Period} = 180°}$$



c) Set $f(x) = -1$ and rearrange for x.

- Tip: Draw $y = -1$ on the graph to see the number of solutions and what they are approximately.

$2\sin(2x) = -1$

$\div 2$

$\sin 2x = -\frac{1}{2}$

inverse sin

$2x = \sin^{-1}\left(-\frac{1}{2}\right)$

$\div 2$

$x = \dfrac{\sin^{-1}\left(-\frac{1}{2}\right)}{2}$

$\boxed{x = 105°, 165°, 285°, 345°}$

Alternative GDC method

- Graph $f(x)$ and $y = -1$ and find intersections.

Question 10

a) Graph $f(x)$ on your GDC and find the axes intercepts.

A, D and F are the x-intercepts ("zeros")

C is the y-intercept, which is when $x = 0$.

i) $\boxed{A(-1.7, 0)}$

ii) $\boxed{C(0, 1)}$

iii) $\boxed{D(0.234, 0)}$

iv) $\boxed{F(2.46, 0)}$

b) Find the local maximum and minimum of $f(x)$ on your GDC.

i) $\boxed{B(-0.869, 3.06)}$

ii) $\boxed{E(1.54, -3.88)}$

Question 11

a) i) Sub $x = 2$ into $f(x)$.

$f(2) = 2^{(2)} - 3$

$\boxed{f(2) = 1}$

ii) Set $f(x) = -1$ and rearrange for x.

$f(x) = -1$

$2^x - 3 = -1$

$2^x = 2$

$\boxed{x = 1}$

b) y-intercept is when $x = 0$.

Sub $x = 0$ into $f(x)$.

$f(0) = 2^{(0)} - 3$

$f(0) = 1 - 3$

$f(0) = -2$

$\boxed{P(0, -2)}$

c) x-intercept is when $f(x) = 0$.

Set $f(x) = 0$ and rearrange for x.

$2^x - 3 = 0$

algebraic solver on GDC

$x \approx 1.58$

$\boxed{Q(1.58, 0)}$

c) $f(x) = 2^x - 3$ and $2^x > 0$.

$\boxed{\therefore f(x) = -3 \text{ has no solutions}}$

N.B the line $y = -3$ is the horizontal asymptote of $f(x)$.