EXAM PAPERS PRACTICE

A Level Physics CIE

19. Capacitance

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19.1 Capacitors

19.1.1 Capacitance

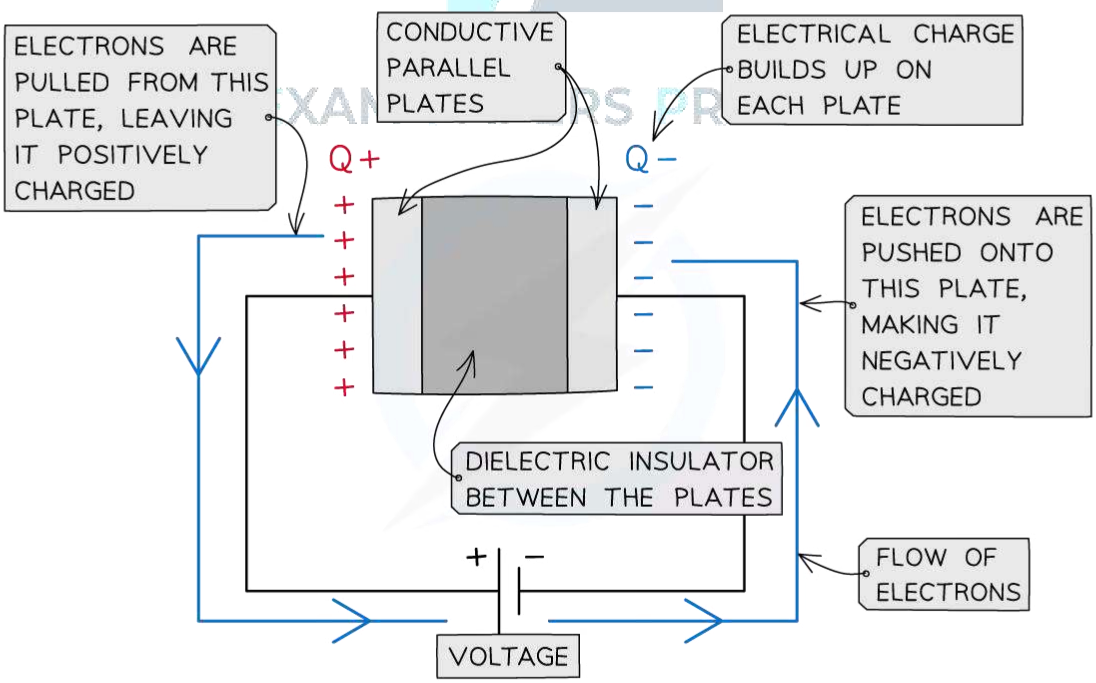
Defining Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- They can be in the form of:
  - An isolated spherical conductor
  - Parallel plates

- Capacitors are marked with a value of their **capacitance.** This is defined as:

  The charge stored per unit potential difference

- The greater the capacitance, the greater the energy stored in the capacitor
- A parallel plate capacitor is made up of two conductive metal plates connected to a voltage supply
  - The negative terminal of the voltage supply pushes electrons onto one plate, making it negatively charged
  - The electrons are repelled from the opposite plate, making it positively charged
  - There is commonly a dielectric in between the plates, this is to ensure charge does not freely flow between the plates



A parallel plate capacitor is made up of two conductive metal plates with opposite charges building up on each plate

Exam Tip

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The capacitor itself **does not** store charge.

Calculating Capacitance

- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
  - C = capacitance (F)
  - Q = charge (C)
  - V = potential difference (V)
- It is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Capacitance will often be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, Q is the charge on the plates and V is the potential difference across the capacitor
- The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates or spherical conductor
- This capacitance equation shows that an object's capacitance is the **ratio of the charge on an object to its potential**

Capacitance of a Spherical Conductor

- The capacitance of a charged sphere is defined by the charge per unit potential at the surface of the sphere
- The potential *V* is defined by the potential of an isolated point charge (since the charge on the surface of a spherical conductor can be considered as a point charge at its centre):

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Substituting this into the capacitance equation means the capacitance *C* of a sphere is given by the expression:

$$\mathbf{C = 4\pi\varepsilon_0 r}$$

> **?** Worked Example
>
> A parallel plate capacitor has a capacitance of 1 nF and connected to a voltage supply of 0.3 kV. Calculate the charge on the plates.

Step 1: Write down the known quantities

Capacitance, C = 1 nF = 1 × 10^{-9} F

Potential difference, V = 0.3 kV = 0.3 × 10^{3} V

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

$$\mathbf{Q = CV}$$

Step 4: Substitute in values

$$\mathbf{Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\ C = 300\ nC}$$

Exam Tip

The letter ‘C’ is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

19.1.2 Derivation of C = Q/V

Derivation of C = Q/V

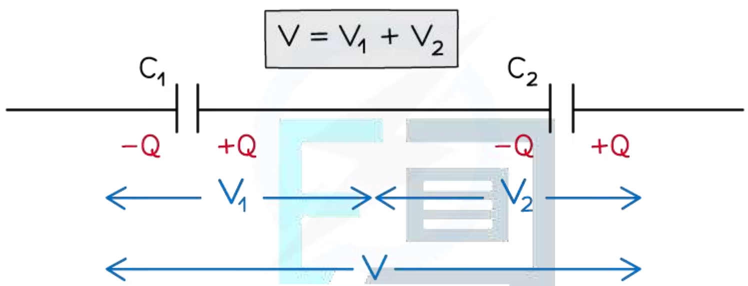
- The circuit symbol for a parallel plate capacitor is two parallel lines

Circuit symbol for a capacitor

- Capacitors can be combined in series and parallel circuits
- The combined capacitance depends on whether the capacitors are connected in series or parallel

Capacitors in Series

- Consider two parallel plate capacitors C_1 and C_2 connected in series, with a potential difference (p.d) V across them



Capacitors connected in series have different p.d across them but have the same charge

- In a series circuit, p.d is **shared** between all the components in the circuit
  - Therefore, if the capacitors store the same charge on their plates but have different p.ds, the p.d across C_1 is V_1 and across C_2 is V_2
- The total potential difference V is the sum of V_1 and V_2

$$\mathbf{V = V_1 + V_2}$$

- Rearranging the capacitance equation for the p.d V means V_1 and V_2 can be written as:

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

- Where the total p.d V is defined by the total capacitance

$$V = \frac{Q}{C_{total}}$$

- Substituting these into the equation $V = V_1 + V_2$ equals:

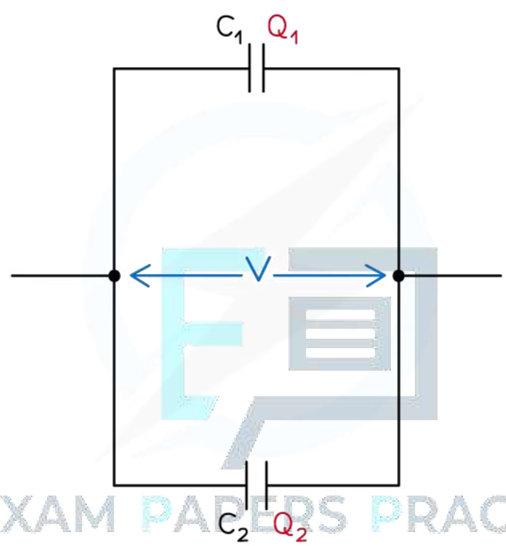
$$\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

- Since the current is the **same** through all components in a series circuit, the charge Q is the same through each capacitor and cancels out
- Therefore, the equation for combined capacitance of capacitors in **series** is:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

Capacitors in Parallel

- Consider two parallel plate capacitors C_1 and C_2 connected in parallel, each with p.d V



Capacitors connected in parallel have the same p.d across them, but different charge

- Since the current is **split** across each junction in a parallel circuit, the charge stored on each capacitor is **different**
- Therefore, the charge on capacitor C_1 is Q_1 and on C_2 is Q_2
- The total charge Q is the sum of Q_1 and Q_2

$$Q = Q_1 + Q_2$$

- Rearranging the capacitance equation for the charge Q means Q_1 and Q_2 can be written as:

$$Q_1 = C_1V \quad \text{and} \quad Q_2 = C_2V$$

- Where the total charge Q is defined by the total capacitance:

$$Q = C_{total}V$$

- Substituting these into the $Q = Q_1 + Q_2$ equals:

$$C_{total}V = C_1V + C_2V = (C_1 + C_2)\ V$$

- Since the p.d is the **same** through all components in each branch of a parallel circuit, the p.d V cancels out
- Therefore, the equation for combined capacitance of capacitors in **parallel** is:

$$C_{total} = C_1 + C_2 + C_3 \ldots$$

Exam Tip

You will be expected to remember these derivations for your exam, therefore, make sure you understand each step. You should especially make sure to revise how the current and potential difference varies in a series and parallel circuit.

Capacitors in Series & Parallel

- Recall the formula for the combined capacitance of capacitors in series:

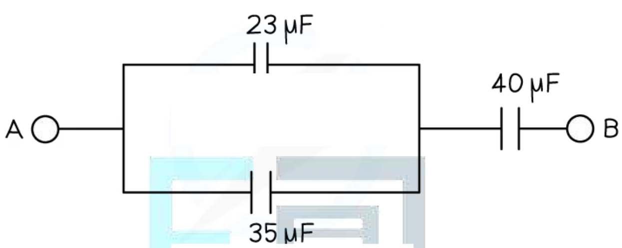
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

- In parallel:

$$C_{total} = C_1 + C_2 + C_3 \ldots$$

Worked Example

Three capacitors with capacitance of 23 μF, 35 μF and 40 μF are connected as shown below



Calculate the total capacitance between points A and B

Step 1: Calculate the combined capacitance of the two capacitors in parallel

Capacitors in parallel: $C_{total} = C_1 + C_2 + C_3 \ldots$

$$C_{parallel} = 23 + 35 = 58 \ \mu F$$

Step 2: Connect this combined capacitance with the final capacitor in series

Capacitors in series: $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$

$$\frac{1}{C_{total}} = \frac{1}{58} + \frac{1}{40} = \frac{49}{1160}$$

Step 3: Rearrange for the total capacitance

$$C_{total} = \frac{1160}{49} = 23.673\ldots = \mathbf{24\ \mu F}\ (2\ s.f)$$

Exam Tip

Both the combined capacitance equations look similar to the equations for combined resistance in series and parallel circuits. However, take note that they are the **opposite way** around to each other!

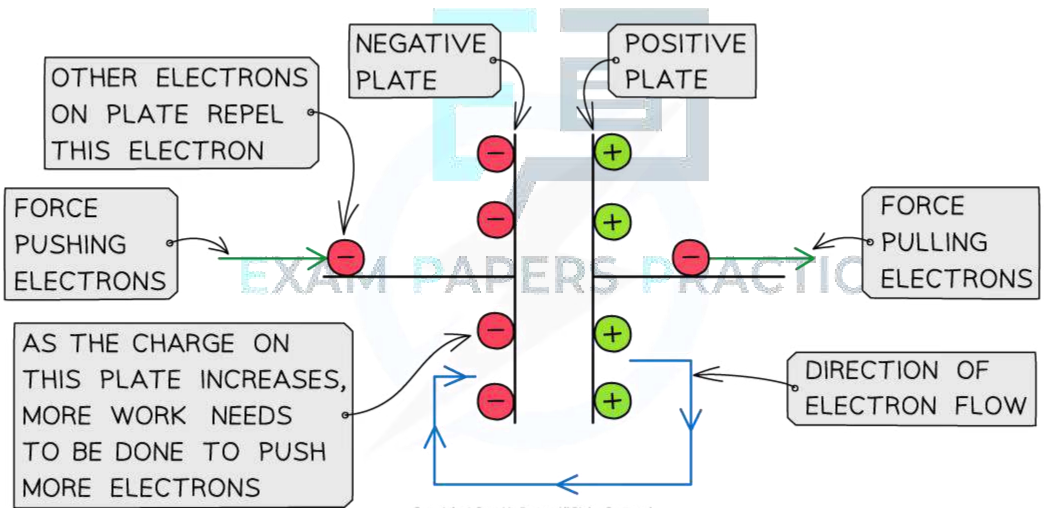
Area Under a Potential-Charge Graph

- When charging a capacitor, the power supply pushes electrons from the positive to the negative plate
  - It therefore does **work** on the electrons, which increase their **electric potential energy**

- At first, a small amount of charge is pushed from the positive to the negative plate, then gradually, this builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion

- As the charge of the negative plate increases ie. becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

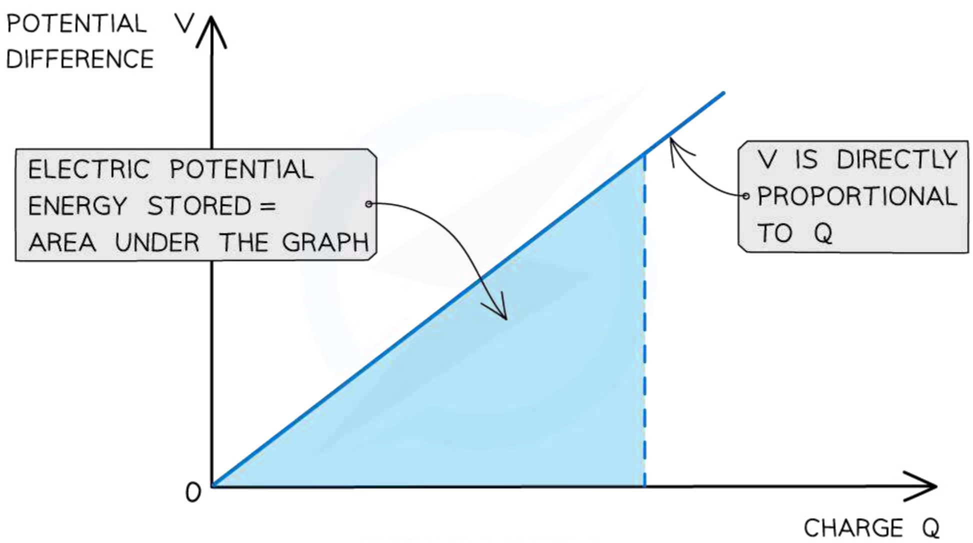
The potential difference V across the capacitor increases as the amount of charge Q increases



As the charge on the negative plate builds up, more work needs to be done to add more charge

- The charge Q on the capacitor is **directly proportional** to its potential difference V
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The electric potential energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the area of a right-angled triangle:

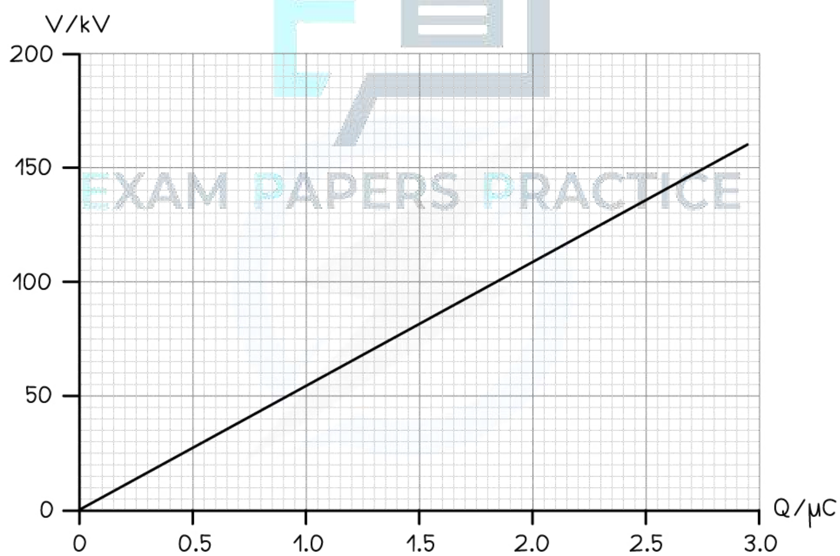
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$



The electric potential energy stored in the capacitor is the area under the potential-charge graph

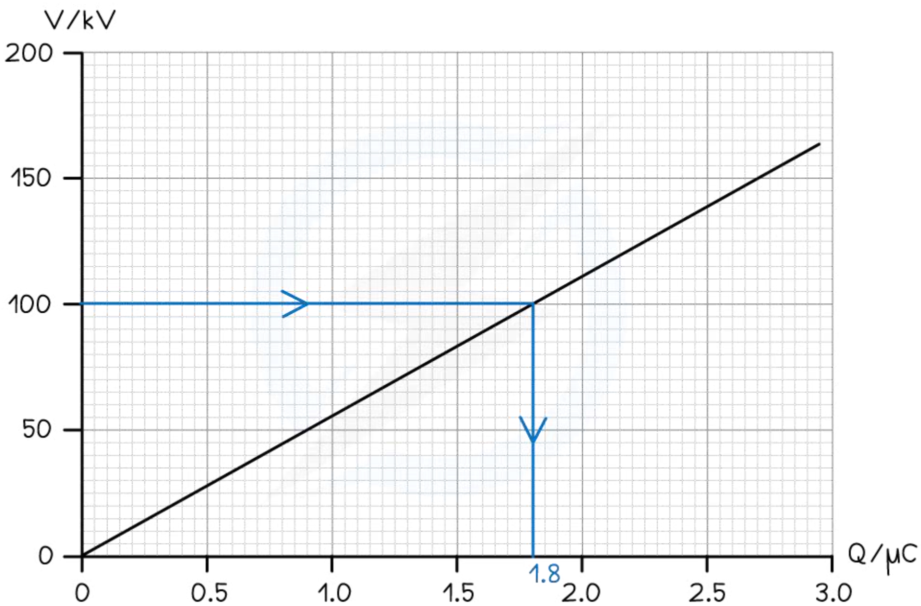
Worked Example

The variation of the potential V of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

Step 1: Determine the charge on the sphere at the potential of 100 kV



- From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2: Calculate the electric potential energy stored

- The electric potential energy stored is the area under the graph at 100 kV
- The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

- Substituting in the values gives:

$$\text{Area} = \frac{1}{2} \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{E.P.E} = \frac{1}{2} \times 1.8 \times 10^{-6} \times 100 \times 10^{3} = \mathbf{0.09\ J}$$

Exam Tip

Remember to always check the **units** of the charge-potential difference graphs. The charges can often be in μC or the potential difference in kV! The units must be in C and V to get a work done in J.

Calculating Energy Stored in a Capacitor

- Recall the **electric potential energy is the area under a potential-charge graph**
- This is equal to the **work done** in charging the capacitor to a particular potential difference
  - The shape of this area is a right angled triangle
- Therefore the work done, or **energy stored** in a capacitor is defined by the equation:

$$W = \frac{1}{2}QV$$

- Substituting the charge with the capacitance equation Q = CV, the work done can also be defined as:

$$W = \frac{1}{2}CV^2$$

- Where:
  - W = work done/energy stored (J)
  - Q = charge on the capacitor (C)
  - V = potential difference (V)
  - C = capacitance (F)
- By substituting the potential V, the work done can also be defined in terms of just the charge and the capacitance:

$$W = \frac{Q^2}{2C}$$

Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Step 1: Write down the equation for energy stored in terms of capacitance C and p.d V

$$W = \frac{1}{2}CV^2$$

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C(V_2 - V_1)^2$$

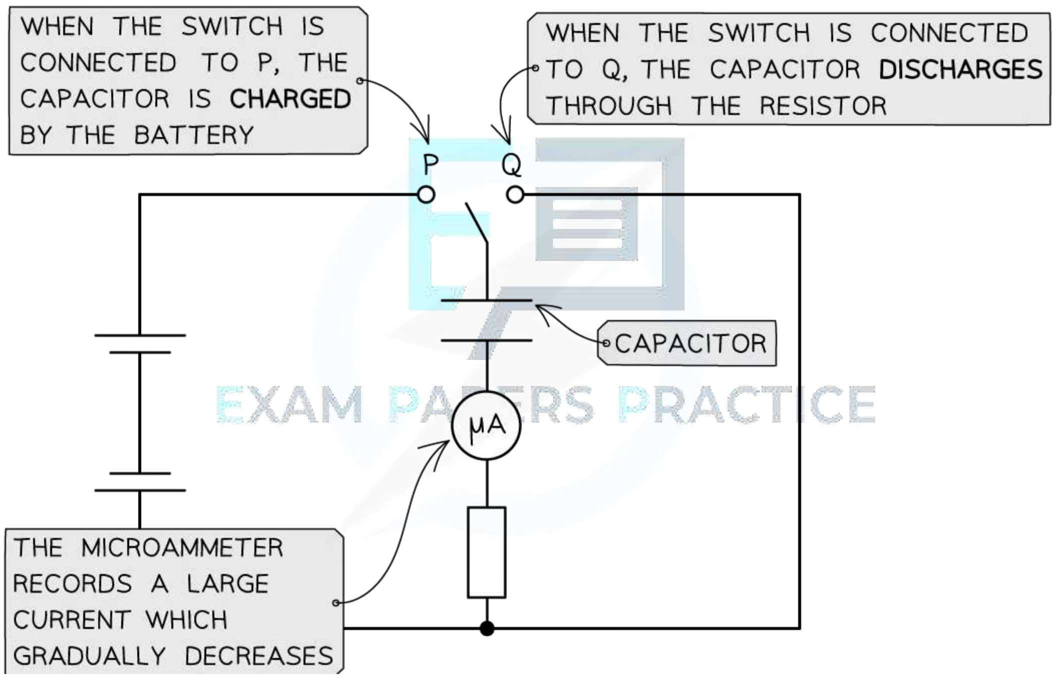
Step 3: Substitute in values

$$\Delta W = \frac{1}{2} \times 1500 \times 10^{-6} \times (30 - 10)^2 = 0.3 \text{ J}$$

19.2.1 Capacitor Discharge Graphs

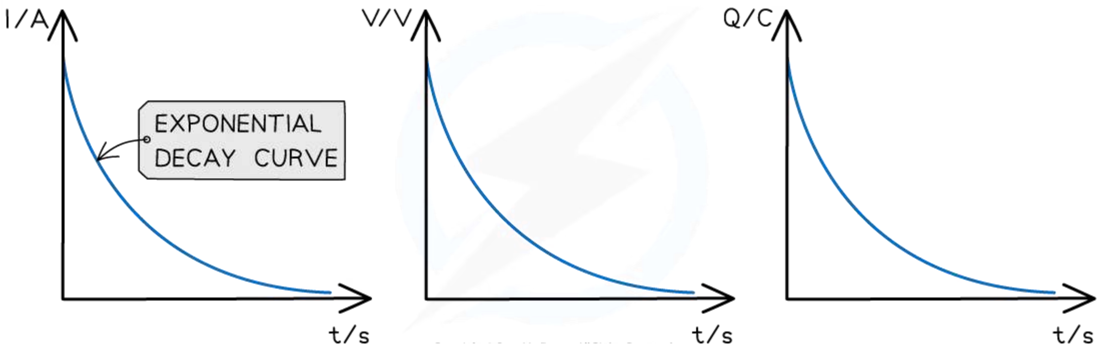
Capacitor Discharge Graphs

- So far, only capacitors charged by a battery have been considered
  - This is when the electrons flow from the positive to negative plate
  - At the start, when the capacitor is charging, the current is large and then gradually falls to zero
- Capacitors are **discharged** through a resistor
  - The electrons now flow back from the negative plate to the positive plate until there are equal numbers on each plate
- At the start of discharge, the current is large (but in the opposite direction to when it was charging) and gradually falls to zero



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
- The means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and represent an **exponential decay**



Graphs of variation of current, p.d and charge with time for a capacitor discharging through a resistor

- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient
  - The initial value starts on the y axis and decreases exponentially

- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease and charge will flow from the capacitor plates more slowly, meaning the capacitor will take longer to discharge
  - If the resistance is **low**, the current will increase and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge faster

The Time Constant

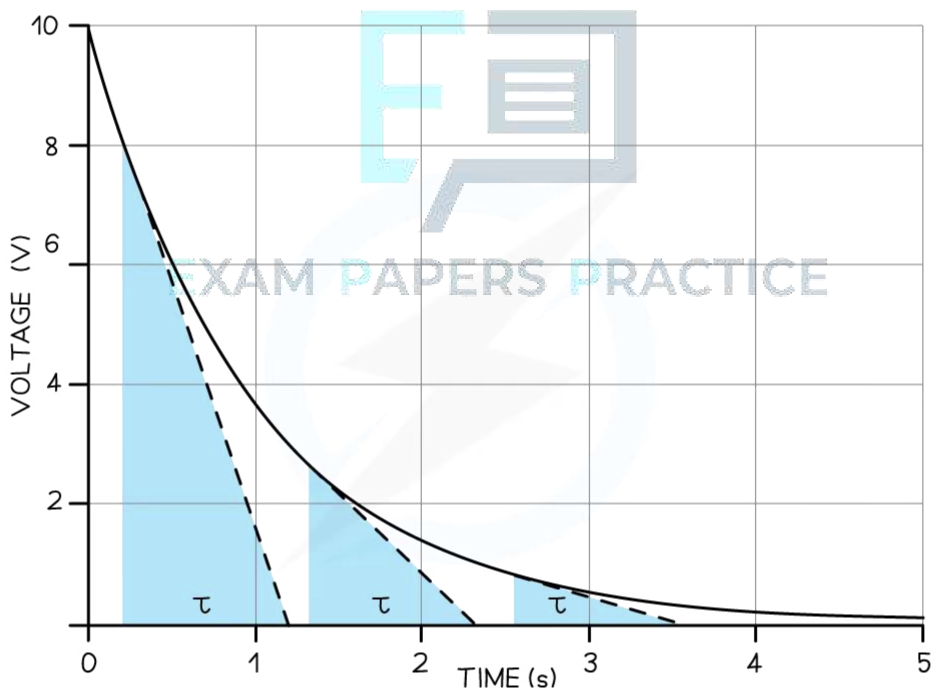
- The time constant of a capacitor discharging through a resistor is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant is:

The time taken for the charge of a capacitor to decrease to 0.37 of its original value

- This is represented by the greek letter tau (τ) and measured in units of **seconds** (s)
- The time constant gives an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- The time constant is defined by the equation:

$$\tau = RC$$

- Where:
  - τ = time constant (s)
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)



The graph of voltage-time for a discharging capacitor showing the positions of the first three time constants

Worked Example

A capacitor of 7 nF is discharged through a resistor of resistance R. The time constant of the discharge is 5.6×10^{-3} s.Calculate the value of R.

Step 1: Write out the known quantities

Capacitance, C = 7 nF = 7×10^{-9} F

Time constant, $\tau = 5.6 \times 10^{-3}$ s

Step 2: Write down the time constant equation

$$\tau = RC$$

Step 3: Rearrange for resistance R

$$R = \frac{\tau}{C}$$

Step 4: Substitute in values and calculate

$$R = \frac{5.6 \times 10^{-3}}{7 \times 10^{-9}} = 8 \times 10^5\ \Omega = \mathbf{800\ k\Omega}$$

Using the Capacitor Discharge Equation

- The time constant is used in the exponential decay equations for the current, charge or potential difference (p.d) for a capacitor discharging through a resistor
  - These can be used to determine the amount of current, charge or p.d left after a certain amount of time when a capacitor is discharging

- The exponential decay of current on a discharging capacitor is defined by the equation:

$$I = I_0 e^{-\frac{t}{RC}}$$

- Where:
  - I = current (A)
  - I_0 = initial current before discharge (A)
  - e = the exponential function
  - t = time (s)
  - RC = resistance (Ω) × capacitance (F) = the time constant τ (s)

- This equation shows that the faster the time constant τ, the quicker the exponential decay of the current when discharging
- Also, how big the initial current is affects the rate of discharge
  - If I_0 is large, the capacitor will take longer to discharge

- **Note:** during capacitor discharge, I_0 is always larger than I, this is because the current I will always be decreasing
- The current at any time is directly proportional to the p.d across the capacitor and the charge across the parallel plates

- Therefore, this equation also describes the change in p.d and charge on the capacitor:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

- Where:
  - Q = charge on the capacitor plates (C)
  - Q_0 = initial charge on the capacitor plates (C)

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
  - V = p.d across the capacitor (C)
  - V_0 = initial p.d across the capacitor (C)

The Exponential Function *e*

- The symbol e represents the exponential constant, a number which is approximately equal to e = 2.718...
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if e^x = y, then x = ln (y)

- The 0.37 in the definition of the **time constant** arises as a result of the exponential constant, the true definition is:

The time taken for the charge of a capacitor to decrease to $\frac{1}{e}$ of its original value

- Where $\frac{1}{e}$ = 0.3678...

Worked Example

The initial current through a circuit with a capacitor of 620 μF is 0.6 A. The capacitor is connected across the terminals of a 450 Ω resistor. Calculate the time taken for the current to fall to 0.4 A.

Step 1: Write out the known quantities

Initial current before discharge, I_0 = 0.6 A

Current, I = 0.4 A

Resistance, R = 450 Ω

Capacitance, C = 620 μF = 620 × 10^{-6} F

Step 2: Write down the equation for the exponential decay of current

$$I = I_0 e^{-\frac{t}{RC}}$$

Step 3: Calculate the time constant

τ = RC

τ = 450 × (620 × 10^{-6}) = 0.279 s

Step 4: Substitute into the current equation

$$0.4 = 0.6 \times e^{-\frac{t}{0.279}}$$

Step 5: Rearrange for the time *t*

$$\frac{0.4}{0.6} = e^{-\frac{t}{0.279}}$$

The exponential can be removed by taking the natural log of both sides:

$$\ln\left(\frac{0.4}{0.6}\right) = -\frac{t}{0.279}$$

$$t = -0.279 \times \ln\left(\frac{0.4}{0.6}\right) = 0.1131 = 0.1 \text{ s}$$

Exam Tip

Make sure you're confident in rearranging equations with natural logs (ln) and the exponential function (e). To refresh your knowledge of this, have a look at the AS Maths revision notes on Exponentials & Logarithms