Exam Papers Practice

2.1 Linear Functions & Graphs

Mark Schemes

Course	DP IB Maths
Section	2. Functions
Topic	2.1 Linear Functions & Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) i) The y-intercept is when $x = 0$.

$2(0) - y + 6 = 0$

$y = 6$

$\therefore$ The y-intercept is at $(0, 6)$.

ii) The x-intercept is when $y = 0$.

$2x - (0) + 6 = 0$

$x = -3$

$\therefore$ The x-intercept is at $(-3, 0)$.

iii) Rearrange L_1 into the form $y = mx + c$, where m is the gradient.

$2x - y + 6 = 0$

$y = 2x + 6$) $+y$ and rearrange

$\therefore$ The gradient of L_1 is 2.

b) i) Perpendicular gradients

$m_2 = -\frac{1}{m_1}$

$\boxed{m_2 = -\frac{1}{2}}$

ii) Point - gradient formula

$y - y_1 = m(x - x_1)$ (in formula booklet)

point $(4, 0)$ and $m_2 = -\frac{1}{2}$

Sub x_1, y_1 and m_2 into $y - y_1 = m(x - x_1)$.

$y - 0 = -\frac{1}{2}(x - 4)$ → expand RHS

$y = -\frac{1}{2}x + 2$ → $\times 2$

$2y = -x + 4$ → rearrange to make a, b and c integers

$\boxed{x + 2y - 4 = 0}$

Question 2

a) Midpoint formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

A(2, 8) B(-8, 2)

Sub A and B into formula to find M.

$M = \left(\frac{2 + (-8)}{2}, \frac{8 + 2}{2}\right)$

$M = \left(\frac{-6}{2}, \frac{10}{2}\right)$

$\boxed{M = (-3, 5)}$

b) Gradient formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (in formula booklet)

A(2, 8) B(-8, 2)

Sub A and B into formula to find m_1.

$m_1 = \frac{2 - 8}{-8 - 2}$

$m_1 = \frac{-6}{-10}$

$\boxed{m_1 = \frac{3}{5}}$

c) Point - gradient formula

$y - y_1 = m(x - x_1)$ (in formula booklet)

*A(2, 8) $m_1 = \frac{3}{5}$

Sub A and m_1 into $y - y_1 = m(x - x_1)$.

$y - 8 = \frac{3}{5}(x - 2)$

expand RHS

$y - 8 = \frac{3}{5}x - \frac{6}{5}$

x4

$5y - 40 = 3x - 6$

rearrange to make a, b and c integers

$3x - 5y + 34 = 0$

*N.B. You could also use B.

Question 3 a) Midpoint formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

A(1, 7) B(5, 5)

Sub A and B into formula to find M.

$M = \left(\frac{1+5}{2}, \frac{7+5}{2}\right)$

$M = \left(\frac{6}{2}, \frac{12}{2}\right)$

$M = (3, 6)$

b) Gradient formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (in formula booklet)

A(1, 7) B(5, 5)

Sub A and B into formula to find m_1.

$m_1 = \frac{5-7}{5-1}$ $\therefore m_1 = -\frac{1}{2}$

Sub A and m_1 into $y - y_1 = m(x - x_1)$.

$y - 7 = -\frac{1}{2}(x - 1)$ → expand RHS

$y - 7 = -\frac{1}{2}x + \frac{1}{2}$ → + 7

$y = -\frac{1}{2}x + \frac{15}{2}$

c) l_2 is perpendicular to l_1 and passes through M.

Perpendicular gradients

$m_2 = -\frac{1}{m_1}$ $\quad m_1 = -\frac{1}{2}$

$\therefore m_2 = 2$

$M(3, 6)$ and $m_2 = 2$

Sub M and m_2 into $y - y_1 = m(x - x_1)$.

$y - 6 = 2(x - 3)$

$y - 6 = 2x - 6$

$\boxed{y = 2x}$

Question 4

a) Identify the linear function.

$y = mx + c$

$y = C_A$

$m = \$15/\text{hour}$ (hourly rate)

$x = t$ hours

$c = \$25$ (fixed fee)

$\boxed{C_A = 15t + 25}$

b) Sub $t = 7$ into C_A.

$C_A = 15(7) + 25$

$C_A = \$130$

c) Identify the linear function.

$y = mx + c$

$y = C_B$

$m = \$16/\text{hour}$ (hourly rate)

$x = t$ hours

$c = \$20$ (fixed fee)

$C_B = 16t + 20$

d) Sub $t = 6$ into C_A and C_B.

$C_A = 15(6) + 25$ $\quad$ $C_B = 16(6) + 20$

$C_A = \$115$ $\quad$ $C_B = \$116$

$\therefore$ Plumber A is cheaper.

Question 5

a) Gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ (in formula booklet)

A(0, 10) B(5, 0)

Sub A and B into formula.

$$m_1 = \frac{0 - 10}{5 - 0} \quad \therefore m_1 = -2$$

Sub A and m_1 into $y - y_1 = m(x - x_1)$.

$y - 10 = -2(x - 0)$ → expand RHS

$y - 10 = -2x$ → +10

$$y = -2x + 10$$

b) Distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$ (in formula booklet)

A(0, 10) B(5, 0)

Sub A and B into formula.

$$d = \sqrt{(0 - 5)^2 + (10 - 0)^2}$$

$$d = 11.2 \text{ units}$$

c) Parallel lines have the same gradient.

$C(8,0)$ and $m_1 = m_2 = -2$

Sub C and m_2 into $y - y_1 = m(x - x_1)$.

$y - 0 = -2(x - 8)$

$y = -2x + 16$

$$2x + y - 16 = 0$$

d) y-intersect happens when $x = 0$.

Sub $x = 0$ into L_2.

$2(0) + y - 16 = 0$

$y - 16 = 0$

$y = 16$

$\therefore$ y-intersect at $(0, 16)$

Question 6

a) Linear relationship: $y = mx + c$

$122 = m(115) + c$ $\quad$ $190 = m(200) + c$

solve simultaneous equations using your GDC.

$m = 0.8$ and $c = 30$

$\therefore y_A = 0.8x + 30$

i) Sub $x = 180$.

$y_A = 0.8(180) + 30$

$y_A = \$174$

ii) Sub $y_A = 385.20$.

$385.20 = 0.8x + 30$

$x = 444$ copies

b) $y_A = 0.8x + 30$ $\quad$ $y_B = 0.82x + 25.50$

Sub $x = 220$ into y_A and y_B.

$y_A = 0.8(220) + 30$ $\quad$ $y_B = 0.82(220) + 25.50$

$y_A = \$206$ $\quad$ $y_B = \$205.90$

$\therefore$ Photocopy shop B is cheaper.

Question 7

a) Linear relationship: $y = mx + c$

Company A: $y = 0.2x + 25$

Company B: $y = 0.22x + 10$

b) Set both cost functions equal to each other.

Company A = Company B

$0.2x + 25 = 0.22x + 10$

Solve using your GDC.

$x = 750$

Monthly energy consumption = 750 kWh

Question 8

a) Linear function: $C(x) = mx + c$

$m = \frac{\text{change in expenditure}}{\text{change in income}} = \frac{\text{change in } C(x)}{\text{change in } x}$

$m = \frac{*\Delta C(x)}{\Delta x}$ *Δ = "change in..."

$\Delta C(x) = 60$ $\Delta x = 150$

Sub $\Delta C(x)$ and Δx into formula.

$m = \frac{60}{150}$ $\therefore m = 0.4$

$C(x) = 1000$ $x = 1200$ $m = 0.4$ $(C(1200) = 1000)$

Sub $C(x)$, x and m into formula.

$0.4(1200) + c = 1000$ → rearrange for c

$c = 520$

$$C(x) = 0.4x + 520$$

b) Sub $x = 1885$ into $C(x)$.

$C(1885) = 0.4(1885) + 520$

$$C(1885) = \$1274$$

c) Sub $C(x) = 1070$ and solve for x.

$0.4x + 520 = 1070$ → -520

$0.4x = 550$ → $\div 0.4$

$x = \frac{550}{0.4}$

$$x = \$1375$$

Question 9

a) Two points on W are (0,0) and (20,8).

Sub points into gradient formula.

$$m = \frac{8-0}{20-0}$$

$$\boxed{m = 0.4}$$

b) Find the equation of the east slope.

point (20,8) and $m = -\frac{3}{10}$

$$y - 8 = -\frac{3}{10}(x - 20)$$) expand RHS and + 8

$$y = -\frac{3}{10}x + 14$$

Find the x-intersect of the east slope.

$$0 = -\frac{3}{10}x + 14$$) $+\frac{3}{10}x$

$$\frac{3}{10}x = 14$$) $\times \frac{10}{3}$

$$x = \frac{140}{3}$$

x-intersect at $\left(\frac{140}{3}, 0\right)$

Distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$ (in formula booklet)

Total distance = west slope + east slope.

West points are (0,0) and (20,8).

East points are (20,8) and $\left(\frac{140}{3}, 0\right)$.

$$d = \sqrt{(0-20)^2 + (0-8)^2} + \sqrt{(20-\frac{140}{3})^2 + (8-0)^2}$$

$d = 49.4$ units

$$\boxed{d = 4.94 \text{ km}}$$

c) Real life vs. mathematical model

Any valid reason with an explanation is needed.

The actual total distance hiked may be greater than the answer in part (b) because the slope of a mountain is not constant.

Question 10

a) i) Sub (17, 0) and (0, 17) into gradient formula.

$m_1 = \frac{17-0}{0-17}$ $\therefore m_1 = -1$

Sub (17, 0) and m_1 into $y - y_1 = m(x - x_1)$.

$y - 0 = -1(x - 17)$

$y = -x + 17$

ii) Sub (2, 0) and (0, −1) into gradient formula.

$m_2 = \frac{-1-0}{0-2}$ $\therefore m_2 = \frac{1}{2}$

Sub (2, 0) and m_2 into $y - y_1 = m(x - x_1)$.

$y - 0 = \frac{1}{2}(x - 2)$

$y = \frac{1}{2}x - 1$

b) Shaded region forms a triangle.

Area of a triangle formula

$A = \frac{1}{2}bh$ (in formula booklet)

b is the base, h is the perpendicular height

b is formed by the x-intercepts of l_1 and l_2, (17, 0) and (2, 0) respectively.

$b = 17 - 2$ $\therefore b = 15$ units

h is the y-coordinate where l_1 and l_2 intersect

Find where l_1 and l_2 intersect.

Intersection = (12, 5) $\therefore h = 5$

Sub b and h into formula.

$A = \frac{1}{2}(15)(5)$

$A = 37.5 \text{ units}^2$

Question 11

a) PERPENDICULAR GRADIENT

$m_1 \times m_2 = -1$

$L_1 = x + y = 16$

$y = mx + c$ ↑ GRADIENT

$y = -x + 16$

$m_1 = -1$ $\quad m = 1$

$y - y_1 = m(x - x_1)$ $\quad (0,0)$ $\quad m = 1$

L_2 $y = 1x$ $\quad y = x$

POINT R $L_1 = L_2$

SUB L_2 INTO L_1 $x + x = 16$

$2x = 16$

$x = 8$

$R = (8,8)$

b) $(8,8)$ IN THE RATIO $3:1$

$6:2$

$P = (6,6)$