Exam Papers Practice

12.1 The Interaction of Matter with Radiation

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Course	DP IB Physics
Section	12. Quantum & Nuclear Physics (HL only)
Topic	12.1 The Interaction of Matter with Radiation
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **C** because:

- The uncertainty principle is stated as $\Delta p \Delta x \geq \frac{h}{4\pi}$ where *p* is the momentum of the particle and *x* is its position
- The position of the particle in this question is given the symbol *r*
- Momentum *p* is defined as:
  - $p = mv$
  - Where *m* is the particle's mass and *v* is its velocity
- The question states there is uncertainty in the particle's velocity (i.e., not in its mass)
  - Therefore, the uncertainty in the momentum $\Delta p = \Delta(mv) = m\Delta v$
- Hence, the uncertainty principle becomes:
  - $m \Delta v \Delta r \geq \frac{h}{4\pi}$
- Therefore, the uncertainty in the particle's velocity is:
  - $\Delta v \geq \frac{h}{4\pi m \Delta r}$
- The correct answer is **C**

You are expected to be comfortable with language like "the uncertainty in" some quantity. Remember the uncertainty principle is given in terms of **momentum** and **position**, therefore, make sure you can manipulate expressions involving these quantities!

2

The correct answer is **D** because:

- Red light has a longer wavelength than violet light
- The energy of each photon is:
  - $E = \frac{hc}{\lambda}$
  - Where *c* is the speed of light and λ is the wavelength
- *E* is inversely proportional to λ ($E \propto \frac{1}{\lambda}$)

- Therefore, the photons of red light carry **less** energy than photons of violet light
- The stopping potential V_S is the potential difference at which there is **no photocurrent** detected
    - This occurs when the potential of the collecting plate V is connected to the negative terminal
    - Since photons of red light transfer less maximum kinetic energy to each photoelectron, then the stopping potential must become less negative (closer to zero)
- This eliminates option C
- The intensity of incident light is kept constant
    - Therefore, the number of incident photons per second stays constant for both violet and red light
- Hence, the maximum photocurrent remains constant
    - This eliminates option B
- If the maximum photocurrent remains constant, then the potential difference at this maximum photocurrent will remain constant
    - This eliminates option A
- Therefore, the correct answer is option D

3

The correct answer is **B** because:

- The wave function ψ describes the **probability distribution** of a particle's position through all space
    - This is a function of position x and time t, so is written as $\psi(x,t)$
- The probability $\mathbb{P}(x,t)$ of finding an electron within a small region Δx near position x at time t is given by:
    - $\mathbb{P}(x,t) = |\psi(x,t)|^2 \Delta x$
- Therefore, the probability is proportional to the **square** of the magnitude of the wave function, $|\psi(x,t)|^2$
    - Hence the correct answer is **B**

A is incorrect as	the uncertainty principle as applied to uncertainty in energy ΔE and lifetime Δt is $\Delta E \Delta t \geq \frac{h}{4\pi}$. Therefore, $\frac{h}{4\pi \Delta E}$ is an expression for the uncertainty in **lifetime** (of some particular particle's state)
C is incorrect as	the uncertainty principle as applied to position Δx and momentum Δp is $\Delta p \Delta x \geq \frac{h}{4\pi}$. Therefore, $\frac{h}{4\pi \Delta p}$ is an expression for the uncertainty in measurements of the **position** of a particle
D is incorrect as	the magnitude of the wave function $\lvert\psi(x,t)\rvert$ is not proportional to a probability. Remember, the magnitude of the wave function must be **squared**, in order to calculate probabilities

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The correct answer is **D** because:

- The uncertainty principle as applied to measurements of momentum p and position x is given by:
  - $\Delta p \Delta x \geq \frac{h}{4\pi}$
  - Where Δp is the uncertainty in momentum, Δx is the uncertainty in position and h is Planck's constant
- Therefore, Δp is inversely proportional to Δx ($\Delta p \propto \frac{1}{\Delta x}$)
  - Hence, the particle with the largest uncertainty in momentum has the smallest uncertainty in position
- The quantity ψ^2 is proportional to the probability of finding the particle in some region in space
- The graph that shows the smallest uncertainty in position is graph **D**
  - This is because the probability (proportional to ψ^2) is localised over the smallest region Δx

A is incorrect as	the probability (proportional to ψ^2) is constant as a function of distance *x*. This means the particle has an equal chance of being found anywhere (i.e., the uncertainty in its exact location is **large**!)
B is incorrect as	the probability (proportional to ψ^2) varies over the entire region Δx. In some locations, the probability of finding the particle is maximum (i.e., a very small uncertainty in its position) and in some locations the probability of finding the particle is minimum (i.e., a large uncertainty in its position)
C is incorrect as	the probability (proportional to ψ^2) is not as localised (i.e., 'squeezed') as the probability of graph **D**. Therefore, the particle's position is less certain than that in graph **D** (and hence, the uncertainty in momentum is not as great as that in graph **D**)

5

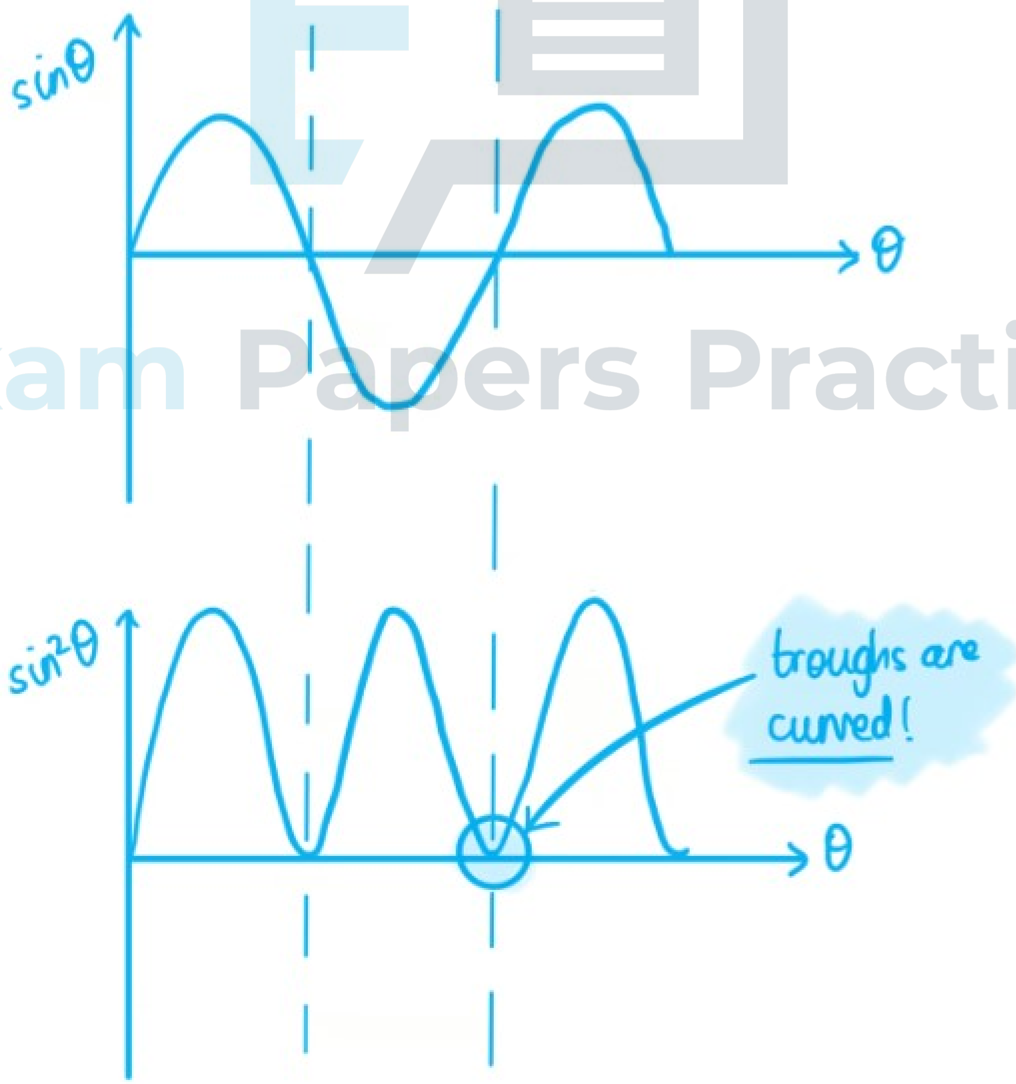
The correct answer is **B** because:

- The probability $\mathbb{P}$ is proportional to the **square** of the magnitude of the wave function (Ψ^2)
- Therefore, the correct graph must be the square of the wave function given in the question
  - This is given by graph **B**

A is incorrect as	this is the original wave function, it has not been squared to determine a probability. You can tell this is the case because probability **cannot be negative**
C is incorrect as	the troughs are **pointed**, not **curved**. This is not the correct shape for the square of the function given in the question

D is incorrect as	this graph shows **extended** regions of zero probability, whereas the wave function is only zero **instantaneously.** Therefore, the probability (corresponding to ψ^2) should also be zero only instantaneously (in other words, the a **turning point** on the graph of ψ^2)

The difference between graphs **B** and **C** are barely noticeable, but examinable! You should be able to determine the square of a function (e.g. ψ^2). Firstly, by understanding that the squared function can **never be negative**, and secondly, by remembering that the curve is **continuous** (i.e. there are no 'sudden pointed changes'). This is illustrated below by comparing the shape of sin θ and $\sin^2\theta$.



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The correct answer is **C** because:

Method 1: Momentum and Position

- Two quantities that cannot be known simultaneously with unlimited precision are momentum *p* of some particle and its position *x*
- The uncertainty principle relates the uncertainties in these two conjugate quantities as:
  - $\Delta p \Delta x \geq \frac{h}{4\pi}$
- Therefore, the product of their units $[\Delta p][\Delta x]$ = (kg m s^{-1}) × m
  - This is by recalling the definition of momentum $p = mv$ (mass × velocity)
  - Hence, the units of momentum can be expressed as a product of the kilogram, kg and the units of velocity, m s^{-1}
- Hence, $[\Delta p][\Delta x]$ = (kg m s^{-1}) × m = kg m^2 s^{-1}

Method 2: Energy and Time

- Two quantities that cannot be known simultaneously with unlimited precision are energy *E* (of a particular particle's state) and lifetime *t* of that state
- The uncertainty principle relates the uncertainties in these two conjugate quantities as:
  - $\Delta E \Delta t \geq \frac{h}{4\pi}$
- Therefore, the product of their units $[\Delta E][\Delta t]$ = (N m) × s
  - This is by recalling the definition of work (energy transferred) is $E = Fs$ (force × displacement)
  - Hence, the units of energy is the product of the Newton, N and the metre, m (N m)
- Hence, $[\Delta E][\Delta t]$ = (kg m s^{-2})(m) × s
  - This is by recalling $F = ma$, such that the Newton N is expressed as the product of the kilogram, kg and units of acceleration, m s^{-2} (N = kg m s^{-2})

- Therefore:
  - $[\Delta E][\Delta t]$ = kg × m^2 × s^{-1} = kg m^2 s^{-1}

You are expected to remember to examples of conjugate quantities, as expressed by Heisenberg's uncertainty principle. These are energy *E* and lifetime *t* related by their uncertainties as $\Delta E \Delta t \geq \frac{h}{4\pi}$, and momentum *p* and position *x*, related by their uncertainties as $\Delta p \Delta x \geq \frac{h}{4\pi}$. It's very interesting that the units of these two products are equivalent, which hints at something fundamental about uncertainty for pairs of quantities with similar units!

Make sure you're comfortable with combining the same quantity with different powers.

- If the quantities are multiplied, their powers are **added**: m^2 × m^{-1} = m$^{2-1}$ = m

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The correct answer is **D** because:

- The de Broglie wavelength *λ* is given by:
  - $\lambda = \frac{h}{mv}$
  - Where *v* is the velocity of the alpha particle of mass *m*
- The work done *W* is given by:
  - $W = q\Delta V$
  - Where Δ*V* is the change in potential difference *q* is the charge of the alpha particle
- Therefore, since the work is transferred as kinetic energy:
  - $E_k = \frac{1}{2}mv^2$, then $\frac{1}{2}mv^2 = q\,\Delta V$
- Rearranging for *v* gives:
  - $mv^2 = 2q\,\Delta V$
  - $v^2 = \frac{2q\,\Delta V}{m}$
  - $v = \sqrt{\frac{2q\,\Delta V}{m}}$

- This is the velocity v of the alpha particle due to acceleration across a potential difference ΔV
- The charge q of an alpha particle is $2e$, since it is comprised of 2 protons (and 2 neutrons)
- Substituting this into the v equation gives:
  - $v = \sqrt{\frac{2(2e)\Delta V}{m}}$
  - $v = \sqrt{\frac{4e\Delta V}{m}}$
- Substituting this equation for v into the equation for the de Broglie wavelength gives:
  - $\lambda = \frac{h}{m\left(\sqrt{\frac{4e\Delta V}{m}}\right)}$
  - $\lambda = \frac{h}{\sqrt{4m\Delta Ve}}$
- Therefore, the correct answer is **D**

Acceleration of charged particles across a potential difference is a **very** commonly examined bit of physics, and it pops up in a variety of contexts. You should make sure you are very comfortable with equating the energy transferred (or **work done**) by the electric field, which is given by $W = q\Delta V_e$ in your data booklet, with the kinetic energy of the charged particle. From here, you can easily derive an equation for the final velocity of the particle (assuming it starts from rest), as required in this question.

Combining the final variable m outside and inside the square root look like:

$$\lambda = \frac{h}{m\left(\sqrt{\frac{4e\Delta V}{m}}\right)} = \frac{h}{\sqrt{\frac{m^2 4e\Delta V}{m}}} = \frac{h}{\sqrt{m4e\Delta V}}$$

This is using the fact that $m = \sqrt{m^2}$ to since you can multiply two surds e.g. ($\sqrt{m^2} \times \sqrt{m} = \sqrt{m^3}$)

8

The correct answer is **C** because:

- The de Broglie wavelength λ is:
  - $\lambda = \frac{h}{p}$
  - Where p is the momentum of the electron and h is Planck's constant
- The kinetic energy of the electron E is given by:
  - $E = \frac{p^2}{2m}$ where m is its mass
- Therefore, $p = \sqrt{2mE}$
- Hence, the de Broglie wavelength can be written in terms of the electron's mass m and energy E as:
  - $\lambda = \frac{h}{\sqrt{2mE}}$
- The energy E_n (in electron volts) of an electron in the n^{th} energy level is given by:
  - $E_n = -\frac{13.6\ eV}{n^2}$
- Hence, if the electron exists in the $n = 2$ energy level, then:
  - $E_2 = -\frac{13.6\ eV}{2^2} = -\frac{13.6\ eV}{4} = -3.4\ eV$
- The magnitude of energy possessed by the electron is 3.4 eV
  - In joules, this energy $E = 3.4e$ J (where e is the charge of an electron)
- Substituting this magnitude of energy in to the equation for the de Broglie wavelength gives:
  - $\lambda = \frac{h}{\sqrt{2m(3.4e)}} = \frac{h}{\sqrt{6.8me}}$
- Therefore, **C** is the correct answer

Two key equations are used for this question: the de Broglie wavelength $\lambda = \frac{h}{p}$ (not given in your data booklet!) and the energy of a particle in terms of momentum $E = \frac{p^2}{2m}$. Make sure you are familiar with both!

$E = \frac{p^2}{2m}$ is derived from $E = \frac{1}{2}mv^2$ and $p = mv$.

9

The correct answer is **D** because:

- The quantity $|\psi|^2$ is a probability **density**
- Density is defined as the amount of mass per unit of volume
    - In a similar way, probability density is like the amount of probability in a volume
- Schrödinger's theory suggests probabilities of discovering an electron at different positions in space is described by a spread, or a probability density
    - Therefore, to determine the probability itself in any given volume ΔV, you must evaluate the product of the density and the volume
- Hence, the probability is given by $|\psi|^2 \Delta V$

A is incorrect as	ψ is simply the wave function of the electron. This is a mathematical function of position and time; it does not, in and of itself, represent any measurable quantity

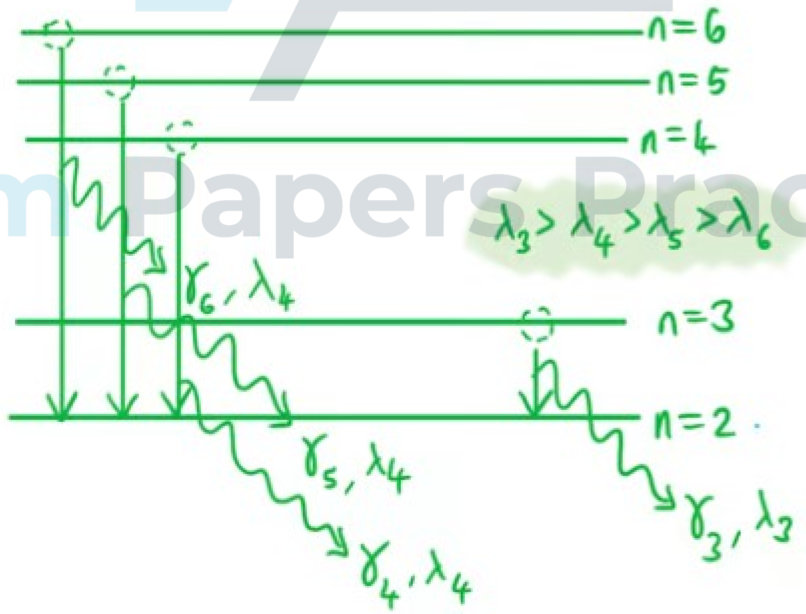
B is incorrect as	many wave functions turn out to be complex functions. For example, imagine a wave function described by $\psi = i\sin\theta$. Since the laws of complex algebra say that the imaginary number $i^2 = -1$, then the square of the wave function would be $\psi^2 = -\sin^2\theta$. This does not give you a sensible probability (density) because probability cannot be negative. Hence, the square of the **magnitude** of the wave function gives the correct probability (density), as $\|\psi\|^2 = \sin^2\theta$ (noting that the absolute value of i is 1). For your exam, you will not need to do explicit complex algebra, just remember the rule that the **magnitude** of the wave function must be squared in order to determine a probability
C is incorrect as	even though this correctly starts with the square of the **magnitude** of the wave function, this is not a probability. The quantity $\|\psi\|^2$ is **proportional** to a probability. Strictly speaking, $\|\psi\|^2$ is a **probability density**. In order to calculate probabilities from a probability density, you must multiply by a **volume** (or a **line** for a particle in 1D, or an **area** for a particle in 2D)

10

The correct answer is **B** because:

- Energy levels $n = 4$, $n = 5$, and $n = 6$ exist above the energy level $n = 3$ in the hydrogen atom

- Each of these energy levels provide transitions for excited electrons down to the $n = 2$ energy level
  - Hence, these levels contribute to the visible light spectrum for hydrogen
- By inspection, the photon emitted during an electron transition from $n = 4$, 5 or 6 down to $n = 2$ will have a **greater frequency** than the photon emitted during an electron transition from $n = 3$ to $n = 2$
  - This is because the energy gap is larger for such transitions
  - Therefore, since the energy of photons emitted $E = hf$, then larger energy transitions produce photons with a greater frequency
- Since the speed of each photon c is related to its frequency f by the speed equation $c = f\lambda$, then:
  - Photons with larger frequency must have a shorter wavelength
  - Hence, each of the photons emitted during transitions from $n = 4$, 5, or 6 to $n = 2$ have a shorter wavelength than that emitted from $n = 3$ to $n = 2$
- This is shown below:



- Hence, each of the additional spectral lines corresponding to visible light must have a **shorter wavelength** than the line corresponding to the transition from $n = 3$ to $n = 2$
  - This eliminates option **C** and **D**
- Since the energy levels $E_n = -\frac{13.6}{n^2}$ eV, then the gap between each energy E_n gets smaller as n increases
  - This is because $E_n \propto -\frac{1}{n^2}$
- Therefore, the correct answer is **B**

A is incorrect as	the spectral lines according to transitions to $n = 2$, from $n = 4$, 5 and 6, are equally spaced. Energy levels should vary in shape according to $E_n \propto -\frac{1}{n^2}$, such that the gap between them becomes smaller as n increases. Hence, spectral lines should be closer together for shorter wavelengths (higher frequency photon emission)
C is incorrect as	all three spectral lines according to transitions to $n = 2$, from $n = 4$, 5 and 6, are shown as having a **larger** wavelength than the transition $n = 3$ to $n = 2$. This cannot be true: the photons emitted due to transitions from higher energy levels have a greater energy, greater frequency, and hence shorter wavelengths
D is incorrect as	one of the spectral lines is shown as having a **longer** wavelength than the transition $n = 3$ to $n = 2$. This cannot be true: the photons emitted due to transitions from any of the higher energy levels must have a greater energy, greater frequency, and hence shorter wavelengths