A Level Physics CIE

18. Electric Fields

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18.1 Electric Fields

18.1.1 Electric Fields & Forces on Charges

Electric Field Definition

- An electric field is a region of space in which an electric charge "feels" a force
- **Electric field strength** at a point is defined as:

__The electrostatic force per unit positive charge acting on a stationary point charge at that point__

- Electric field strength can be calculated using the equation:

$$E = \frac{F}{Q}$$

- Where:
  - E = electric field strength ($N\ C^{-1}$)
  - F = electrostatic force on the charge (N)
  - Q = charge (C)

- It is important to use a positive test charge in this definition, as this determines the direction of the electric field
- The electric field strength is a **vector** quantity, it is always directed:
  - **Away** from a positive charge
  - **Towards** a negative charge

- Recall that **opposite charges** (positive and negative) charges **attract** each other
- Conversely, **like charges** (positive and positive or negative and negative) **repel** each other

Worked Example

A charged particle is in an electric field with electric field strength 3.5×10^4 $N\ C^{-1}$ where it experiences a force of 0.3 N.Calculate the charge of the particle.

Step 1: Write down the equation for electric field strength

$$E = \frac{F}{Q}$$

Step 2: Rearrange for charge Q

$$Q = \frac{F}{E}$$

Step 3: Substitute in values and calculate

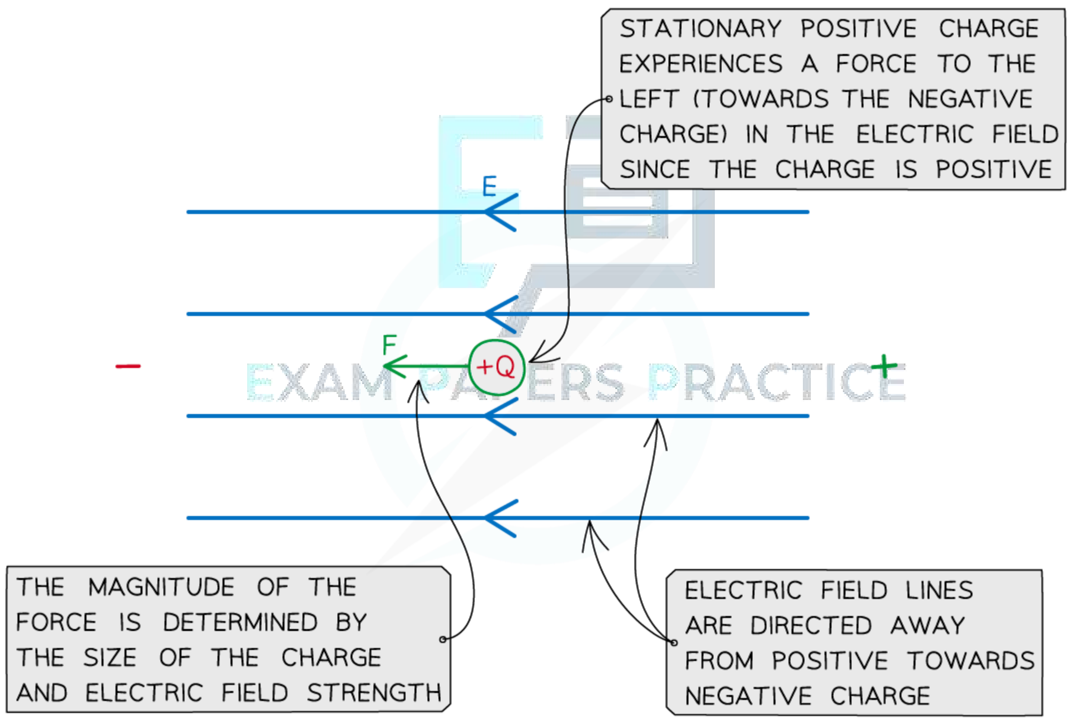
$Q = \frac{0.3}{3.5 \times 10^4} = 8.571 \times 10^{-6}$ C = $\mathbf{8.6 \times 10^{-6}}$ **C** (2 s.f)

Forces on Charges

- The electric field strength equation can be rearranged for the force F on a charge Q in an electric field E:

$$F = QE$$

- Where:
  - F = electrostatic force on the charge (N)
  - Q = charge (C)
  - E = electric field strength ($N\ C^{-1}$)
- The direction of the force is determined by the charge:
  - If the charge is **positive** (+) the force is in the **same** direction as the E field
  - If the charge is **negative** (–) the force is in the **opposite** direction to the E field
- The force on the charge will cause the charged particle to **accelerate** if its in the same direction as the E field, or **decelerate** if in the opposite



An electric field strength E exerts a force F on a charge +Q in a uniform electric field

- **Note:** the force will always be **parallel** to the electric field lines

Worked Example

An electron is stationary in an electric field with an electric field strength of 5000 N C^{-1}. Calculate the magnitude of the electric force that acts on the electron and state which direction the force will act in relation to the electric field.Electron charge $e = 1.60 \times 10^{-19}$ C.

Step 1: Write out the equation for the force on a charged particle

$$\mathbf{F = QE}$$

Step 2: Substitute in values

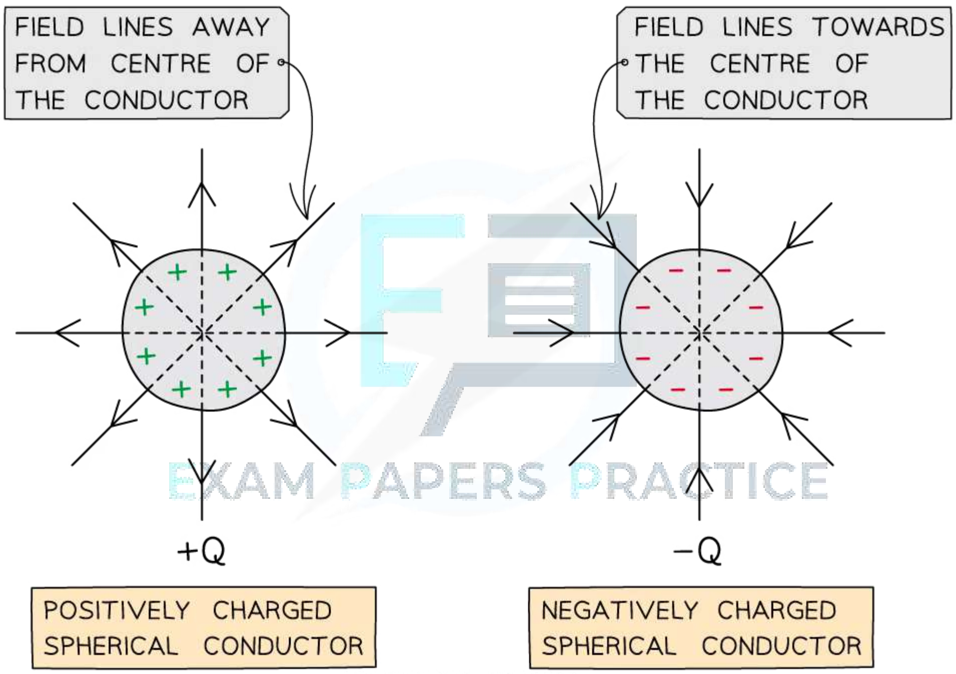
$$\mathbf{F = (1.60 \times 10^{-19}) \times 5000 = 8 \times 10^{-16}\ N}$$

Step 3: State the direction of the force

Since the charge is negative, the force is **directed against the electric field lines** and decelerates the electron.

Point Charge Approximation

- For a point outside a spherical conductor, the charge of the sphere may be considered to be a **point charge** at its centre
    - A **uniform** spherical conductor is one where its charge is **distributed evenly**
- The electric field lines around a spherical conductor are therefore **identical to those around a point charge**
- An example of a spherical conductor is a **charged sphere**
- The field lines are **radial** and their direction depends on the charge of the sphere
    - If the spherical conductor is **positively** charged, the field lines are directed **away** from the centre of the sphere
    - If the spherical conductor is **negatively** charged, the field lines are directed **towards** the centre of the sphere



Electric field lines around a uniform spherical conductor are identical to those on a point charge

Exam Tip

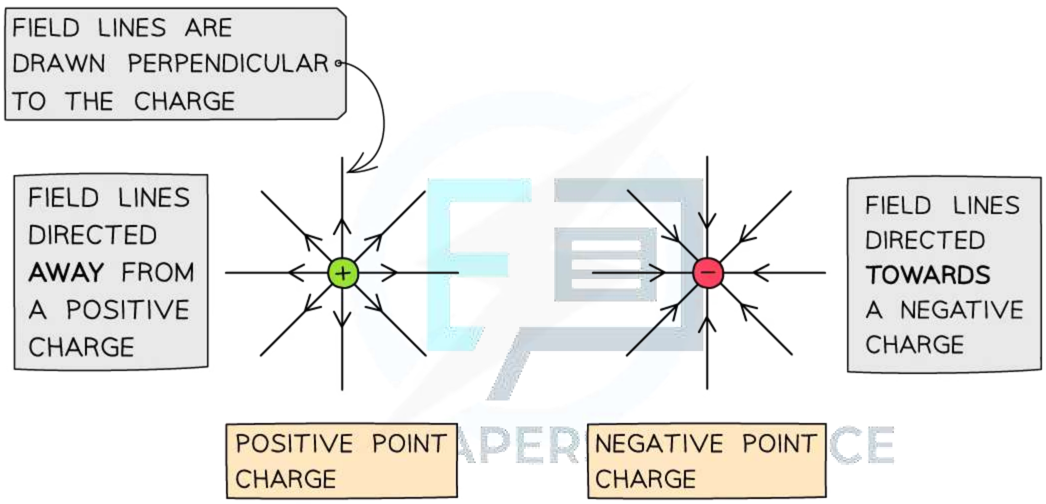
You might have noticed that the electric fields share many similarities to the gravitational fields. The main difference being the gravitational force is always attractive, whilst electrostatic forces can be attractive or repulsive.You should make a list of all the similarities and differences you can find, as this could come up in an exam question.

Representing Electric Fields

- The direction of electric fields is represented by electric field lines
- Electric field lines are directed from positive to negative
  - Therefore, the field lines must be pointed **away** from the **positive** charge and **towards** the **negative** charge

- A radial field spreads uniformly to or from the charge in all directions
  - e.g. the field around a point charge or sphere

- Around a **point charge**, the electric field lines are directly radially inwards or outwards:
  - If the charge is **positive** (+), the field lines are radially **outwards**
  - If the charge is **negative** (–), the field lines are radially **inwards**



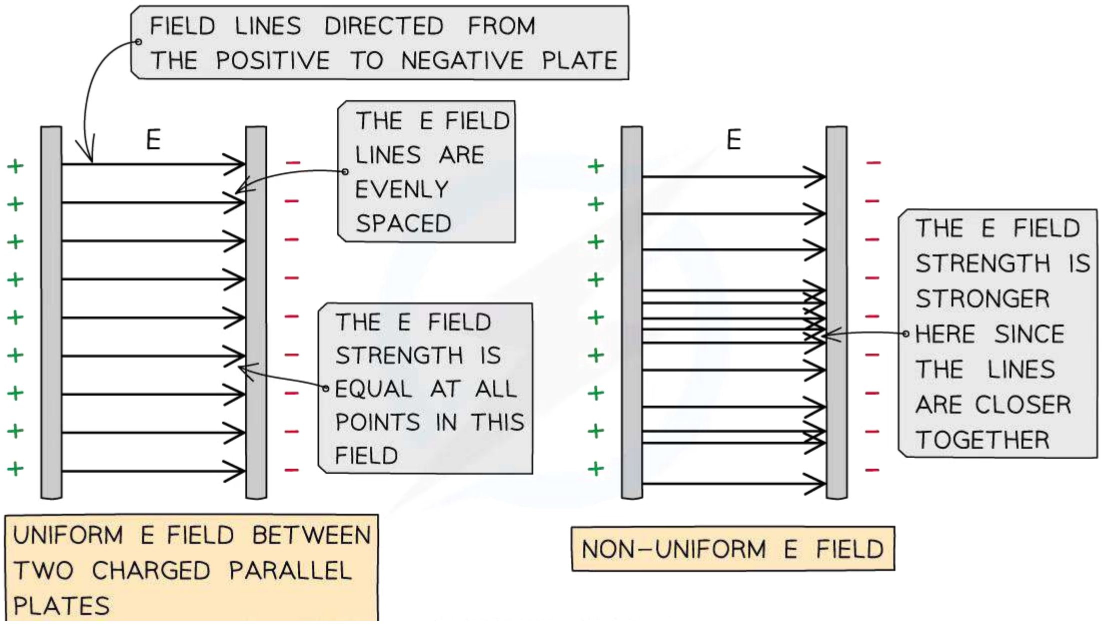
Electric field lines point away from a positive charge and point towards a negative charge

- This shares many similarities to radial gravitational field lines around a point mass
- Since gravity is only an attractive force, the field lines will look similar to the negative point charge, whilst electric field lines can be in either direction

- A uniform electric field has the same electric field strength throughout the field
  - For example, the field between oppositely charged parallel plates

- This is represented by **equally spaced** field lines
  - This shares many similarities to uniform gravitational field lines on the surface of a planet

- A **non-uniform** electric field has varying electric field strength throughout
- The strength of an electric field is determined by the spacing of the field lines:
  - A **stronger** field is represented by the field lines **closer** together
  - A **weaker** field is represented by the field lines **further** apart



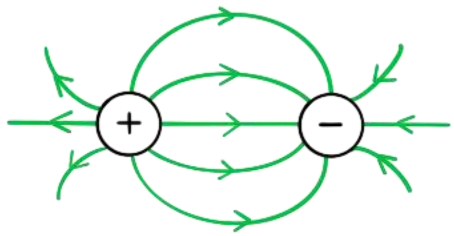
The electric field between two parallel plates is directed from the positive to the negative plate. A uniform E field has equally spaced field lines

- The electric field lines are directed from the **positive** to the **negative** plate
- A radial field is considered a **non-uniform** field
  - So, the electric field strength E is different depending on how far you are from a charged particle

Worked Example

Sketch the electric field lines between the two point charges in the diagram below.

- Electric field lines around point charges are radially outwards for positive charges and radially inwards for negative charges
- The field lines must be drawn with arrows **from the positive charge to the negative charge**



Exam Tip

Always label the arrows on the field lines! The lines must also touch the surface of the source charge or plates.

Electric Field Strength

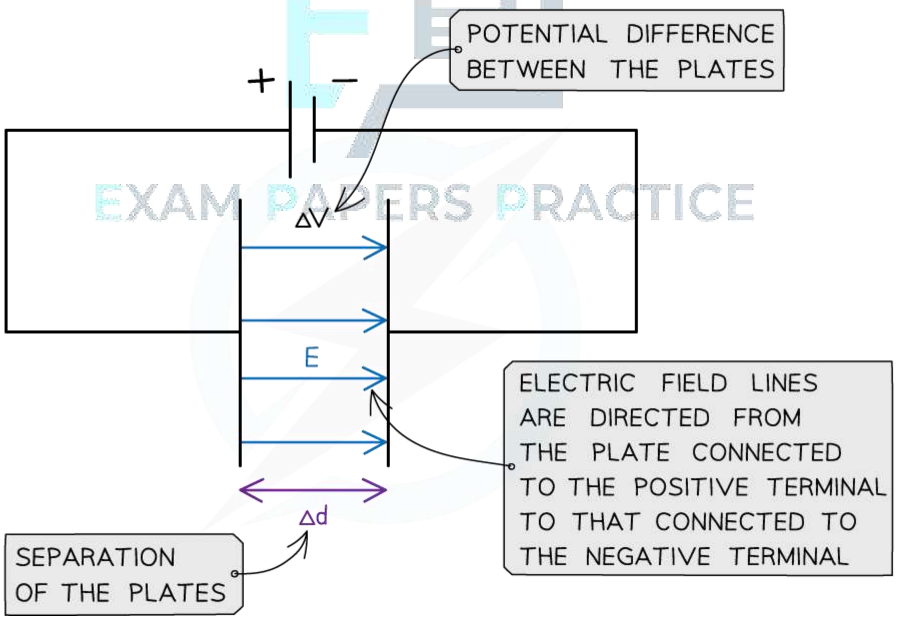
- The electric field strength of a uniform field between two charged parallel plates is defined as:

$$E = \frac{\Delta V}{\Delta d}$$

- Where:
  - E = electric field strength ($V\ m^{-1}$)
  - ΔV = potential difference between the plates (V)
  - Δd = separation between the plates (m)

- **Note:** the electric field strength is now also defined by the units **$V\ m^{-1}$**
- The equation shows:
  - The greater the voltage between the plates, the stronger the field
  - The greater the separation between the plates, the weaker the field

- Remember this equation cannot be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal



The E field strength between two charged parallel plates is the ratio of the potential difference and separation of the plates

- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

Worked Example

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV. Calculate the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Step 1: Write down the known values

Potential difference, ΔV = 7.9 kV = 7.9×10^3 V

Distance between plates, Δd = 3.5 cm = 3.5×10^{-2} m

Charge, Q = 2.6×10^{-15} C

Step 2: Calculate the electric field strength between the parallel plates

$$E = \frac{\Delta V}{\Delta d}$$

$$E = \frac{7.9 \times 10^3}{3.5 \times 10^{-2}} = 2.257 \times 10^5 \text{ V m}^{-1}$$

Step 3: Write out the equation for electric force on a charged particle

$$F = QE$$

Step 4: Substitute electric field strength and charge into electric force equation

$F = QE = (2.6 \times 10^{-15}) \times (2.257 \times 10^5) = 5.87 \times 10^{-10}$ N = 5.9×10^{-10} N (2 s.f.)

Electric Field of a Point Charge

- The electric field strength at a point describes how strong or weak an electric field is at that point
- The electric field strength E at a distance r due to a point charge Q in free space is defined by:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

- Where:
  - Q = the charge producing the electric field (C)
  - r = distance from the centre of the charge (m)
  - ϵ_0 = permittivity of free space (F m^{-1})
- This equation shows:
  - Electric field strength is **not constant**
  - As the distance from the charge r increases, E decreases by a factor of $1/r^2$
- This is an **inverse square law relationship** with distance
- This means the field strength decreases by a factor of **four** when the distance is **doubled**
- **Note:** this equation is only for the field strength around a **point charge** since it produces a radial field
- The electric field strength is a **vector** Its direction is the same as the electric field lines
  - If the charge is negative, the E field strength is negative and points **towards** the centre of the charge
  - If the charge is positive, the E field strength is positive and points **away** from the centre of the charge
- This equation is analogous to the gravitational field strength around a point mass

Worked Example

A metal sphere of diameter 15 cm is negatively charged. The electric field strength at the surface of the sphere is 1.5×10^5 V m^{-1}. Determine the total surface charge of the sphere.

Step 1: Write down the known values

Electric field strength, E = 1.5×10^5 V m^{-1}

Radius of sphere, r = 15 / 2 = 7.5 cm = 7.5×10^{-2} m

Step 2: Write out the equation for electric field strength

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Step 3: Rearrange for charge Q

$$Q = 4\pi\varepsilon_0 E r^2$$

Step 4: Substitute in values

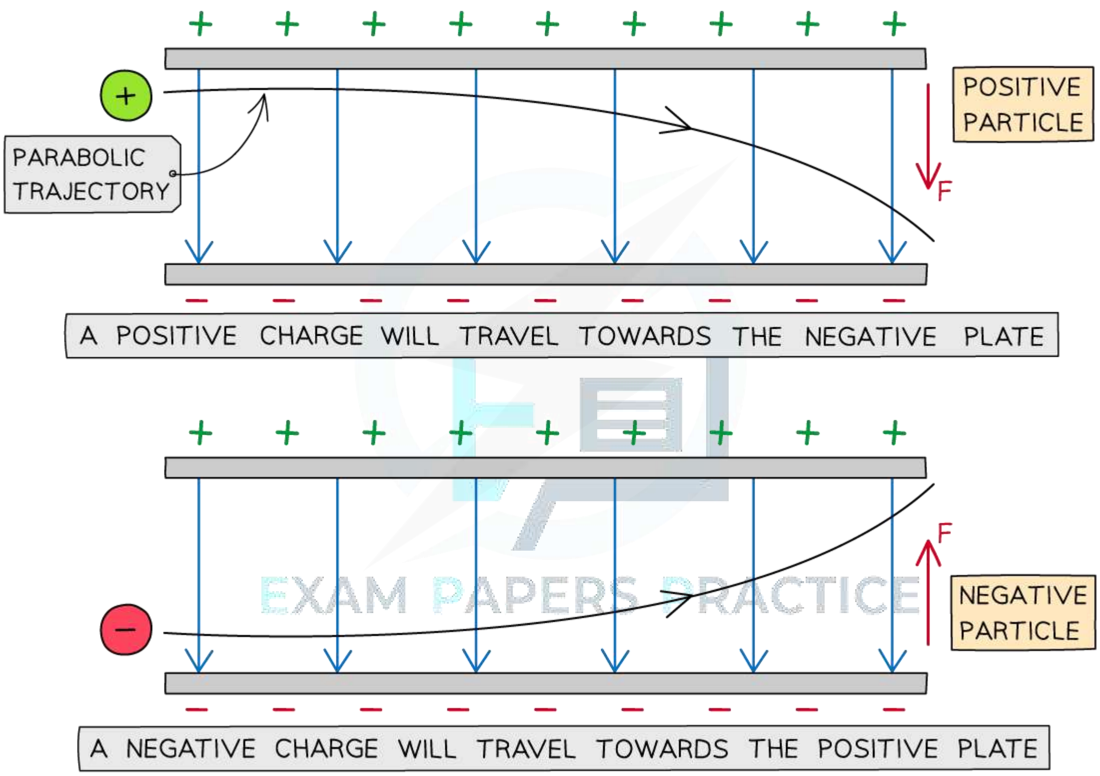
$$Q = (4\pi \times 8.85 \times 10^{-12}) \times (1.5 \times 10^5) \times (7.5 \times 10^{-2})^2 = 9.38 \times 10^{-8} \text{ C} = 94 \text{ nC (2 s.f)}$$

Exam Tip

Remember to always **square** the distance!

Motion of Charged Particles

- A charged particle in an electric field will experience a force on it that will cause it to move
- If a charged particle remains still in a uniform electric field, it will move parallel to the electric field lines (along or against the field lines depending on its charge)
- If a charged particle is in **motion** through a uniform electric field (e.g. between two charged parallel plates), it will experience a constant electric force and travel in a **parabolic trajectory**



The parabolic path of charged particles in a uniform electric field

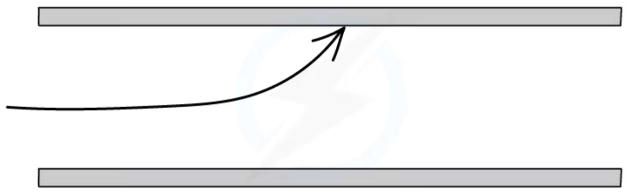
- The direction of the parabola will depend on the charge of the particle
  - A **positive** charge will be deflected towards the **negative** plate
  - A **negative** charge will be deflected towards the **positive** plate

- The force on the particle is the same at all points and is always in the same direction
- **Note:** an uncharged particle, such as a neutron experiences no force in an electric field and will therefore travel straight through the plates undeflected
- The amount of deflection depends on the following properties of the particles:
  - **Mass** – the greater the mass, the smaller the deflection and vice versa
  - **Charge** – the greater the magnitude of the charge of the particle, the greater the deflection and vice versa

- **Speed** – the greater the speed of the particle, the smaller the deflection and vice versa

Worked Example

A single proton travelling with a constant horizontal velocity enters a uniform electric field between two parallel charged plates.The diagram shows the path taken by the proton.



Draw the path taken by a boron nucleus that enters the electric field at the same point and with the same velocity as the proton.Atomic number of boron = 5

Mass number of boron = 11

Step 1:

Compare the charge of the boron nucleus to the proton

- Boron has 5 protons, meaning it has a charge 5 × greater than the proton
- The force on boron will therefore be 5 × greater than on the proton

Step 2:

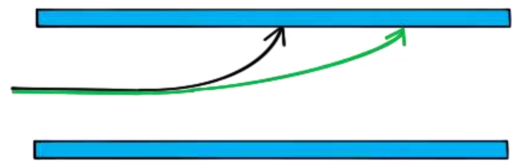
Compare the mass of the boron nucleus to the proton

- The boron nucleus has a mass of 11 nucleons meaning its mass is 11 × greater than the proton
- The boron nucleus will therefore be less deflected than the proton

Step 3:

Draw the trajectory of the boron nucleus

- Since the mass comparison is much greater than the charge comparison, the boron nucleus will be **much less deflected** than the proton
- The nucleus is positively charged since the neutrons in the nucleus have no charge
    - Therefore, the shape of the path will be the same as the proton



18.1.5 Electric Force Between Two Point Charges

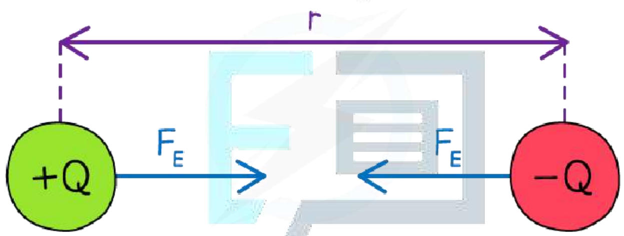
Coulomb's Law

- All charged particles produce an electric field around it
  - This field exerts a force on any other charged particle within range
- The electrostatic force between two charges is defined by **Coulomb's Law**
  - Recall that the charge of a uniform spherical conductor can be considered as a point charge at its centre
- Coulomb's Law states that:

> ***The electrostatic force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation***

- The Coulomb equation is defined as:

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$



The electrostatic force between two charges is defined by Coulomb's Law

- Where:
  - F_E = electrostatic force between two charges (N)
  - Q_1 and Q_2 = two point charges (C)
  - ϵ_0 = permittivity of free space
  - r = distance between the centre of the charges (m)
- The $1/r^2$ relation is called the inverse square law
  - This means that when a charge is twice as far as away from another, the electrostatic force between them reduces by $(½)^2 = ¼$
- If there is a positive and negative charge, then the electrostatic force is negative, this can be interpreted as an **attractive force**
- If the charges are the same, the electrostatic force is positive, this can be interpreted as a **repulsive force**
- Since uniformly charged spheres can be considered as point charges, Coulomb's law can be applied to find the electrostatic force between them as long as the separation is taken from the **centre** of both spheres

Worked Example

An alpha particle is situated 2.0 mm away from a gold nucleus in a vacuum. Assuming them to be point charges, calculate the magnitude of the electrostatic force acting on each of the charges.Atomic number of helium = 2Atomic number of gold = 79Charge of an electron = 1.60×10^{-19} C

Step 1: Write down the known quantities

- **Distance, r = 2.0 mm =2.0×10^{-3} m**

The charge of one proton = +1.60×10^{-19} C

An alpha particle (helium nucleus) has 2 protons

- **Charge of alpha particle, Q_1 = $2 \times 1.60 \times 10^{-19}$ = +3.2×10^{-19} C**

The gold nucleus has 79 protons

- **Charge of gold nucleus, Q_2 = $79 \times 1.60 \times 10^{-19}$ = +1.264×10^{-17} C**

Step 2: The electrostatic force between two point charges is given by Coulomb's Law

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Step 3: Substitute values into Coulomb's Law

$$F_E = \frac{(3.2 \times 10^{-19}) \times (1.264 \times 10^{-17})}{(4\pi \times 8.85 \times 10^{-12}) \times (2.0 \times 10^{-3})^2} = 9.092.. \times 10^{-21} = \mathbf{9.1 \times 10^{-21}\ N}\ (2\text{ s.f.})$$

18.2 Electric Potential

18.2.1 Electric Potential

Electric Potential

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
- Energy is therefore transferred to the charge that is being pushed upon
  - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
  - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lift upwards and decreasing and it falls
- The electric potential at a point is defined as:

The work done per unit positive charge in bringing a small test charge from infinity to a defined point

- Electric potential is a **scalar** quantity
  - This means it doesn't have a direction
- However, you will still see the electric potential with a positive or negative sign. This is because the electric potential is:
  - **Positive** when near an isolated positive charge
  - **Negative** when near an isolated negative charges
  - **Zero** at infinity
- Positive work is done by the mass from infinity to a point around a positive charge and negative work is done around a negative charge. This means:
  - When a test charge moves closer to a **negative** charge, its electric potential **decreases**
  - When a test charge moves closer to a **positive** charge, its electric potential **increases**
- To find the potential at a point caused by multiple charges, add up each potential separately

Electric Potential Due to a Point Charge

- The electric potential in the field due to a point charge is defined as:

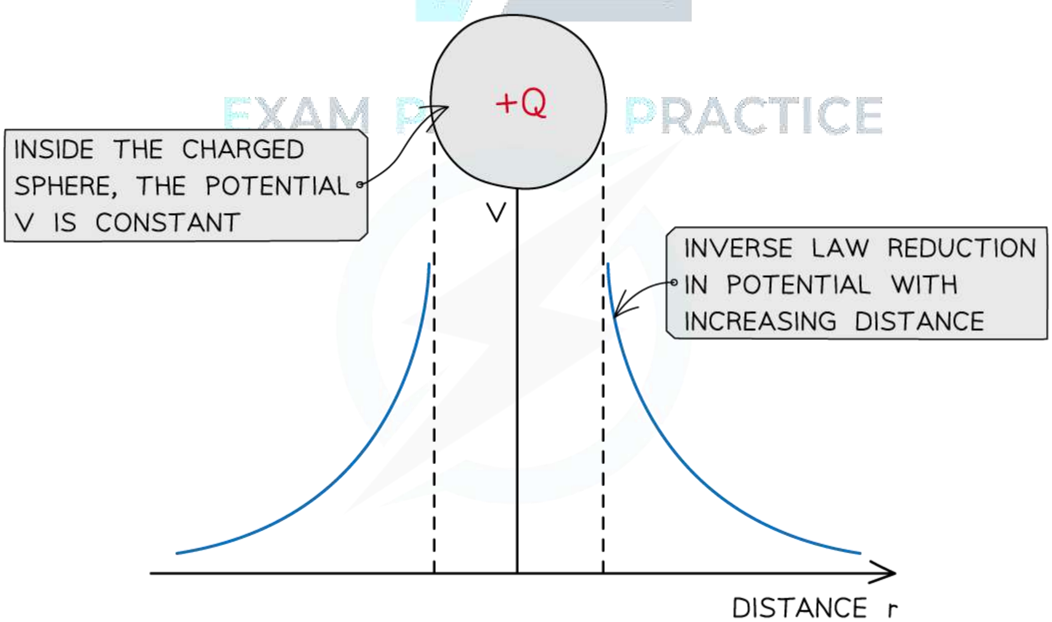
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Where:
  - V = the electric potential (V)
  - Q = the point charge producing the potential (C)
  - ϵ_0 = permittivity of free space (F m^{-1})
  - r = distance from the centre of the point charge (m)

- This equation shows that for a positive (+) charge:
  - As the distance from the charge *r* **decreases**, the potential *V* **increases**
  - This is because more work has to be done on a positive test charge to overcome the repulsive force

- For a negative (−) charge:
  - As the distance from the charge *r* **decreases**, the potential *V* **decreases**
  - This is because less work has to be done on a positive test charge since the attractive force will make it easier

- Unlike the gravitational potential equation, the minus sign in the electric potential equation will be included in the charge
- The electric potential changes according to an inverse square law with distance



The potential changes as an inverse law with distance near a charged sphere

- **Note:** this equation still applies to a conducting sphere. The charge on the sphere is treated as if it concentrated at a point in the sphere from the point charge approximation

Worked Example

A Van de Graaf generator has a spherical dome of radius 15 cm. It is charged up to a potential of 240 kV.Calculate

(a) How much charge is stored on the dome

(b) The potential a distance of 30 cm from the dome

Part (a)

Step 1: Write down the known quantities

Radius of the dome, $r = 15 \text{ cm} = 15 \times 10^{-2}$ m

Potential difference, $V = 240 \text{ kV} = 240 \times 10^{3}$ V

Step 2: Write down the equation for the electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Rearrange for charge Q

$$Q = V4\pi\varepsilon_0 r$$

Step 4: Substitute in values

$$Q = (240 \times 10^{3}) \times (4\pi \times 8.85 \times 10^{-12}) \times (15 \times 10^{-2}) = 4.0 \times 10^{-6} \text{ C} = 4.0 \text{ μC}$$

Part (b)

Step 1: Write down the known quantities

Q = charge stored in the dome = 4.0 μC = 4.0×10^{-6} C

r = radius of the dome + distance from the dome = 15 + 30 = 45 cm = 45×10^{-2} m

Step 2: Write down the equation for electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Substitute in values

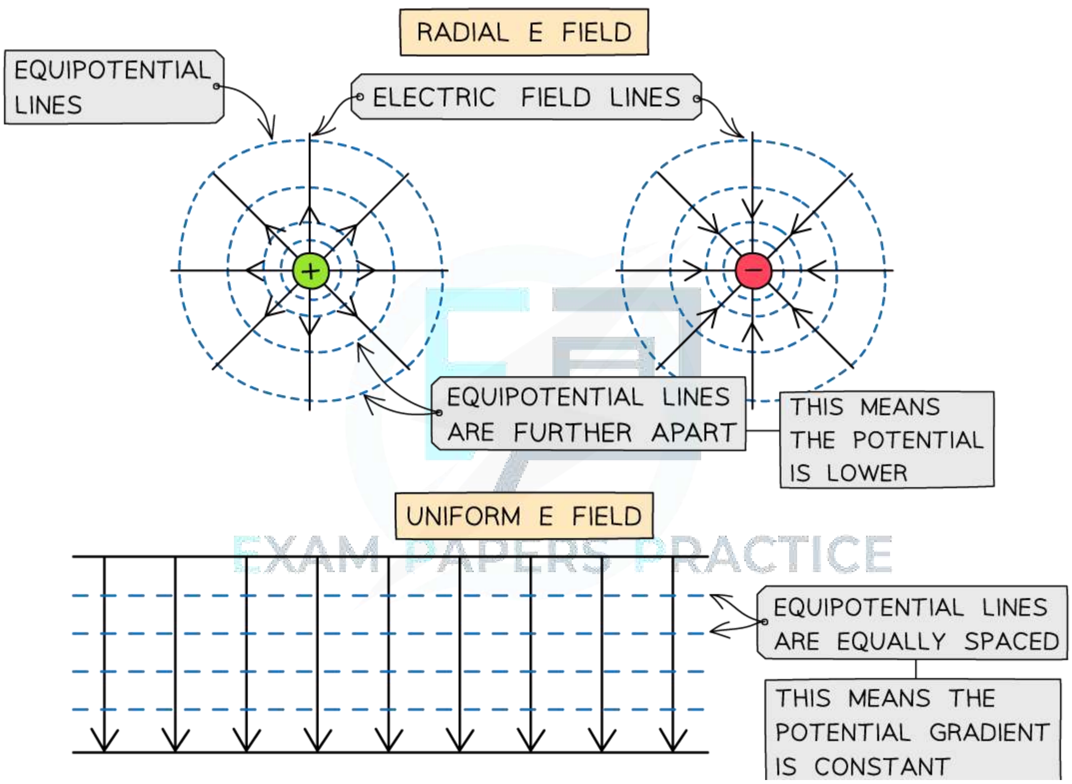
$$V = \frac{(4.0 \times 10^{-6})}{(4\pi \times 8.85 \times 10^{-12}) \times (45 \times 10^{-2})} = 79.93 \times 10^{3} = \mathbf{80\ kV} \text{ (2 s.f.)}$$

Potential Gradient

- An electric field can be defined in terms of the variation of electric potential at different points in the field:

   The electric field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the electric potential in an electric field and are always drawn **perpendicular** to the field lines



Equipotential lines around a radial field or uniform field are perpendicular to the electric field lines

- Equipotential lines are lines of **equal electric potential**
  - Around a radial field, the equipotential lines are represented by concentric circles around the charge with increasing radius
  - The equipotential lines become further away from each other
  - In a uniform electric field, the equipotential lines are equally spaced

- The potential gradient in an electric field is defined as:

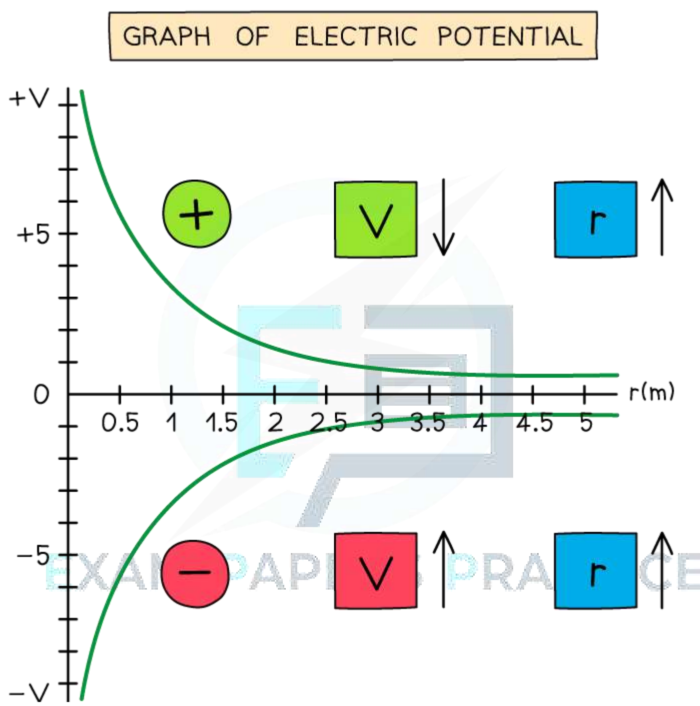
   The rate of change of electric potential with respect to displacement in the direction of the field

- The electric field strength is equivalent to this, except with a negative sign:

$$E = -\frac{\Delta V}{\Delta r}$$

- Where:
  - E = electric field strength (V m^{-1})
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)

- The minus sign is important to obtain an attractive field around a negative charge and repulsive field around a positive charge



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance

- This is because the test charge is positive

EXAM PAPERS PRACTICE

Exam Tip

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines. The potential always **decreases** in the **same** direction as the field lines and vice versa.

Electric Potential Energy of Two Point Charges

- The electric potential energy E_p at point in an electric field is defined as:

  The work done in bringing a charge from infinity to that point

- The electric potential energy of a pair of point charges Q_1 and Q_2 is defined by:

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

- Where:
  - E_p = electric potential energy (J)
  - r = separation of the charges Q_1 and Q_2 (m)
  - ϵ_0 = permittivity of free space (F m^{-1})
- The potential energy equation is defined by the work done in moving point charge Q_2 from infinity towards a point charge Q_1.
- The work done is equal to:

$$W = VQ$$

- Where:
  - W = work done (J)
  - V = electric potential due to a point charge (V)
  - Q = Charge producing the potential (C)
- This equation is relevant to calculate the work done due on a charge in a uniform field
- Unlike the electric potential, the potential energy will always be positive
- Recall that at infinity, V = 0 therefore E_p = 0
- It is more useful to find the change in potential energy eg. as one charge moves away from another
- The change in potential energy from a charge Q_1 at a distance r_1 from the centre of charge Q_2 to a distance r_2 is equal to:

$$\Delta E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- The change in electric potential ΔV is the same, without the charge Q_2

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Both equations are very similar to the change in gravitational potential between two points near a point mass

Worked Example

An α-particle ^4_2He is moving directly towards a stationary gold nucleus $^{197}_{79}Au$.

At a distance of 4.7×10^{-15} m, the α-particle momentarily comes to rest.

Calculate the electric potential energy of the particles at this instant.

Step 1: Write down the known quantities

- **Distance, r = 4.7×10^{-15} m**

The charge of one proton = $+1.60 \times 10^{-19}$ C

An alpha particle (helium nucleus) has 2 protons

- **Charge of alpha particle, $Q_1 = 2 \times 1.60 \times 10^{-19} = +3.2 \times 10^{-19}$ C**

The gold nucleus has 79 protons

- **Charge of gold nucleus, $Q_2 = 79 \times 1.60 \times 10^{-19} = +1.264 \times 10^{-17}$ C**

Step 2: Write down the equation for electric potential energy

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

Step 3: Substitute values into the equation

$$E_p = \frac{(1.264 \times 10^{-17}) \times (3.2 \times 10^{-19})}{(4\pi \times 8.85 \times 10^{-12}) \times (4.7 \times 10^{-15})} = \mathbf{7.7 \times 10^{-12}\ J}\ \text{(2 s.f)}$$

Exam Tip

When calculating electric potential energy, make sure you **do not** square the distance!