A Level Physics CIE

17. Oscillations

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17.1 Simple Harmonic Motion

17.1.1 Describing Oscillations

Describing Oscillations

- An **oscillation** is defined as:

  Repeated back and forth movements on either side of any equilibrium position

- When the object stops oscillating, it returns to its equilibrium position
  - An **oscillation** is a more specific term for a vibration
  - An **oscillator** is a device that works on the principles of oscillations
- Oscillating systems can be represented by displacement-time graphs similar to transverse waves
- The shape of the graph is a sine curve
  - The motion is described as **sinusoidal**

Properties of Oscillations

- **Displacement (x)** of an oscillating system is defined as:

  The distance of an oscillator from its equilibrium position

- **Amplitude (x_0)** is defined as:

  The maximum displacement of an oscillator from its equilibrium position

- **Angular frequency (ω)** is defined as:

  The rate of change of angular displacement with respect to time

- This is a scalar quantity measured in **rad s^{-1}** and is defined by the equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- **Frequency (f)** is defined as:

  The number of complete oscillations per unit time

- It is measured in Hertz (Hz) and is defined by the equation:

$$f = \frac{1}{T}$$

- **Time period (T)** is defined as:

  The time taken for one complete oscillation, in seconds

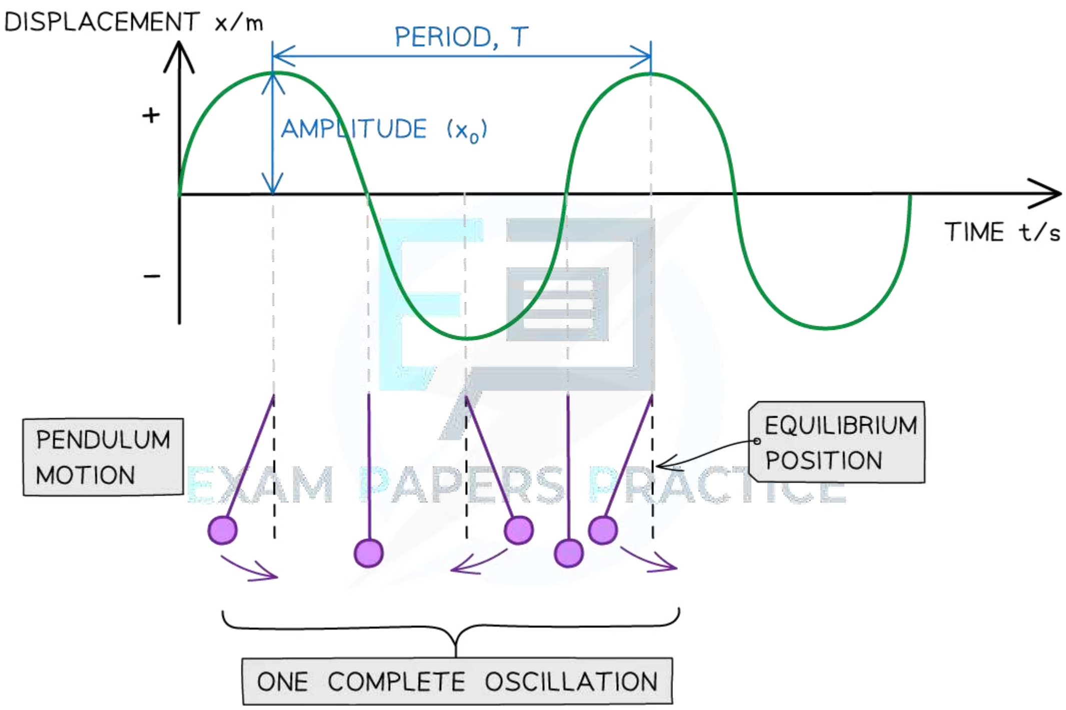
- One complete oscillation is defined as:

  The time taken for the oscillator to pass the equilibrium from one side and back again fully from the other side

- The time period is defined by the equation:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

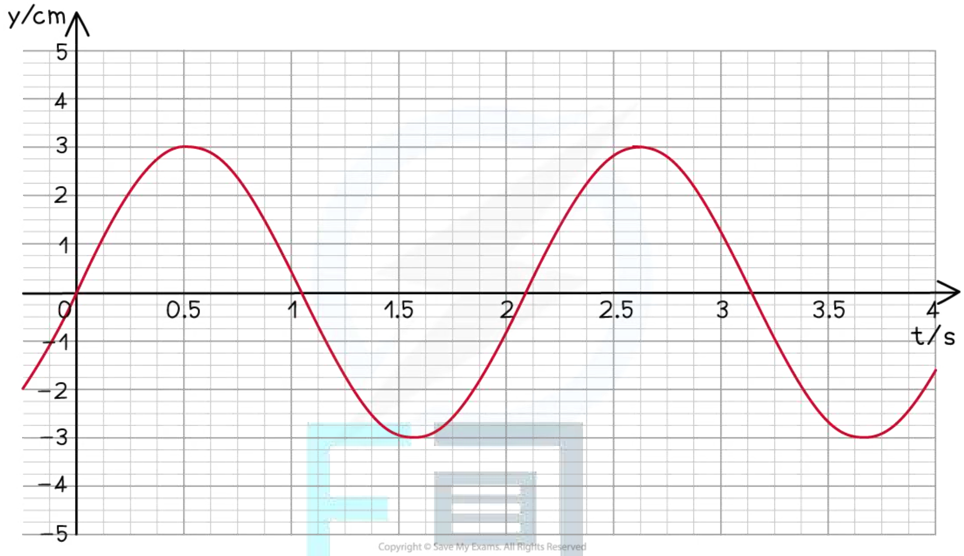
- **Phase difference** is how much one oscillator is in front or behind another
  - When the relative position of two oscillators are equal, they are **in phase**
  - When one oscillator is exactly half a cycle behind another, they are said to be in **anti-phase**
  - Phase difference is normally measured in radians or fractions of a cycle
  - When two oscillators are in antiphase they have a phase difference of **π radians**



Displacement–time graph of an oscillation of a simple pendulum

Worked Example

A student sets out to investigate the oscillation of a mass suspended from the free end of a spring. The mass is pulled downwards and then released. The variation with time *t* of the displacement *y* of the mass is shown in the figure below.



Use the information from the figure to calculate the angular frequency of the oscillations.

Step 1:

Write down the equation for angular frequency

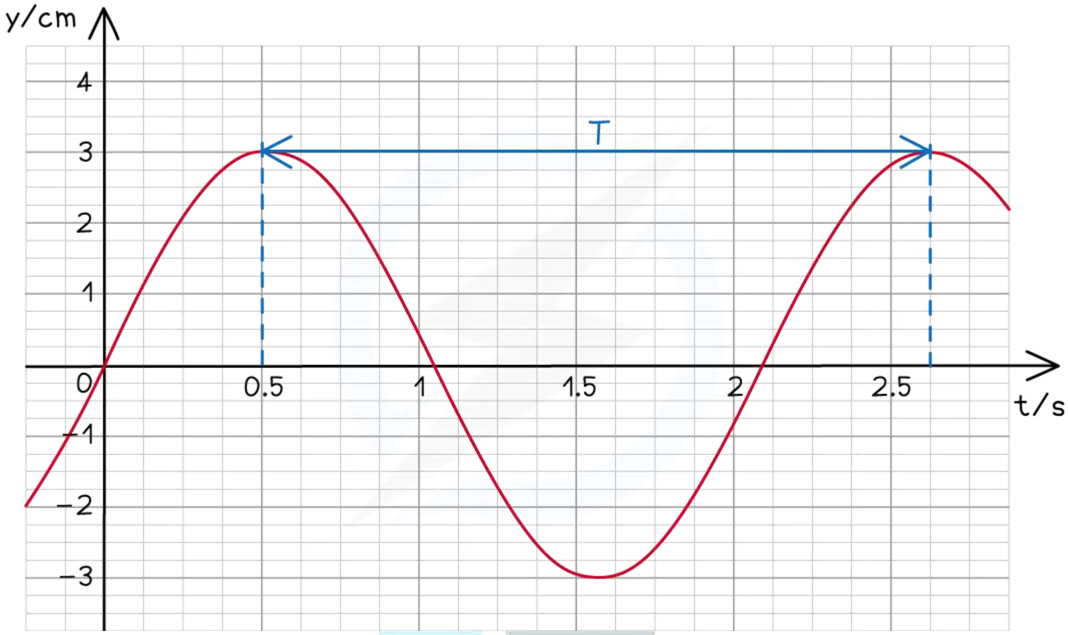
$$\omega = \frac{2\pi}{T}$$

Step 2:

Calculate the time period T from the graph

The time period is defined as the time taken for **one complete oscillation**

This can be read from the graph:



T = 2.6 − 0.5 = 2.1 s

Step 3:

Substitute into angular frequency equation

$$\omega = \frac{2\pi}{2.1} = 2.9919... = 3.0 \text{ rad s}^{-1}$$

Exam Tip

The properties used to describe oscillations are very similar to transverse waves. The key difference is that oscillators do not have a 'wavelength' and their direction of travel is only kept within the oscillations themselves rather than travelling a distance in space.

Conditions for Simple Harmonic Motion

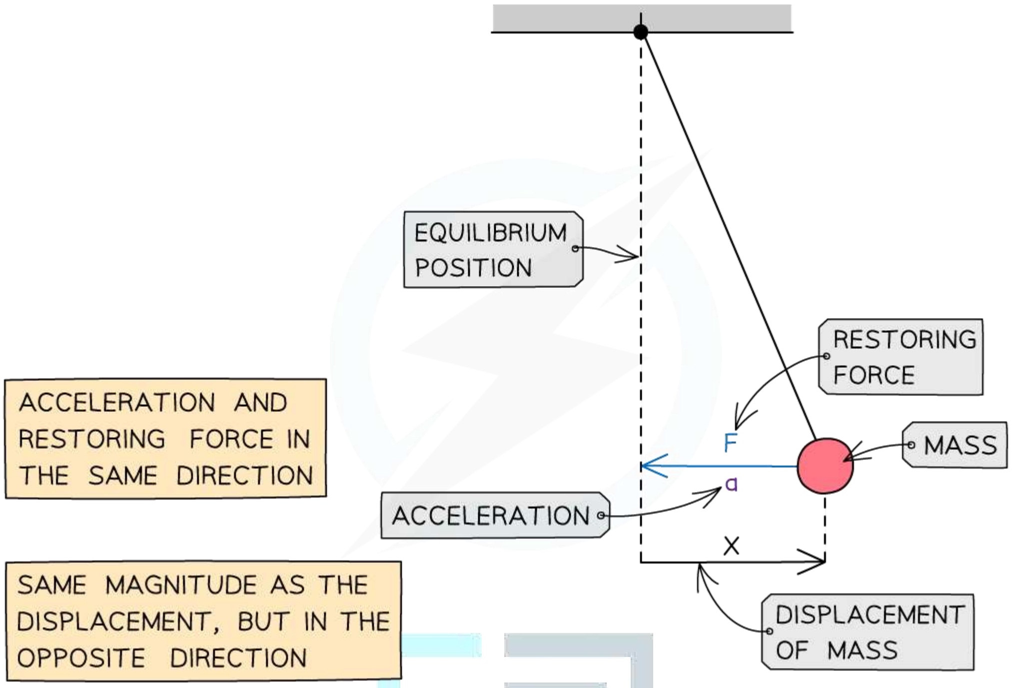
- **Simple harmonic motion (SHM)** is a specific type of oscillation
- SHM is defined as:

A type of oscillation in which the acceleration of a body is proportional to its displacement, but acts in the opposite direction

- Examples of oscillators that undergo SHM are:
  - The pendulum of a clock
  - A mass on a spring
  - Guitar strings
  - The electrons in alternating current flowing through a wire
- This means for an object to oscillate specifically in SHM, it must satisfy the following conditions:
  - Periodic oscillations
  - Acceleration proportional to its displacement
  - Acceleration in the opposite direction to its displacement
- Acceleration a and displacement x can be represented by the defining equation of SHM:

$$a \propto -x$$

- An object in SHM will also have a restoring force to return it to its equilibrium position
- This restoring force will be directly proportional, but in the opposite direction, to the displacement of the object from the equilibrium position
- **Note:** the restoring force and acceleration act in the **same direction**



Force, acceleration and displacement of a pendulum in SHM

- This is why a person jumping on a trampoline is not an example of simple harmonic motion:
  - The restoring force on the person is **not** proportional to their distance from the equilibrium position
  - When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
  - This does not change, even if they jump higher

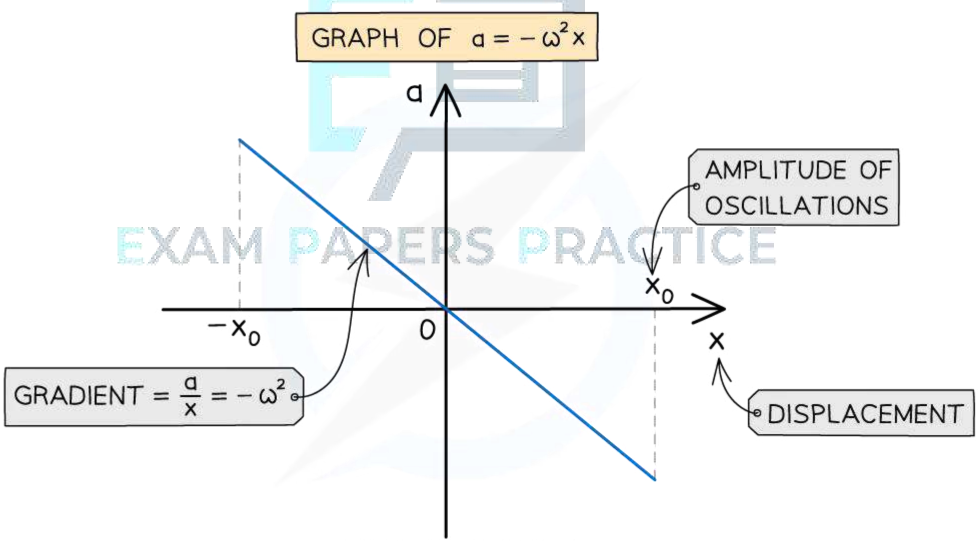
Acceleration & Displacement of an Oscillator

- The acceleration of an object oscillating in simple harmonic motion is:

$$a = -\omega^2 x$$

- Where:
  - a = acceleration (m s^{-2})
  - ω = angular frequency (rad s^{-1})
  - x = displacement (m)

- This is used to find the acceleration of an object in SHM with a particular angular frequency ω at a specific displacement x
- The equation demonstrates:
  - The acceleration reaches its maximum value when the displacement is at a maximum ie. $x = x_0$ (amplitude)
  - The minus sign shows that when the object is displacement to the **right**, the direction of the acceleration is to the **left**



The acceleration of an object in SHM is directly proportional to the negative displacement

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to y = − x)
- Key features of the graph:
  - The gradient is equal to $-\omega^2$
  - The maximum and minimum displacement *x* values are the amplitudes $-x_0$ and $+x_0$

- A solution to the SHM acceleration equation is the displacement equation:

$$x = x_0 \sin(\omega t)$$

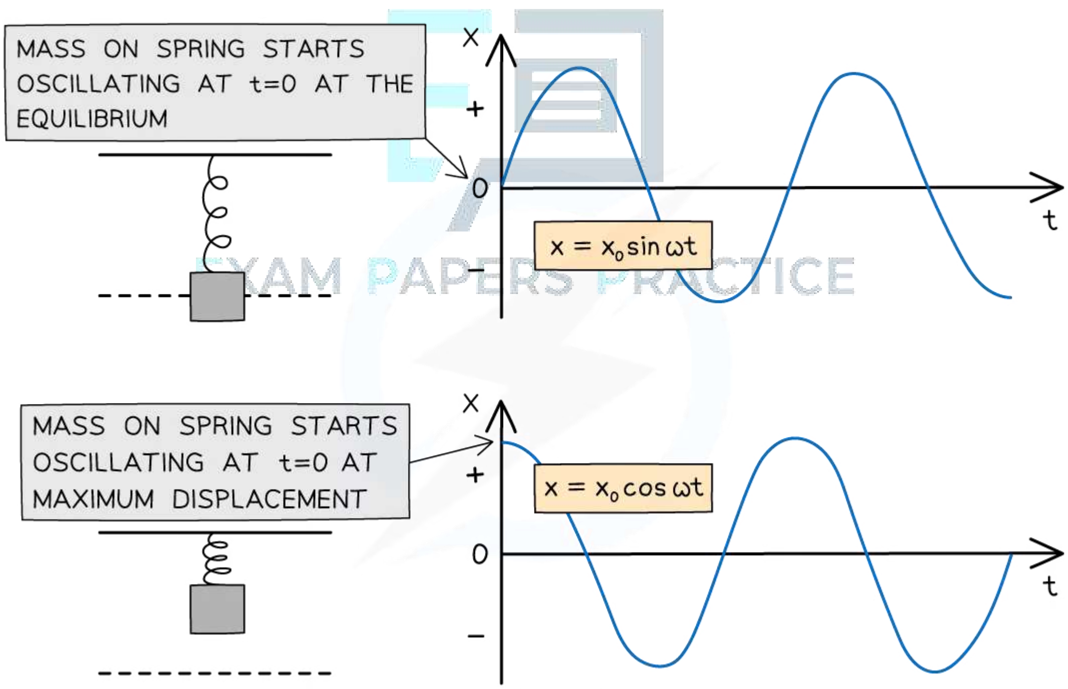
- Where:
  - x = displacement (m)
  - x_0 = amplitude (m)
  - t = time (s)

- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
  - **Note:** This version of the equation is only relevant when an object begins oscillating from the equilibrium position ($x = 0$ at $t = 0$)

- The displacement will be at its maximum when sin(ωt) equals 1 or − 1, when $x = x_0$
- If an object is oscillating from its amplitude position ($x = x_0$ or $x = -x_0$ at $t = 0$) then the displacement equation will be:

$$x = x_0\cos(\omega t)$$

- This is because the cosine graph starts at a maximum, whilst the sine graph starts at 0



These two graphs represent the same SHM. The difference is the starting position

Worked Example

A mass of 55 g is suspended from a fixed point by means of a spring. The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at t = 0.

The mass is observed to perform simple harmonic motion with a period of 0.8 s.

Calculate the displacement x in cm of the mass at time t = 0.3 s.

Step 1: Write down the SHM displacement equation

- Since the mass is released at t = 0 at its maximum displacement, the displacement equation will be with the cosine function:

$$x = x_0 \cos(\omega t)$$

Step 2: Calculate angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 \text{ rad s}^{-1}$$

- Remember to use the value of the time period given, not the time where you are calculating the displacement from

Step 3: Substitute values into the displacement equation

$$x = 4.3 \cos(7.85 \times 0.3) = -3.0369... = \mathbf{-3.0 \text{ cm}} \text{ (2 s.f)}$$

- Make sure the calculator is in **radians mode**
- The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started (3.0 cm above it)

Exam Tip

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative, you could lose a mark if not!

Also, remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees.

You often have to convert between time period *T*, frequency *f* and angular frequency ω for many exam questions – so make sure you revise the equations relating to these.

Calculating Speed of an Oscillator

- The speed of an object in simple harmonic motion varies as it oscillates back and forth
  - Its speed is the magnitude of its velocity
- The greatest speed of an oscillator is at the equilibrium position ie. when its displacement is 0 (x = 0)
- The speed of an oscillator in SHM is defined by:

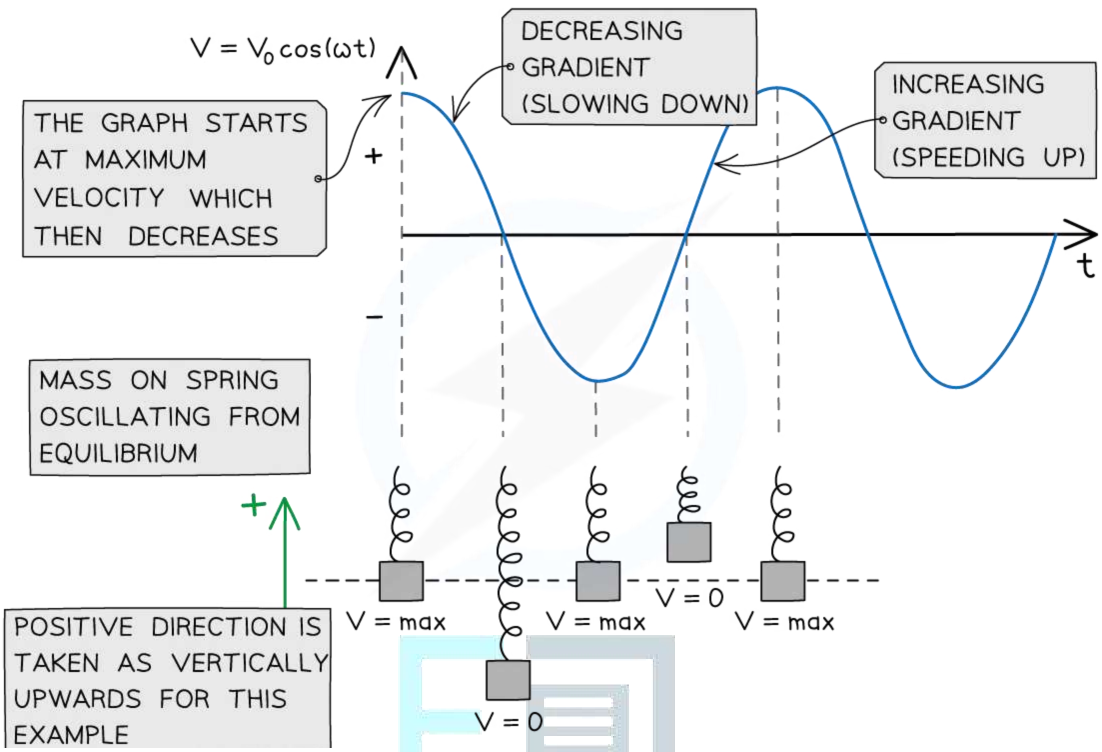
$$v = v_0 \cos(\omega t)$$

- Where:
  - v = speed (m s^{-1})
  - v_0 = maximum speed (m s^{-1})
  - ω = angular frequency (rad s^{-1})
  - t = time (s)
- This is a cosine function if the object starts oscillating from the equilibrium position (x = 0 when t = 0)
- Although the symbol *v* is commonly used to represent velocity, not speed, exam questions focus more on the magnitude of the velocity than its direction in SHM
- How the speed *v* changes with the oscillator's displacement *x* is defined by:

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

- Where:
  - x = displacement (m)
  - x_0 = amplitude (m)
  - ± = 'plus or minus'. The value can be negative or positive
- This equation shows that when an oscillator has a greater amplitude x_0, it has to travel a greater distance in the same time and hence has greater speed *v*
- Both equations for speed will be given on your formulae sheet in the exam
- When the speed is at its maximums (at x = 0), the equation becomes:

$$v_0 = \omega x_0$$



The variation of the speed of a mass on a spring in SHM over one complete cycle

Worked Example

A simple pendulum oscillates with simple harmonic motion with an amplitude of 15 cm. The frequency of the oscillations is 6.7 Hz.Calculate the speed of the pendulum at a position of 12 cm from the equilibrium position.

Step 1: Write out the known quantities

Amplitude of oscillations, **x_0 = 15 cm = 0.15 m**

Displacement at which the speed is to be found, **x = 12 cm = 0.12 m**

Frequency, **f = 6.7 Hz**

Step 2: Oscillator speed with displacement equation

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

Since the speed is being calculated, the ± sign can be removed as direction does not matter in this case

Step 3: Write an expression for the angular frequency

Equation relating angular frequency and normal frequency:

$\omega = 2\pi f = 2\pi \times 6.7 = 42.097...$

Step 4: Substitute in values and calculate

$$v = (2\pi \times 6.7) \times \sqrt{(0.15)^2 - (0.12)^2}$$

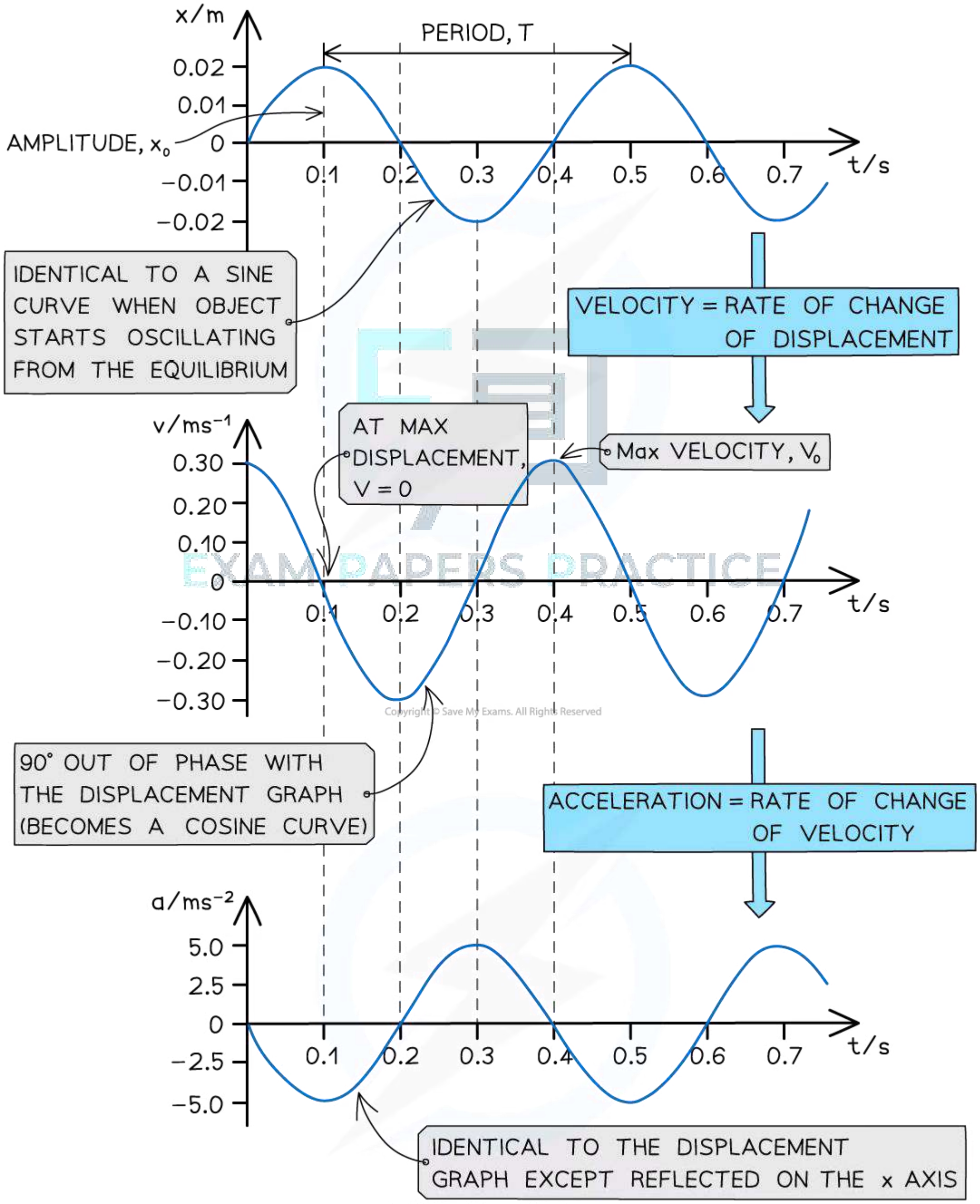
$v = 3.789 = 3.8 \text{ m s}^{-1}$ (2 s.f)

Exam Tip

You often have to convert between time period *T*, frequency *f* and angular frequency ω for many exam questions – so make sure you revise the equations relating to these.

SHM Graphs

- The displacement, velocity and acceleration of an object in simple harmonic motion can be represented by graphs against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves



The displacement, velocity and acceleration graphs in SHM are all 90° out of phase with each other

- **Key features of the displacement–time graph**:
  - The amplitude of oscillations x_0 can be found from the maximum value of x
  - The time period of oscillations T can be found from reading the time taken for one full cycle
  - The graph might not always start at 0
  - If the oscillations starts at the positive or negative amplitude, the displacement will be at its maximum
- **Key features of the velocity–time graph**:
  - It is 90° out of phase with the displacement–time graph
  - Velocity is equal to the rate of change of displacement
  - So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement–time graph**:

$$v = \frac{\Delta x}{\Delta t}$$

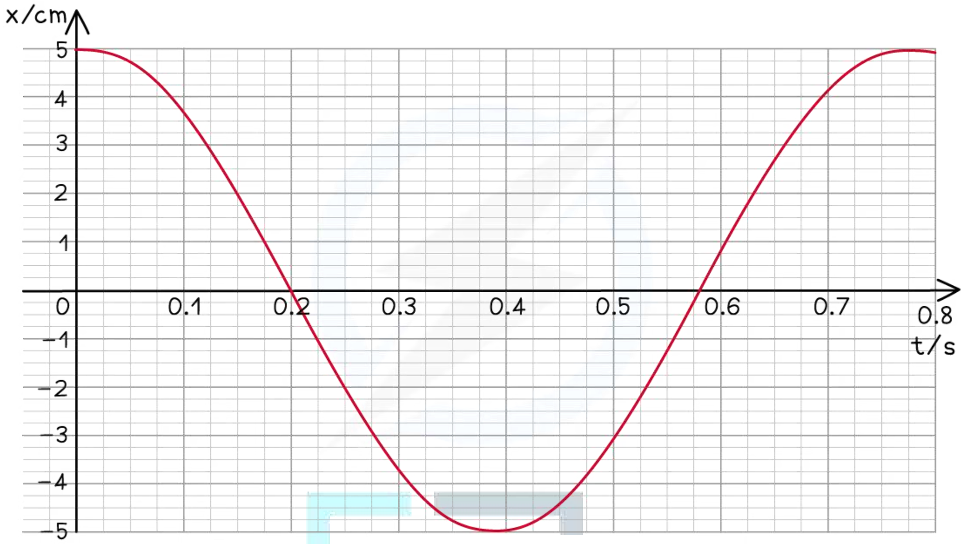
  - An oscillator moves the fastest at its equilibrium position
  - Therefore, the velocity is at its **maximum** when the **displacement is zero**
- **Key features of the acceleration–time graph**:
  - The acceleration graph is a reflection of the displacement graph on the x axis
  - This means when a mass has positive displacement (to the right) the acceleration is in the opposite direction (to the left) and vice versa
  - It is 90° out of phase with the velocity–time graph
  - Acceleration is equal to the rate of change of velocity
  - So, the acceleration of an oscillator at any time can be determined from the **gradient of the velocity–time graph**:

$$a = \frac{\Delta v}{\Delta t}$$

  - The maximum value of the acceleration is when the oscillator is at its **maximum displacement**

Worked Example

A swing is pulled 5 cm and then released.The variation of the horizontal displacement *x* of the swing with time t is shown on the graph below.



The swing exhibits simple harmonic motion.Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

Step 1: The velocity is at its maximum when the displacement x = 0

Step 2: Reading value of time when x = 0

From the graph this is equal to 0.2 s

Exam Tip

These graphs might not look identical to what is in your textbook, depending on where the object starts oscillating from at t = 0 (on either side of the equilibrium, or at the equilibrium). However, if there is no damping, they will all always be a general sine or cosine curves.

Kinetic & Potential Energies

- During simple harmonic motion, energy is constantly exchanged between two forms: kinetic and potential
- The potential energy could be in the form of:
  - Gravitational potential energy (for a pendulum)
  - Elastic potential energy (for a horizontal mass on a spring)
  - Or both (for a vertical mass on a spring)

- Speed, *v,* is at a maximum when displacement, *x*, = 0, so:

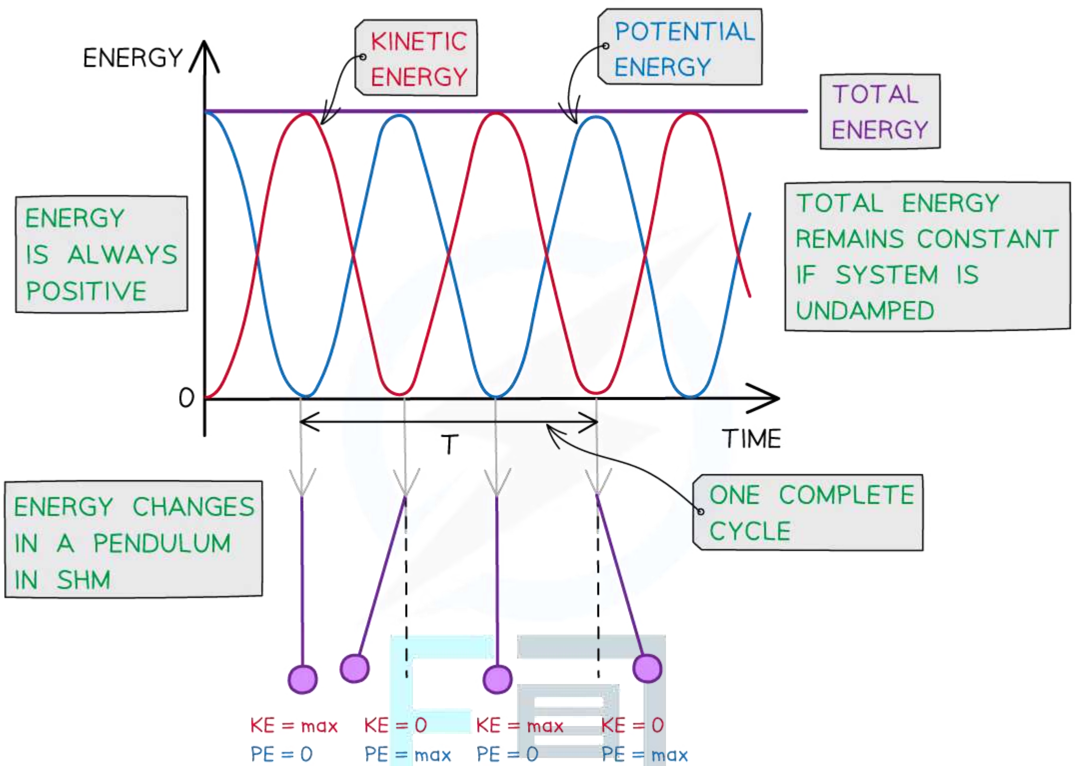
  The system has maximum kinetic energy when the displacement is zero because the oscillator is at its equilibrium position and so moving at maximum velocity.

- Therefore, the kinetic energy is zero at maximum displacement $x = x_0$, so:

  The potential energy is at a maximum when the displacement (both positive and negative) is at a maximum, $x = \pm x_0$ ***(amplitude)***

- A simple harmonic system is therefore constantly converting between kinetic and potential energy
  - When one increases, the other decreases and vice versa, therefore:

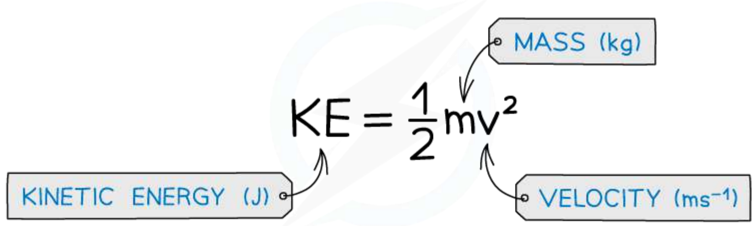
  The total energy of a simple harmonic system always remains constant and is equal to the sum of the kinetic and potential energies



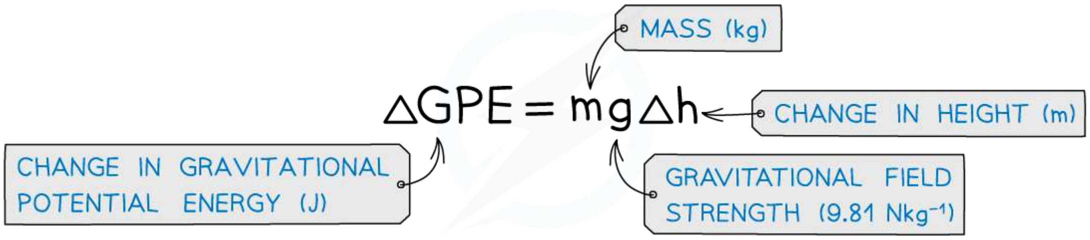
The kinetic and potential energy of an oscillator in SHM vary periodically

- **The key features of the energy-time graph are**:
  - Both the kinetic and potential energies are represented by periodic functions (sine or cosine) which are varying in opposite directions to one another
  - When the potential energy is 0, the kinetic energy is at its maximum point and vice versa
  - The **total energy** is represented by a **horizontal straight line** directly above the curves at the maximum value of both the kinetic or potential energy
  - Energy is **always positive** so there are no negative values on the y axis

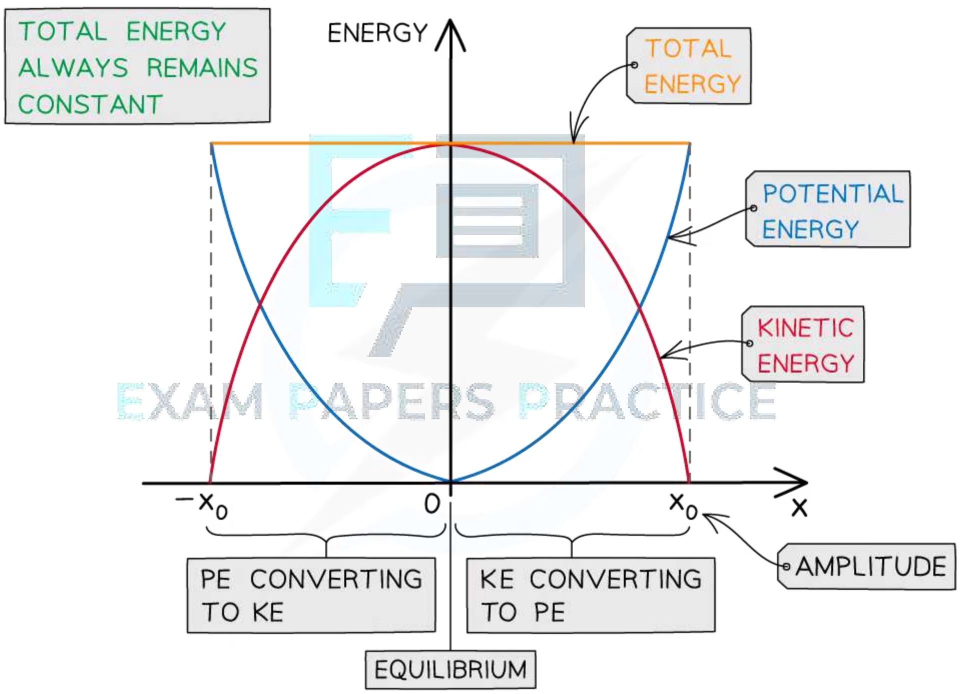
- Recall that the kinetic energy is defined by the equation



- Gravitational potential energy is defined by the equation



- **Note:** kinetic and potential energy go through **two** complete cycles during one **period** of oscillation
  - This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)
- The energy-displacement graph for **half** a cycle looks like:



Potential and kinetic energy v displacement in half a period of an SHM oscillation

- **The key features of the energy-displacement graph**:
  - Displacement is a vector, so, the graph has both **positive** and **negative** x values
  - The potential energy is always at a maximum at the amplitude positions x_0 and 0 at the equilibrium position (x = 0)
  - This is represented by a **'U' shaped curve**
  - The kinetic energy is the opposite: it is 0 at the amplitude positions x_0 and maximum at the equilibrium position x = 0
  - This is represented by a **'n' shaped curve**
  - The total energy is represented by a **horizontal straight line** above the curves

Exam Tip

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

Calculating Total Energy of a Simple Harmonic System

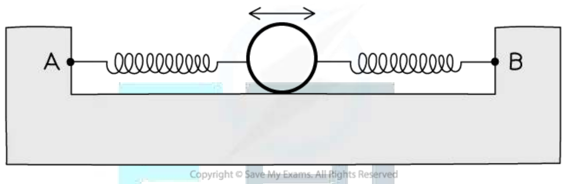
The total energy of system undergoing simple harmonic motion is defined by:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

- Where:
  - E = total energy of a simple harmonic system (J)
  - m = mass of the oscillator (kg)
  - ω = angular frequency (rad s^{-1})
  - x_0 = amplitude (m)

Worked Example

A ball of mass 23 g is held between two fixed points A and B by two stretch helical springs, as shown in the figure below



The ball oscillates along the line AB with simple harmonic motion of frequency 4.8 Hz and amplitude 1.5 cm.Calculate the total energy of the oscillations.

Step 1: Write down all known quantities

Mass, m = 23 g = **23×10^{-3} kg**

Amplitude, x_0 = 1.5 cm = **0.015 m**

Frequency, f = **4.8 Hz**

Step 2: Write down the equation for the total energy of SHM oscillations:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

Step 3: Write an expression for the angular frequency

$$\omega = 2\pi f = 2\pi \times 4.8$$

Step 4: Substitute values into energy equation

$$E = \frac{1}{2} \times (23 \times 10^{-3}) \times (2\pi \times 4.8)^2 \times (0.015)^2$$

$$E = 2.354 \times 10^{-3} = 2.4 \text{ mJ (2 s.f.)}$$

17.2 Damped Oscillations

17.2.1 Damping

Damping

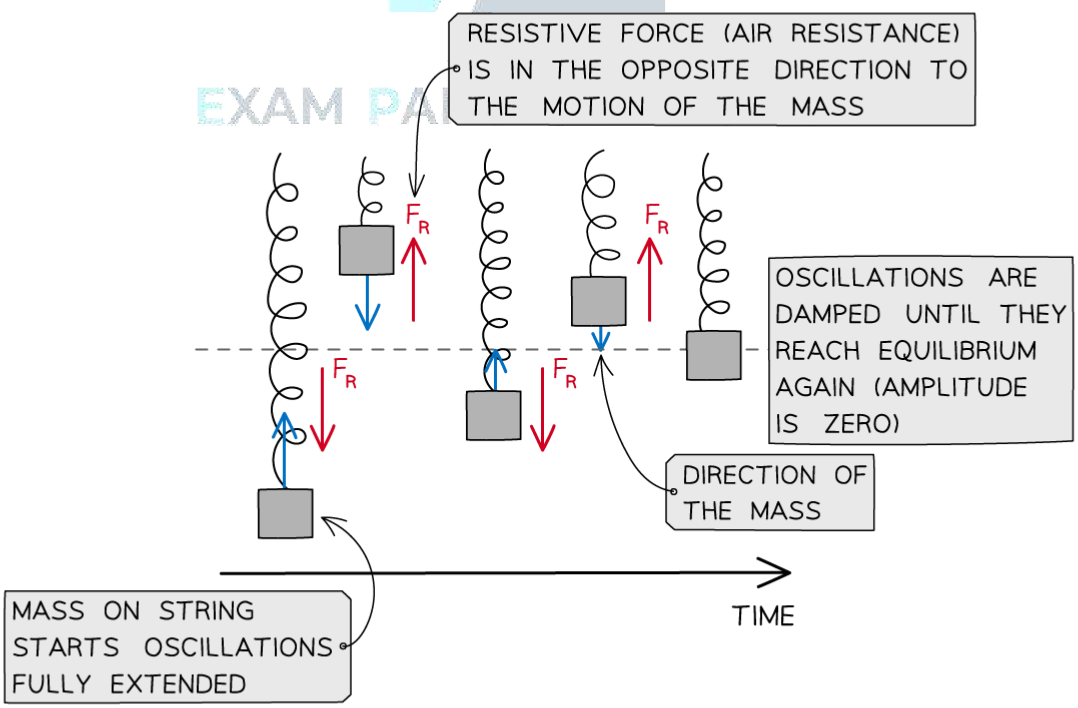
- In practice, all oscillators eventually stop oscillating
  - Their amplitudes decrease rapidly, or gradually

- This happens due to **resistive forces**, such friction or air resistance, which act in the opposite direction to the motion of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
  - These are known as **damped** oscillations

- Damping is defined as:

  The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system

- Damping continues until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
  - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude



Damping on a mass on a spring is caused by a resistive force acting in the opposite direction to the motion. This continues until the amplitude of the oscillations reaches zero

Exam Tip

Make sure not to confuse **resistive** force and **restoring** force:

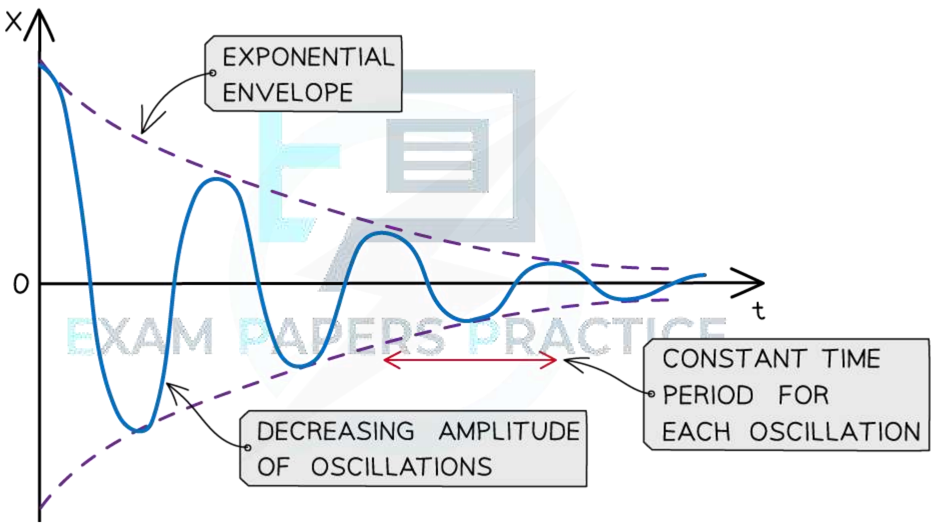
- Resistive force is what **opposes the motion** of the oscillator and causes damping
- Restoring force is what brings the oscillator **back to the equilibrium position**

Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decrease:
  - Light damping
  - Critical damping
  - Heavy damping

Light Damping

- When oscillations are lightly damped, the amplitude does not decrease linearly
  - It decays exponentially with time
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with gradually decreasing amplitude
  - For example, a swinging pendulum decreasing in amplitude until it comes to a stop

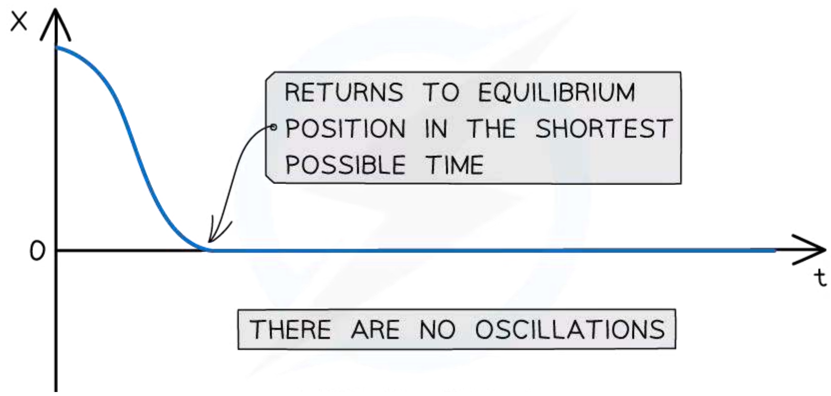


A graph for a lightly damped system consists of oscillations decreasing exponentially

- **Key features of a displacement-time graph for a lightly damped system**:
  - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
  - This is shown by the height of the curve decreasing in both the positive and negative displacement values
  - The amplitude decreases exponentially
  - The frequency of the oscillations remain constant, this means the time period of oscillations must stay the same and each peak and trough is equally spaced

Critical Damping

- When a critically damped oscillator is displaced from the equilibrium, it will return to rest at its equilibrium position in the shortest possible time **without** oscillating
  - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road

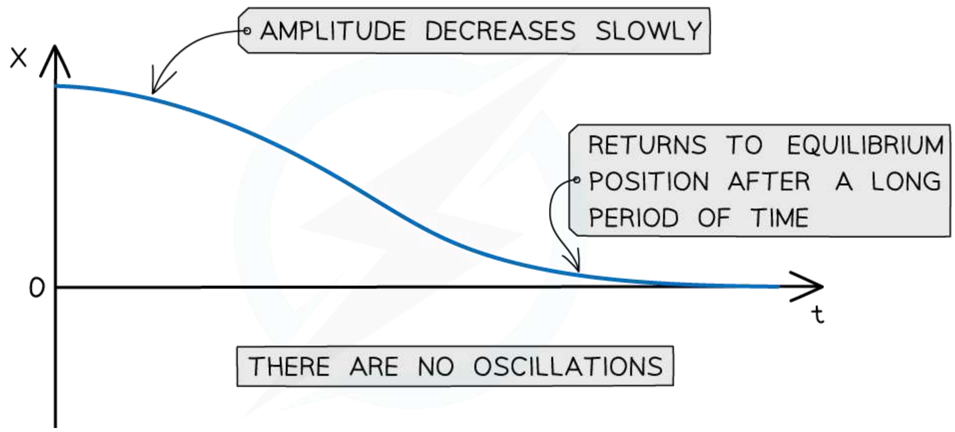


The graph for a critically damped system shows no oscillations and the displacement returns to zero in the quickest possible time

- **Key features of a displacement-time graph for a critically damped system**:
  - This system does **not** oscillate, meaning the displacement falls to 0 straight away
  - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
  - When the oscillator reaches the equilibrium position (x = 0), the graph is a horizontal line at x = 0 for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a long time to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
  - For example, door dampers to prevent them slamming shut



A heavy damping curve has no oscillations and the displacement returns to zero after a long period of time

- **Key features of a displacement-time graph for a heavily damped system**:
  - There are no oscillations. This means the displacement does not pass 0

- The graph has a slow decreasing gradient from when the oscillator is first displaced until it reaches the x axis
- The oscillator reaches the equilibrium position (x = 0) after a long period of time, after which the graph remains a horizontal line for the remaining time

Worked Example

A mechanical weighing scale consists of a needle which moves to a position on a numerical scale depending on the weight applied. Sometimes, the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale to which it is pointing to.Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

ANSWER:

- Ideally, the needle should not oscillate before settling
  - This means the scale should have either **critical** or **heavy damping**
- Since the scale is read straight away after a weight is applied, ideally the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale
- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

Resonance

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
  - This periodic force **does work** on the resistive force decreasing the oscillations
- These are known as **forced oscillations**, and are defined as:

    Periodic forces which are applied in order to sustain oscillations

- For example, when a child is on a swing, they will be pushed at one end after each cycle in order to keep swinging and prevent air resistance from damping the oscillations
  - These extra pushes are the forced oscillations, without them, the child will eventually come to a stop
- The frequency of forced oscillations is referred to as the **driving frequency (f)**
- All oscillating systems have a **natural frequency (f_0)**, this is defined as:

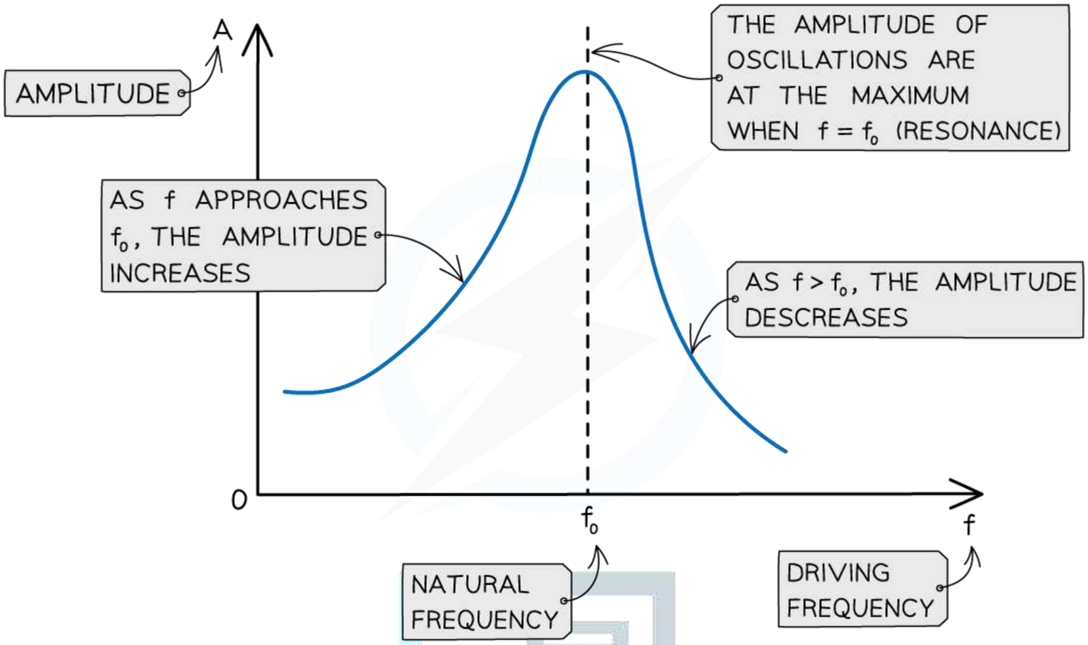
    The frequency of an oscillation when the oscillating system is allowed to oscillate freely

- Oscillating systems can exhibit a property known as **resonance**
- When resonance is achieved, a maximum amplitude of oscillations can be observed
- Resonance is defined as:

    When the driving frequency applied to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly

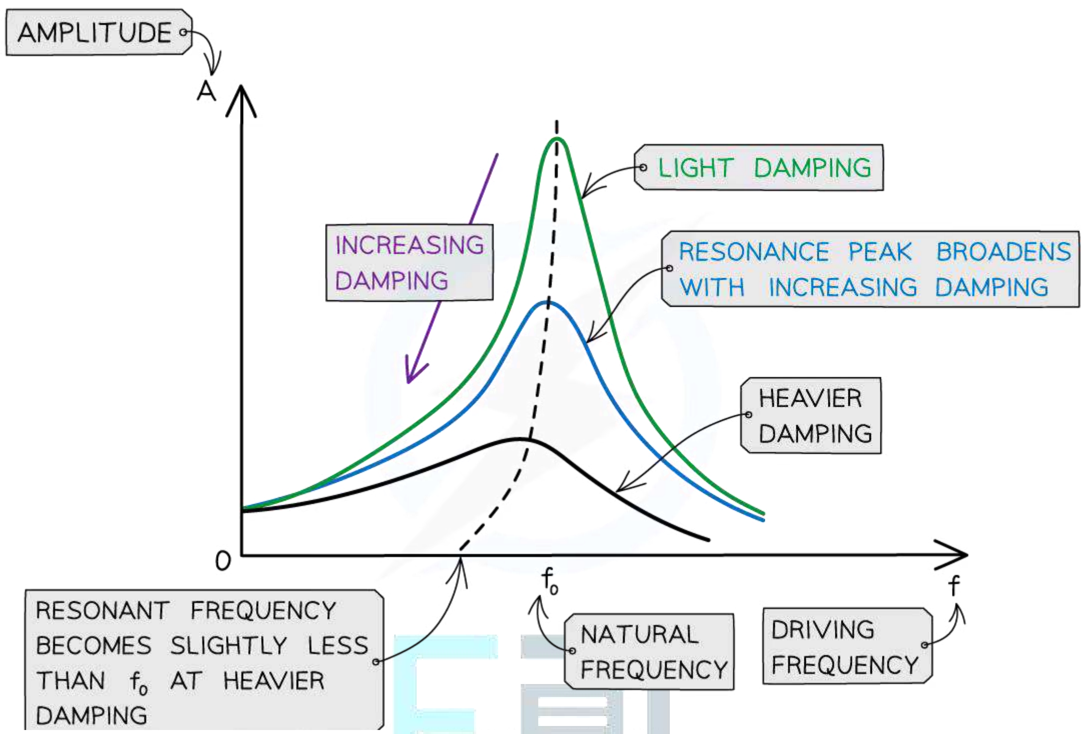
- For example, when a child is pushed on a swing:
  - The swing plus the child has a fixed natural frequency
  - A small push after each cycle increases the amplitude of the oscillations to swing the child higher
  - If the driving frequency does not quite match the natural frequency, the amplitude will increase but not to the same extent at when resonance is achieved
- This is because at resonance, energy is transferred from the driver to the oscillating system **most efficiently**
  - Therefore, at resonance, the system will be transferring the maximum kinetic energy possible
- A graph of driving frequency *f* against amplitude *a* of oscillations is called a **resonance curve**. It has the following key features:
  - When $f < f_0$, the amplitude of oscillations increases
  - At the peak where $f = f_0$, the amplitude is at its maximum. This is **resonance**

- When $f > f_0$, the amplitude of oscillations starts to decrease



The maximum amplitude of the oscillations occurs when the driving frequency is equal to the natural frequency of the oscillator

- Damping reduces the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
    - **Note:** the natural frequency f_0 will remain the same
- As the degree of damping is increased, the resonance graph is altered in the following ways:
    - The amplitude of resonance vibrations decrease, meaning the peak of the curve lowers
    - The resonance peak broadens
    - The resonance peak moves slightly to the left of the natural frequency when heavily damped



As damping is increased, resonance peak lowers, the curve broadens and moves slightly to the left