# DP IB Maths: AI SL

## 2.1 Linear Functions & Graphs

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## 2.1.1 Equations of a Straight Line

# Equations of a Straight Line

### How do I find the gradient of a straight line?

- Find two points that the line passes through with coordinates $(x_1, y_1)$ and $(x_2, y_2)$
- The gradient between these two points is calculated by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

  - This is given in the **formula booklet**
- The gradient of a straight line measures its **slope**
  - A line with gradient 1 will go up 1 unit for every unit it goes to the right
  - A line with gradient -2 will go down two units for every unit it goes to the right

### What are the equations of a straight line?

- $y = mx + c$
  - This is the **gradient-intercept form**
  - It clearly shows the gradient $m$ and the $y$-intercept (0, c)
- $y - y_1 = m(x - x_1)$
  - This is the **point-gradient form**
  - It clearly shows the gradient $m$ and a point on the line $(x_1, y_1)$
- $ax + by + d = 0$
  - This is the **general form**
  - You can quickly get the $x$-intercept $\left(-\frac{d}{a}, 0\right)$ and $y$-intercept $\left(0, -\frac{d}{b}\right)$

### How do I find an equation of a straight line?

- You will need the gradient
  - If you are given two points then first find the gradient
- It is easiest to start with the **point-gradient form**
  - then rearrange into whatever form is required
    - multiplying both sides by any denominators will get rid of fractions
- You can check your answer by using your GDC
  - Graph your answer and check it goes through the point(s)
  - If you have two points then you can enter these in the **statistics mode** and find the regression line $y = ax + b$

## Worked example

The line $l$ passes through the points $(-2, 5)$ and $(6, -7)$.

Find the equation of $l$, giving your answer in the form $ax + by + d = 0$ where $a$, $b$ and $d$ are integers to be found.

Find the gradient between (-2,5) and (6,-7)

Formula booklet

$m = \dfrac{-7 - 5}{6 - -2} = -\dfrac{3}{2}$

| | |
|---|---|
| Gradient formula | $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ |

Use the point-gradient formula

Formula booklet

| | |
|---|---|
| Equations of a straight line | $y - y_1 = m(x - x_1)$ |

$(x_1, y_1) = (-2, 5)$  $m = -\dfrac{3}{2}$

$y - 5 = -\dfrac{3}{2}(x - -2)$ Simplify

$y - 5 = -\dfrac{3}{2}(x + 2)$

Multiply by denominator

$2(y - 5) = -3(x + 2)$

Expand

$2y - 10 = -3x - 6$

Rearrange

$\boxed{3x + 2y - 4 = 0}$

## Parallel Lines

### How are the equations of parallel lines connected?

- **Parallel lines** are always equidistant meaning they never intersect
- Parallel lines have the same gradient
  - If the gradient of line $l_1$ is $m_1$ and gradient of line $l_2$ is $m_2$ then...
    - $m_1 = m_2 \Rightarrow l_1$ & $l_2$ are parallel
    - $l_1$ & $l_2$ are parallel $\Rightarrow m_1 = m_2$
- To determine if two lines are parallel:
  - Rearrange into the gradient-intercept form $y = mx + c$
  - Compare the coefficients of $x$
  - If they are equal then the lines are parallel

### Worked example

The line $l$ passes through the point $(4, -1)$ and is parallel to the line with equation $2x - 5y = 3$.

Find the equation of $l$, giving your answer in the form $y = mx + c$.

Rearrange into $y = mx + c$ to find the gradient

$5y = 2x - 3 \Rightarrow y = \frac{2}{5}x - \frac{3}{5}$ $\therefore$ gradient $= \frac{2}{5}$

Parallel lines $\Rightarrow m_1 = m_2$

$m = \frac{2}{5}$

Use the point-gradient formula

Formula booklet

| | |
|---|---|
| Equations of a straight line | $y - y_1 = m(x - x_1)$ |

$(x_1, y_1) = (4, -1)$ $m = \frac{2}{5}$

$y + 1 = \frac{2}{5}(x - 4)$

$y + 1 = \frac{2}{5}x - \frac{8}{5}$

$\boxed{y = \frac{2}{5}x - \frac{13}{5}}$

## Perpendicular Lines

### How are the equations of perpendicular lines connected?

- **Perpendicular lines** intersect at right angles
- The gradients of two perpendicular lines are negative reciprocals
  - If the gradient of line $l_1$ is $m_1$ and gradient of line $l_2$ is $m_2$ then...
    - $m_1 \times m_2 = -1 \Rightarrow l_1$ & $l_2$ are perpendicular
    - $l_1$ & $l_2$ are perpendicular $\Rightarrow m_1 \times m_2 = -1$
- To determine if two lines are perpendicular:
  - Rearrange into the gradient-intercept form $y = mx + c$
  - Compare the coefficients of $x$
  - If their product is -1 then they are perpendicular
- Be careful with horizontal and vertical lines
  - $x = p$ and $y = q$ are perpendicular where $p$ and $q$ are constants

## Worked example

The line $l_1$ is given by the equation $3x - 5y = 7$.

The line $l_2$ is given by the equation $y = \frac{1}{4} - \frac{5}{3}x$.

Determine whether $l_1$ and $l_2$ are perpendicular. Give a reason for your answer.

Rearrange $l_1$ into $y = mx + c$ form

$5y = 3x - 7 \Rightarrow y = \frac{3}{5}x - \frac{7}{5}$

Identity gradients

$m_1 = \frac{3}{5}$ $m_2 = -\frac{5}{3}$

$m_1 \times m_2 = -1 \Rightarrow$ Perpendicular lines

$\frac{3}{5} \times -\frac{5}{3} = -1$

$\boxed{l_1 \text{ and } l_2 \text{ are perpendicular as } m_1 \times m_2 = -1}$