A Level Physics CIE

15. Ideal Gases

CONTENTS

Amount of Substance

- In thermodynamics, the amount of substance is measured in the SI unit 'mole'
  - This has the symbol **mol**
  - The mole is a unit of **substance**, not a unit of mass
- The mole is defined as:

  The SI base unit of an 'amount of substance'. It is the amount containing as many particles (e.g. atoms or molecules) as there are atoms in 12 g of carbon–12

- The mole is an important unit in thermodynamics
- If we consider the number of moles of two different gases under the same conditions, their physical properties are the same

The Avogadro Constant

- In AS Physics, the atomic mass unit (u) was introduced as approximately the mass of a proton or neutron = 1.66×10^{-27} kg
- This means that an atom or molecule has a mass approximately equal to the number of protons and neutrons it contains
- A carbon-12 atom has a mass of:

$$\mathbf{12\ u = 12 \times 1.66 \times 10^{-27} = 1.99 \times 10^{-26}\ kg}$$

- The exact number for a mole is defined as the number of molecules in exactly 12 g of carbon:

$$1 \text{ mole} = \frac{0.012}{1.99 \times 10^{-26}} = \mathbf{6.02 \times 10^{23}\ molecules}$$

- Avogadro's constant *(N_A)* is defined as:

  The number of atoms of carbon-12 in 12 g of carbon-12; equal to 6.02 × 10^{23} mol^{-1}

- For example, 1 mole of sodium (Na) contains 6.02×10^{23} **atoms** of sodium
- The number of atoms can be determined if the number of moles is known by multiplying by N_A, for example:

2.0 mol of nitrogen contains: $2.0 \times N_A = 2.0 \times 6.02 \times 10^{23} = 1.20 \times 10^{24}$ atoms

Mole and the Atomic Mass

- One mole of any element is equal to the **relative atomic mass** of that element in grams
  - E.g. Helium has an atomic mass of 4 – this means 1 mole of helium has a mass of 4 g
- If the substance is a compound, add up the relative atomic masses, for example, water (H_2O) is made up of
  - 2 hydrogen atoms (each with atomic mass of 1) and 1 oxygen atom (atomic mass of 16)
  - So, 1 mole of water would have a mass of $(2 \times 1) + 16 = 18$ g

Molar Mass

- The molar mass of a substance is the mass, in grams, in one mole
  - Its unit is **g mol^{-1}**
- The number of moles from this can be calculated using the equation:

$$\text{Number of moles} = \frac{mass\ (g)}{molar\ mass\ (g\ mol^{-1})}$$

? Worked Example

How many molecules are there in 6 g of magnesium-24?

Step 1: Calculate the mass of 1 mole of magnesium

One mole of any element is equal to the relative atomic mass of that element in grams

1 mole = 24 g of magnesium

Step 2: Calculate the amount of moles in 6 g

$$\frac{6}{24} = 0.25 \text{ moles}$$

Step 3: Convert the moles to number of molecules

1 mole = 6.02×10^{23} molecules

0.25 moles = $0.25 \times 6.02 \times 10^{23} = 1.51 \times 10^{23}$ molecules

Exam Tip

If you want to find out more about the mole, check out the CIE A Level Chemistry revision notes!

15.1.2 Ideal Gases

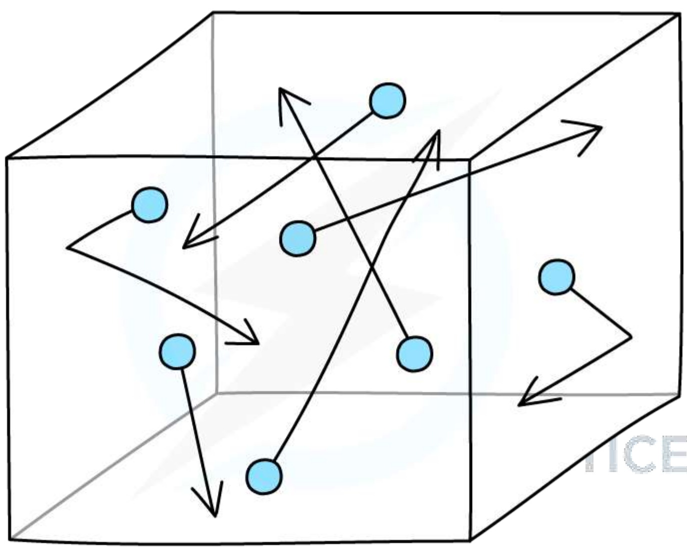
Ideal Gases

- An **ideal gas** is one which obeys the relation:

$$pV \propto T$$

- Where:
  - p = pressure of the gas (Pa)
  - V = volume of the gas (m^3)
  - T = thermodynamic temperature (K)

- The molecules in a gas move around randomly at high speeds, colliding with surfaces and exerting pressure upon them



Gas molecules move about randomly at high speeds

- Imagine molecules of gas free to move around in a box
- The temperature of a gas is related to the average speed of the molecules:
  - The hotter the gas, the faster the molecules move
  - Hence the molecules collide with the surface of the walls more frequently

- Since force is the rate of change of momentum:
  - Each collision applies a **force** across the surface area of the walls
  - The faster the molecules hit the walls, the greater the force on them

- Since pressure is the **force per unit area**
  - **Higher temperature leads to higher pressure**

- If the volume *V* of the box decreases, and the temperature *T* stays constant:
  - There will be a smaller surface area of the walls and hence more collisions
  - This also creates more pressure

- Since this equates to a greater force per unit area, pressure in an ideal gas is therefore defined by:

The frequency of collisions of the gas molecules per unit area of a container

Boyle's Law

- If the temperature T is constant, then **Boyle's Law** is given by:

$$p \propto \frac{1}{V}$$

- This leads to the relationship between the pressure and volume for a fixed mass of gas at constant temperature:

$$P_1V_1 = P_2V_2$$

Charles's Law

- If the pressure P is constant, then **Charles's law** is given by:

$$V \propto T$$

- This leads to the relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

- If the volume V is constant, the the **Pressure law** is given by:

$$P \propto T$$

- This leads to the relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Worked Example

An ideal gas is in a container of volume 4.5×10^{-3} m^3. The gas is at a temperature of 30°C and a pressure of 6.2×10^5 Pa.

Calculate the pressure of the ideal gas in the same container when it is heated to 40 °C.

Step 1: State the known values

- Volume, $V = 4.5 \times 10^{-3}$ m^3
- Initial pressure, $P_1 = 6.2 \times 10^5$ Pa
- Initial temperature, $T_1 = 30°C = 303$ K
- Initial temperature, $T_2 = 40°C = 313$ K

Step 2: Since volume is constant, state the pressure law

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Step 3: Rearrange to make P_2 the subject

$$P_2 = \frac{P_1 \times T_2}{T_1}$$

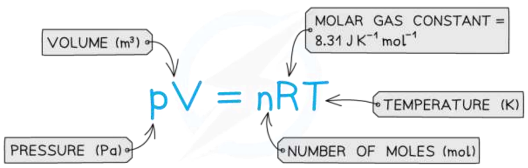
Step 4: Substitute in known values and calculate P_2

$$P_2 = \frac{6.2 \times 10^5 \times 313}{303} = \mathbf{6.4 \times 10^5 \ Pa}$$

Ideal Gas Equation

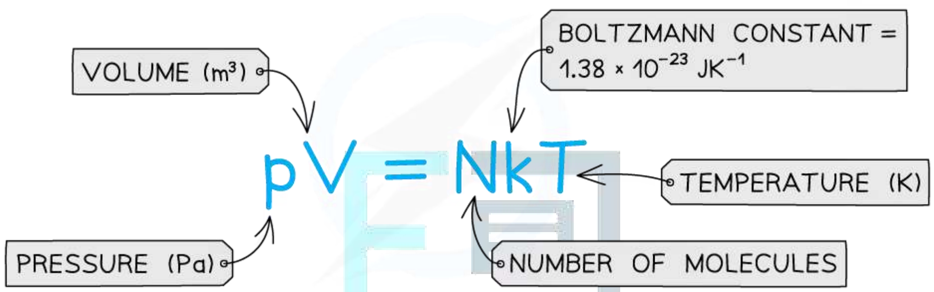
- The equation of state for an ideal gas (or the ideal gas equation) can be expressed as:

$$pV = nRT$$



- The ideal gas equation can also be written in the form:

$$pV = NkT$$



- An ideal gas is therefore defined as:

A gas which obeys the equation of state pV = nRT at all pressures, volumes and temperatures

Worked Example

A storage cylinder of an ideal gas has a volume of 8.3×10^3 cm^3. The gas is at a temperature of 15°C and a pressure of 4.5×10^7 Pa. Calculate the amount of gas in the cylinder, in moles.

Step 1: Write down the ideal gas equation

Since the number of moles (n) is required, use the equation:

$$pV = nRT$$

Step 2: Rearrange for the number of moles n

$$n = \frac{pV}{RT}$$

Step 3: Substitute in values

$$V = 8.3 \times 10^3\ cm^3 = 8.3 \times 10^3 \times 10^{-6} = 8.3 \times 10^{-3}\ m^3$$

$$T = 15\ ^oC + 273.15 = 288.15\ K$$

$$n = \frac{4.5 \times 10^7 \times 8.3 \times 10^{-3}}{8.31 \times 288.15} = 155.98 = \mathbf{160\ mol}\ (2\ s.f.)$$

Exam Tip

Don't worry about remembering the values of R and k, they will both be given in the equation sheet in your exam.

The Boltzmann Constant

- The Boltzmann constant *k* is used in the ideal gas equation and is defined by the equation:

$$k = \frac{R}{N_A}$$

- Where:
  - R = molar gas constant
  - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = \mathbf{1.38 \times 10^{-23}\ J\ K^{-1}}$$

- The Boltzmann constant relates the properties of microscopic particles (e.g. kinetic energy of gas molecules) to their macroscopic properties (e.g. temperature)
  - This is why the units are $J\ K^{-1}$
- Its value is very small because the increase in kinetic energy of a molecule is very small for every incremental increase in temperature

15.2 Kinetic Theory

15.2.1 Kinetic Theory of Gases

Assumptions of the Kinetic Theory of Gases

- Gases consist of atoms or molecules randomly moving around at high speeds
- The kinetic theory of gases models the thermodynamic behaviour of gases by linking the **microscopic properties** of particles (mass and speed) to **macroscopic properties** of particles (pressure and volume)

- The theory is based on a set of the following assumptions:
  - Molecules of gas behave as identical, hard, perfectly elastic spheres
  - The volume of the molecules is negligible compared to the volume of the container
  - The time of a collision is negligible compared to the time between collisions
  - There are no forces of attraction or repulsion between the molecules
  - The molecules are in continuous random motion
- The number of molecules of gas in a container is very large, therefore the **average** behaviour (eg. speed) is usually considered

Exam Tip

Make sure to memorise **all** the assumptions for your exams, as it is a common exam question to be asked to recall them.

Root-Mean-Square Speed

- The pressure of an ideal gas equation includes the **mean square** speed of the particles:

$$\langle c^2 \rangle$$

- Where
  - c = **average** speed of the gas particles
  - $\langle c^2 \rangle$ has the units **m² s⁻²**

- Since particles travel in all directions in 3D space and velocity is a vector, some particles will have a negative direction and others a positive direction
- When there are a large number of particles, the total positive and negative velocity values will cancel out, giving a net zero value overall
- In order to find the pressure of the gas, the **velocities must be squared**
  - This is a more useful method, since a negative or positive number squared is **always positive**

- To calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

$$\sqrt{\langle c^2 \rangle} = c_{r.m.s}$$

- $c_{r.m.s}$ is known as the **root-mean-square** speed and still has the units of **$m\ s^{-1}$**
- The mean square speed is **not** the same as the mean speed

? Worked Example

An ideal gas has a density of 4.5 kg m^{-3} at a pressure of 9.3×10^5 Pa and a temperature of 504 K.Determine the root-mean-square (r.m.s.) speed of the gas atoms at 504 K.

Step 1: Write out the equation for the pressure of an ideal gas with density

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

Step 2: Rearrange for mean square speed

$$\langle c^2 \rangle = \frac{3p}{\rho}$$

Step 3: Substitute in values

$$\langle c^2 \rangle = \frac{3 \times (9.3 \times 10^5)}{4.5} = 6.2 \times 10^5\ m^2\ s^{-2}$$

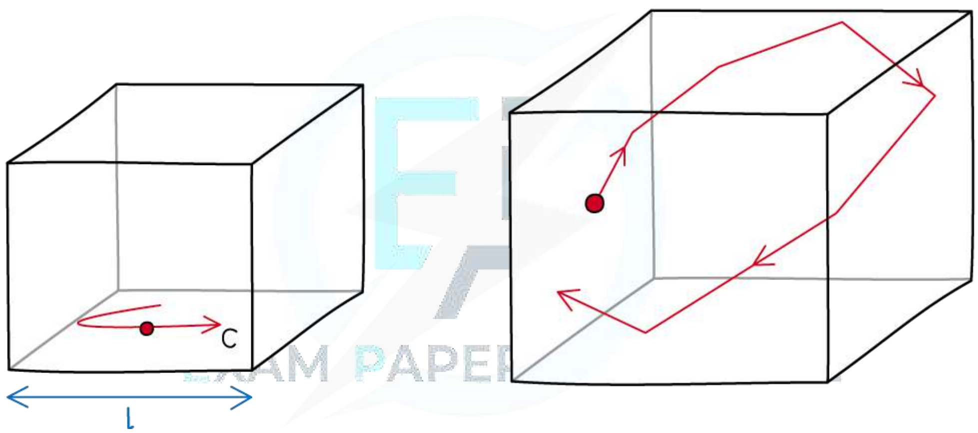
Step 4: To find the r.m.s value, take the square root of the mean square speed

$c_{r.m.s} = \sqrt{<c^2>} = \sqrt{6.2 \times 10^5} = 787.4 =$ **790 m s^{-1}** (2 s.f)

Derivation of the Kinetic Theory of Gases Equation

- When molecules rebound from a wall in a container, the change in momentum gives rise to a force exerted by the particle on the wall
- Many molecules moving in random motion exert forces on the walls which create an average overall **pressure**, since pressure is the force per unit area

- Picture a single molecule in a cube-shaped box with sides of equal length *l*
- The molecule has a mass *m* and moves with speed *c*, parallel to one side of the box
- It collides at regular intervals with the ends of the box, exerting a force and contributing to the pressure of the gas
- By calculating the pressure this one molecule exerts on one end of the box, the total pressure produced by all the molecules can be deduced



A single molecule in a box collides with the walls and exerts a pressure

5 Step Derivation

1. **Find the change in momentum as a single molecule hits a wall perpendicularly**

- One assumption of the kinetic theory is that molecules **rebound elastically**
- This means there is no kinetic energy lost in the collision
- If they rebound in the opposite direction to their initial velocity, their final velocity is −c
- The change in momentum is therefore:

$$\Delta p = -mc - (+mc) = -mc - mc = -2mc$$

2. **Calculate the number of collisions per second by the molecule on a wall**

- The time between collisions of the molecule travelling to one wall and back is calculated by travelling a distance of 2*l* with speed c:

$$\text{Time between collisions} = \frac{distance}{speed} = \frac{2l}{c}$$

- **Note:** c is **not** taken as the speed of light in this scenario

3. **Find the change in momentum per second**

- The force the molecule exerts on one wall is found using Newton's second law of motion:

$$\text{Force = rate of change of momentum} = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{mc^2}{l}$$

- The change in momentum is +2mc since the force on the molecule from the wall is in the opposite direction to its change in momentum

4. **Calculate the total pressure from N molecules**

- The area of one wall is l^2
- The pressure is defined using the force and area:

$$\text{Pressure p} = \frac{Force}{Area} = \frac{\frac{mc^2}{l}}{l^2} = \frac{mc^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of *N* molecules, the pressure can now be written as:

$$p = \frac{Nmc^2}{l^3}$$

- Each molecule has a different velocity and they all contribute to the pressure
- The mean squared speed of c^2 is written with left and right-angled brackets $<c^2>$
- The pressure is now defined as:

$$p = \frac{Nm<c^2>}{l^3}$$

5. **Consider the effect of the molecule moving in 3D space**

- The pressure equation still assumes all the molecules are travelling in the same direction and colliding with the same pair of opposite faces of the cube
- In reality, all molecules will be moving in three dimensions equally
- Splitting the velocity into its components c_x, c_y and c_z to denote the amount in the x, y and z directions, c^2 can be defined using pythagoras' theorem in 3D:

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

- Since there is nothing special about any particular direction, it can be determined that:

$$<c_x^2> = <c_y^2> = <c_z^2>$$

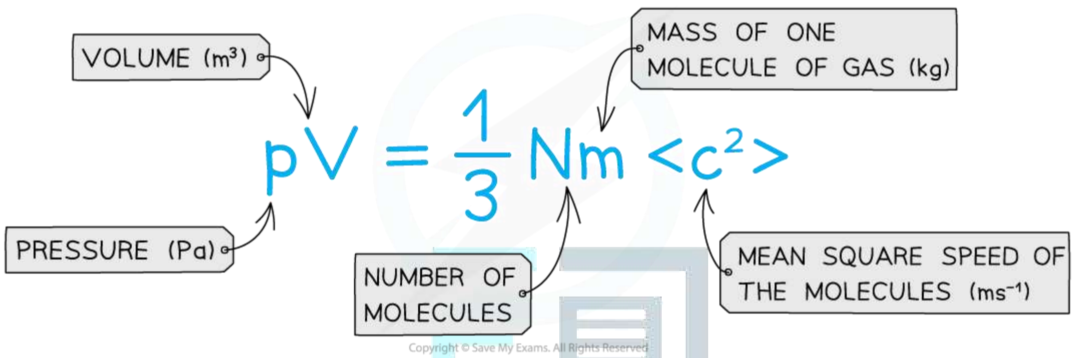
- Therefore, $\langle c_x^2 \rangle$ can be defined as:

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$$

- The box is a cube and all the sides are of length *l*
  - This means l^3 is equal to the volume of the cube, *V*
- Substituting the new values for $\langle c^2 \rangle$ and l^3 back into the pressure equation obtains the final equation:

$$pV = \frac{1}{3} Nm\langle c^2 \rangle$$

- This is known as the **Kinetic Theory of Gases equation**



- This can also be written using the density ρ of the gas:

$$\rho = \frac{mass}{volume} = \frac{Nm}{V}$$

- Rearranging the pressure equation for p and substituting the density ρ:

$$p = \frac{1}{3} \rho\langle c^2 \rangle$$

Exam Tip

Make sure to revise and understand each step for the whole of the derivation, as you may be asked to derive all, or part, of the equation in an exam question.

Average Kinetic Energy of a Molecule

- An important property of molecules in a gas is their **average kinetic energy**
- This can be deduced from the ideal gas equations relating pressure, volume, temperature and speed
- Recall the ideal gas equation:

$$pV = NkT$$

- Also recall the equation linking pressure and mean square speed of the molecules:

$$pV = \frac{1}{3} Nm\langle c^2 \rangle$$

- The left hand side of both equations are equal (pV)
- This means the right hand sides are also equal:

$$\frac{1}{3} Nm\langle c^2 \rangle = NkT$$

- N will cancel out on both sides and multiplying by 3 obtains the equation:

$$m\langle c^2 \rangle = 3kT$$

- Recall the familiar kinetic energy equation from mechanics:

$$\text{Kinetic energy} = \frac{1}{2} mv^2$$

- Instead of v^2 for the velocity of one particle, $\langle c^2 \rangle$ is the average speed of all molecules

- Multiplying both sides of the equation by ½ obtains the **average translational kinetic energy** of the molecules of an ideal gas:

$$E_K = \frac{1}{2} m\langle c^2 \rangle = \frac{3}{2} kT$$

- Where:
  - E_K = kinetic energy of a molecule (J)
  - m = mass of one molecule (kg)
  - $\langle c^2 \rangle$ = mean square speed of a molecule ($m^2\ s^{-2}$)
  - k = Boltzmann constant
  - T = temperature of the gas (K)

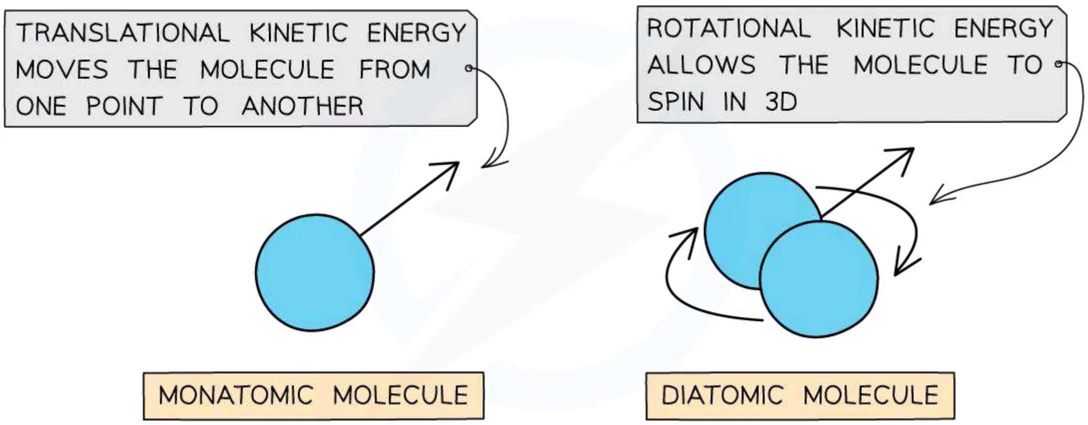
- **Note:** this is the average kinetic energy for only **one** molecule of the gas
- A key feature of this equation is that the mean kinetic energy of an ideal gas molecule is proportional to its thermodynamic temperature

$$E_K \propto T$$

- **Translational** kinetic energy is defined as:

The energy a molecule has as it moves from one point to another

- A monatomic (one atom) molecule only has translational energy, whilst a diatomic (two-atom) molecule has both **translational** and **rotational energy**



A diatomic molecule has both rotational and translational kinetic energy

Worked Example

Helium can be treated like an ideal gas.Helium molecules have a root-mean-square (r.m.s) speed of 730 m s^{-1} at a temperature of 45 ^{o}C.Calculate the r.m.s speed of the molecules at a temperature of 80 ^{o}C.

Step 1: Write down the equation for the average translational kinetic energy:

$$E_K = \frac{1}{2} m\langle c^2\rangle = \frac{3}{2} kT$$

Step 2: Find the relation between $c_{r.m.s}$ and temperature T

Since m and k are constant, $\langle c^2\rangle$ is directly proportional to T

$$\langle c^2\rangle \propto T$$

Therefore, the relation between $c_{r.m.s}$ and T is:

$$c_{r.m.s} = \sqrt{\langle c^2 \rangle} \propto \sqrt{T}$$

Step 3: Write the equation in full

$$c_{r.m.s} = a\sqrt{T}$$

where a is the constant of proportionality

Step 4: Calculate the constant of proportionality from values given by rearranging for a:

$$T = 45\ ^{o}C + 273.15 = 318.15\ K$$

$$a = \frac{c_{r.m.s}}{\sqrt{T}} = \frac{730}{\sqrt{318.15}} = 40.92...$$

Step 5: Calculate $c_{r.m.s}$ at 80 ^oC by substituting the value of a and new value of T

$$\mathbf{T = 80\ ^oC + 273.15 = 353.15\ K}$$

$$c_{r.m.s} = \frac{730}{\sqrt{318.15}} \times \sqrt{353.15} = 769 = \mathbf{770\ m\ s^{-1}}\ (2\ s.f.)$$

Exam Tip

Keep in mind this particular equation for kinetic energy is only for **one** molecule in the gas. If you want to find the kinetic energy for all the molecules, remember to multiply by ***N***, the total number of molecules.You can remember the equation through the rhyme 'Average K.E is three-halves kT'.