Algebra and functions - Composite and inverse functions

Name: _____

Class: _____

Date: _____

Time:

Total marks available:

Total marks achieved: _____

A Level Mathematics : Pure Mathematics

Subject: Mathematics

Topic 2 : Algebra and functions - Composite and inverse functions

Type: Topic Questions

To be used by all students preparing for Edexcel A Level Mathematics - Students of other Boards may also find this useful

Q1.

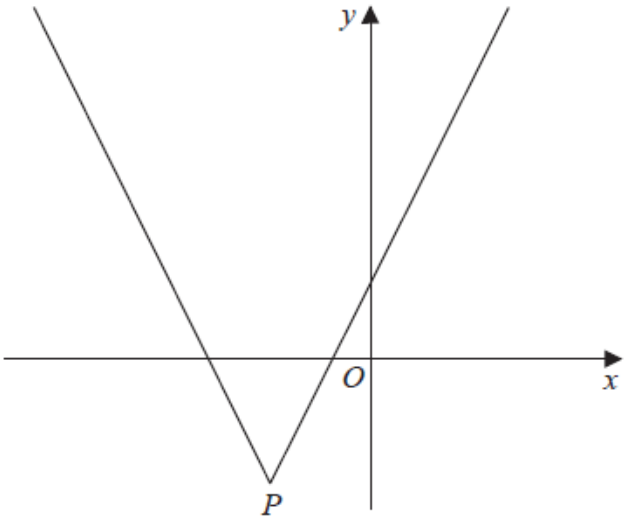

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

(Total for question = 7 marks)

Q2.

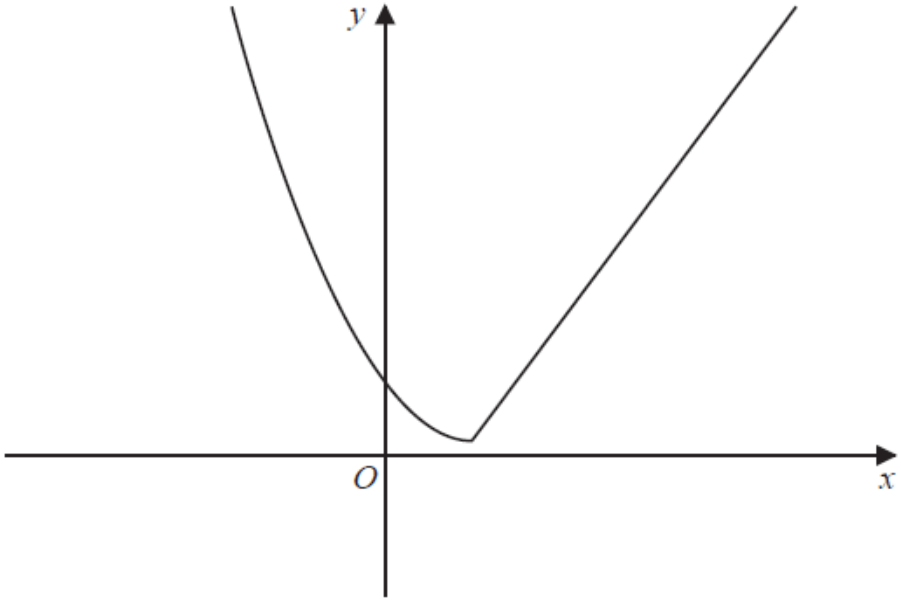

Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \qquad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for question = 10 marks)

Q3.

Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for gf(x), simplifying your answer.

(2)

(b) Show that there is only one real value of x for which gf(x) = fg(x)

(3)

(Total for question = 5 marks)

Q4.

$$g(x) = \frac{2x+5}{x-3} \quad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

(Total for question = 6 marks)

Q5.

The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(Total for question = 4 marks)