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11.3 Special Relavility

Turning Points in Physics

AQA A Level Revision Notes

A Level Physics AQA

12.3 Special Relativity

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12.3.1 The Michelson-Morley Interferometer

The Michelson-Morley Interferometer

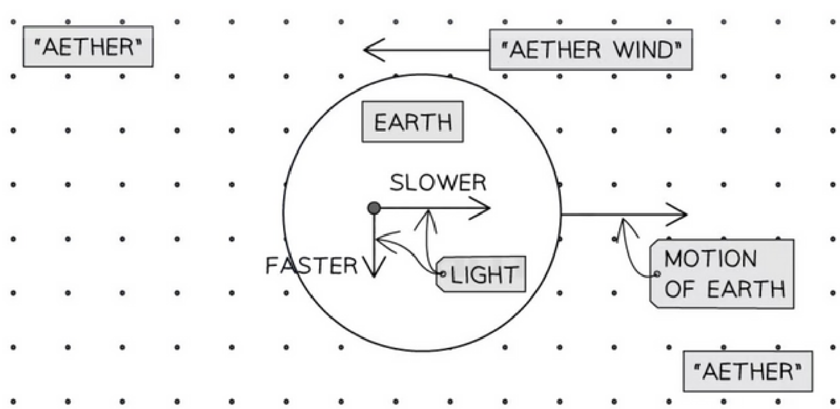
What was the Luminiferous Aether?

- The Dutch physicist Christiaan Huygens had developed the wave theory of light
  - All other known waves at the time (sound and water) travelled through a **medium**, so physicists assumed light did too
  - Huygens called this medium for light waves the "luminiferous aether" (or just aether) and physicists wanted to study its properties

What was the Aim of the Michelson-Morley Experiment?

- In 1849, Hippolyte Fizeau measured the speed of light in **moving** water
  - One beam of light travelled **with** the current and the other travelled **against** the current of the water
  - Through the interference between the two beams, he found that light moving in the same direction as the medium travelled faster than light in a direction opposing the motion of the medium
  - Interestingly, the speed decrease when travelling against the medium was greater than the speed increase when travelling with the medium
- In the 1880s, Michelson and Morley wished to use a similar method to prove the existence of the aether
  - If the aether existed, then the Earth was travelling through it
- Light travelling in the direction of the Earth's motion would be travelling **against** the aether
  - The motion of the aether against the light was called the **aether wind**
  - Like with water, if light was travelling into the aether wind, it would be travelling more slowly than light travelling perpendicular to the aether wind

Theory of motion of light relative to the "aether wind" as a result of the Earth's motion through it



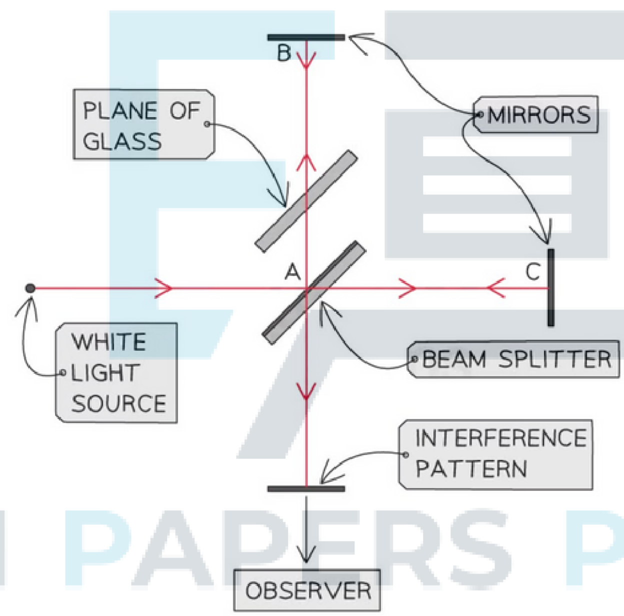
Light travelling into the aether wind was predicted to travel a small fraction slower than light travelling perpendicular to the aether wind

- The aim of the Michelson-Morley experiment was to use this difference in speeds of light to determine the **absolute motion** of the Earth relative to the aether
  - Absolute motion refers to the idea that the motion of all bodies in the Universe could be measured relative to the aether
  - The phase difference in the two beams of light can be used to determine the Earth's motion relative to the aether

What was the Michelson-Morley Interferometer?

- Michelson and Morley created a device called an interferometer
  - This consisted of two arms of identical lengths at right angles to each other, with mirrors at each end and a **beam splitter** (a semi-silvered mirror which allows some light to pass through and reflects some light) at their intersection

A diagram showing the arrangement of the Michelson-Morley Interferometer



Upon reaching the beam splitter, some light was reflected and some was transmitted, forming two beams that where initially coherent. The plane of glass ensured both beams travelled through the same distances of glass and air.

- White light travelled from a source to the beam splitter, where some travelled along path AB and some travelled along path AC
  - For the reflected beam, a plane of glass was placed in its path to ensure both beams travel through the same amount of air and the same amount of glass
  - Both beams are reflected by mirrors at B and C and meet at an eyepiece
- Both beams are from the same source and are therefore **coherent**
  - This means they will form an interference pattern
- The Michelson-Morley interferometer was set up to float on a bath of mercury
  - This allowed it to be rotated with minimal friction

- An interference pattern would be observed with one beam of light being slowed by the aether wind
  - Rotating the interferometer would then affect the phase difference of the beams differently, causing a **phase shift** in the interference pattern
  - The predicted shift was 0.4× the width of one fringe in the interference pattern, so the equipment was designed to detect changes of 0.01 fringe widths

The Detection of Absolute Motion

The Results

- Michelson and Morley performed the experiment at different angles and at different times of the day (so the Earth had also rotated relative to the aether)
  - However, their results only ever showed displacements of the interference project around 0.02 fringe widths and not even in the expected orientations
  - These values were far too small to be significant and most likely experimental noise
- This **null result** led scientists to the following conclusions:
  - The aether does not exist and therefore light is a wave able to travel without a medium
  - The speed of light is unchanged by the Earth's motion - it is **invariant**
- If there is no medium from which to measure the motion of the Earth, there is no absolute motion - everything is moving with respect to everything else

Worked Example

Explain why Michelson and Morley predicted that the fringes in the interference pattern would shift when the interferometer was rotated 90 degrees.

Answer:

- They predicted the speed of light depended on the motion of the Earth (relative to the aether)
- (Therefore) the <u>time</u> difference would change between the two beams when they were rotated
- (So) there would be a change in the <u>phase</u> difference, shifting the fringes

Exam Tip

When referring to the expected change in the interference pattern, make sure to call it an expected **phase shift** between the two beams, **not** a path difference. When the interferometer is rotated, the length of the path of each beam remains the same.

The Invariance of the Speed of Light

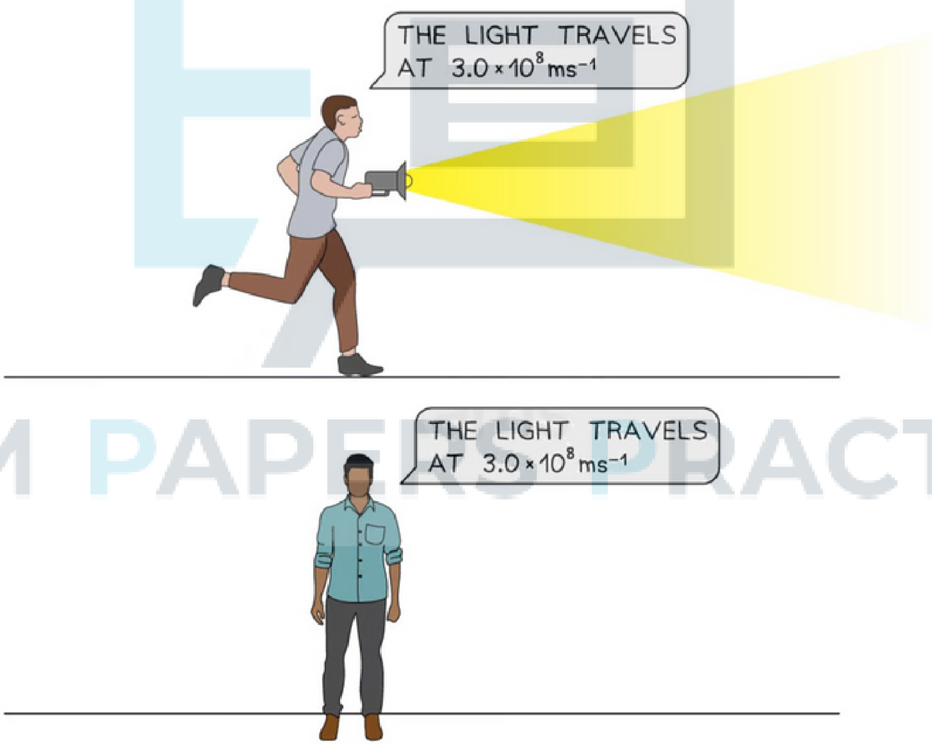
The Speed of Light

- In a vacuum, light travels at a speed of 3.0×10^8 ms^{-1}, which is given the symbol c
- When travelling through different materials, the speed of light may **reduce** but it can never **exceed** c

What does invariance mean?

- The invariance of light means that when in a vacuum, the speed of light is always 3.0×10^8 ms^{-1} for **every** observer
  - Light still travels at this speed even when emitted from a **moving** object

Diagram demonstrating the invariance of the speed of light



Light does not travel at 3.0×10^8 ms^{-1} plus the speed of the person running, to all observers it just travels at 3.0×10^8 ms^{-1}

- This concept only applies to light, as it is the only thing capable of traveling at the speed of light
  - The invariance of the speed of light has some very **surprising** consequences, which will be explored in the following revision notes...

Exam Tip

The invariance of the speed of light may feel counter-intuitive at first, but remember that nothing can travel faster than the speed of light, so if you ever end up with an answer for velocity bigger than 3.0×10^8 ms^{-1}, something has gone wrong!

12.3.3 Inertial Frames of Reference

Inertial Frames of Reference

- The term **relative** is used often in Physics to make it clear which point of view we are referring to
  - For example, the velocity of a car **relative** to someone stationary is different from the velocity measured by another car travelling alongside the initial car at the same speed
- A **reference frame**, or a **frame of reference**, refers to the **position** of an object, it is defined as:

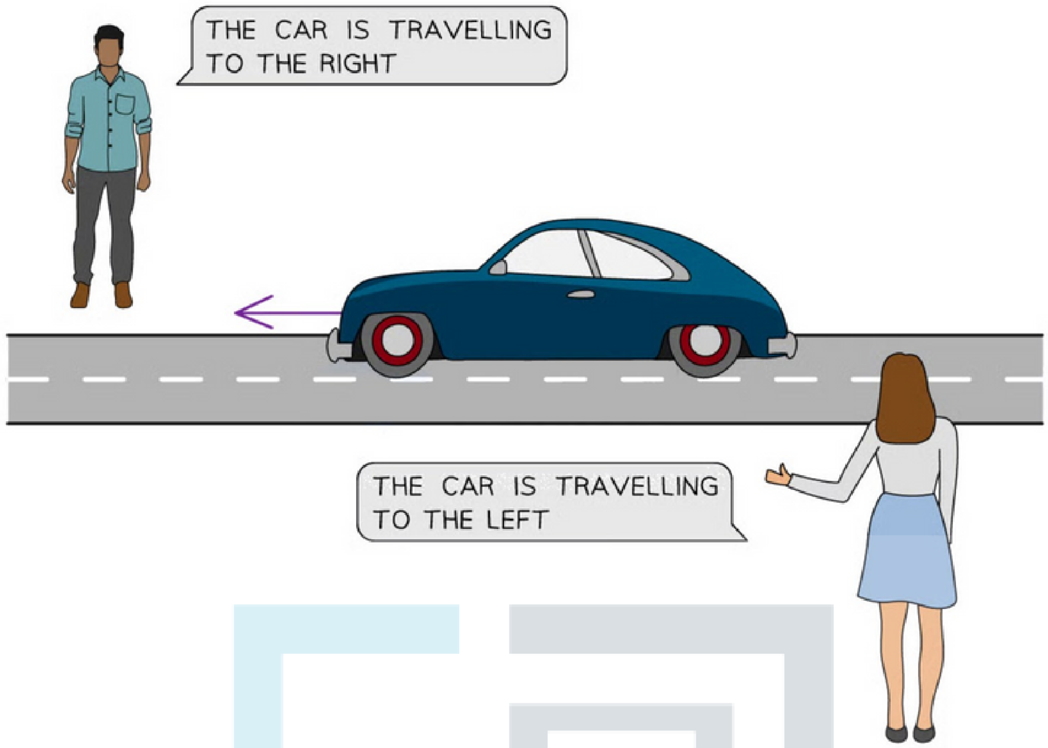
  A set of coordinates to record the position and time of events

- For example, you, currently sitting on your chair at your desk, is your current reference frame
  - You feel as if you are stationary, despite the fact the Earth is revolving on its axis and orbiting the Sun
- A reference frame is the point of view where an object, at a specific co-ordinate, is **at rest**

Examples of Reference Frames

- An everyday example is the **direction** of an object from your point of view in comparison to someone else
- In this example, a car is driving down a road and two people are standing on opposite sides of that road
- Despite the car moving in one direction, each person will view its direction **relative to them** differently
  - The person on one side of the road would say the car is moving to the right, and the person on the other side of the road would say the car is moving to the left
  - Both are correct, but they are viewing the car's motion from **different points of reference**

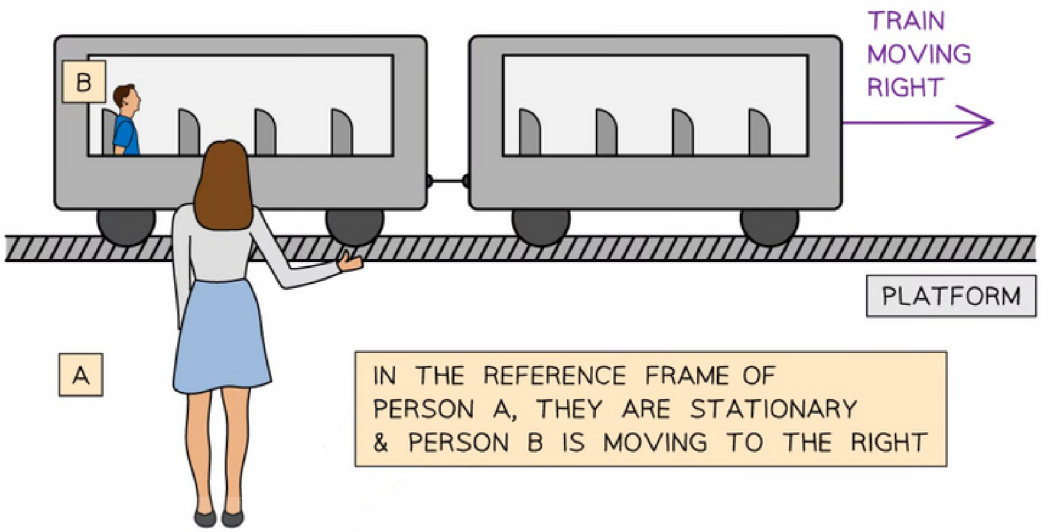
Diagram showing different points of reference for a moving car

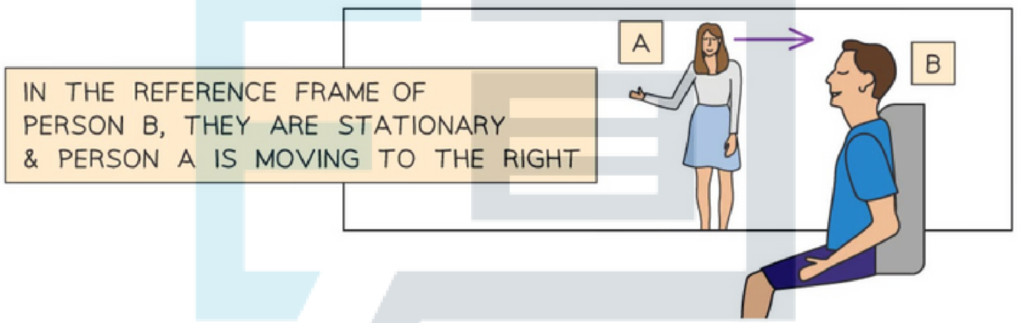


Each person has a different frame of reference, so they interpret the direction of the car differently relative to themselves

- Another common example is of a train pulling out of a station, where Person A is on the platform and Person B is on the train
- As the train begins to move, Person A, on the platform, views Person B, on the train, moving to the right
  - Therefore, according to Person A, they, themselves, are stationary and Person B is moving to the right
- Things look a little different from Person B's perspective
- As the train begins to move (to the left from Person B's perspective), Person B, on the train, views Person A, on the platform, moving to the right
  - Therefore, according to Person B, they, themselves, are stationary (as they cannot feel the train moving to the left) and Person A is moving to the right

Diagram demonstrating different reference frames for a train leaving a station





Person A and B are both stationary in their own reference frames and see the other as moving

- Therefore, frames of reference are used to specify the relationship between a moving and stationary object

Inertial Frames of Reference

- An **inertial** reference frame is

    A reference frame that is non-accelerating

- Therefore, all inertial reference frames are moving at **constant velocity** with **respect** to each other
- There is no such thing as an absolute reference frame in our Universe
    - In other words, there is no place in the Universe that is completely stationary
    - Everything is always moving relative to everything else

Worked Example

A student is cycling to school with their friend who is also cycling exactly in line. As they cycle past a bus stop, they wave to their aunt who is stationary at the bus stop as she waits for her bus.

The student's aunt estimates the speed of the students to be 5 m s^{-1}.

At what speed would the friend measure the student to be travelling?

A 5 m s^{-1}

B -5 m s^{-1}

C 0 m s^{-1}

D 2 m s^{-1}

Answer:

The correct answer is **C** because:

- We must think about the friend's reference frame for this question, in which they are stationary (according to them)
- Since the friend is cycling **in line** with the student, this means they measure the student to be travelling at **0 m s^{-1}** relative to them

Exam Tip

In exam questions, look out for terms such as 'for the reference frame of...', 'in the reference frame of...' or 'relative to ...' to know which reference frame is being referred to. You can think of it as 'What do they see from their **point of view?'.** This becomes important when you learn about special relativity.

You will not come across non-inertial reference frames (i.e. ones where a frame is accelerating) in your exam.

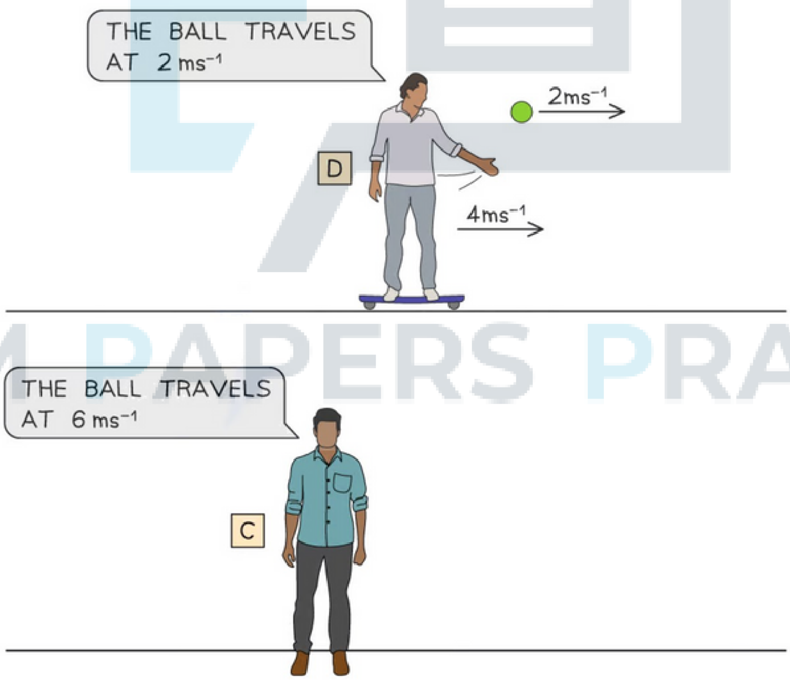
12.3.4 Einstein's Postulates

Einstein's Postulates

Galilean Relativity

- **Galilean relativity** is the type of relativity we are most familiar with
- At everyday speeds, an object's speed in one frame of reference may be different in another frame of reference
  - This is because the **velocity**, **position** and **time** of an event appear differently from different reference frames
- For example, Person D is on a skateboard travelling at 4 m s^{-1} when they throw a ball in a straight line at a constant velocity of 2 m s^{-1}. Person C is a stationary observer of the event.
  - In Person D's reference frame, the ball is travelling at 2 m s^{-1}
  - In Person C's reference frame, the ball is travelling at 4 + 2 = 6 m s^{-1}
- So the speed of the ball depends on the reference frame.

Diagram showing the difference in velocity of an object in two reference frames



Person C measures the ball to be travelling faster than when measured by Person D

When does Galilean Relativity Stop Working?

- Galilean relativity states that Newton's laws of motion are the same in **all** inertial reference frames
  - Therefore space and time are treated as fixed and absolute

- This means the time interval between two events in one frame (x, y) is the same as the time interval between another frame (x', y')

- However, this is **not** what happens when we are close to the speed of light
  - Space and time become **relative**, meaning, the length of an object or a time interval **depends** on the frame of reference
- Velocity addition works with speeds much **lower** than the speed of light (c)
- It **doesn't** work for objects travelling closer to the speed of light
  - According to Galilean relativity, if a rocket ship travels at 0.7c and releases a probe directly in front of it at 0.5c, a stationary observer would view this at 0.7c + 0.5c = 1.2c
  - **However**, we know that **nothing** can travel faster than the speed of light, so this is **not** possible
- Einstein proposed two **postulates of special relativity**

The First Postulate of Relativity

- Einstein's first postulate of special relativity states:

The laws of physics are the same in all inertial frames of reference

- In our own reference frame, we are always **stationary**
- This means in practice, we should **not** be able to tell whether we are moving or not
  - Someone conducting a physics experiment on a moving train versus on a stationary platform should produce the **exact same** results

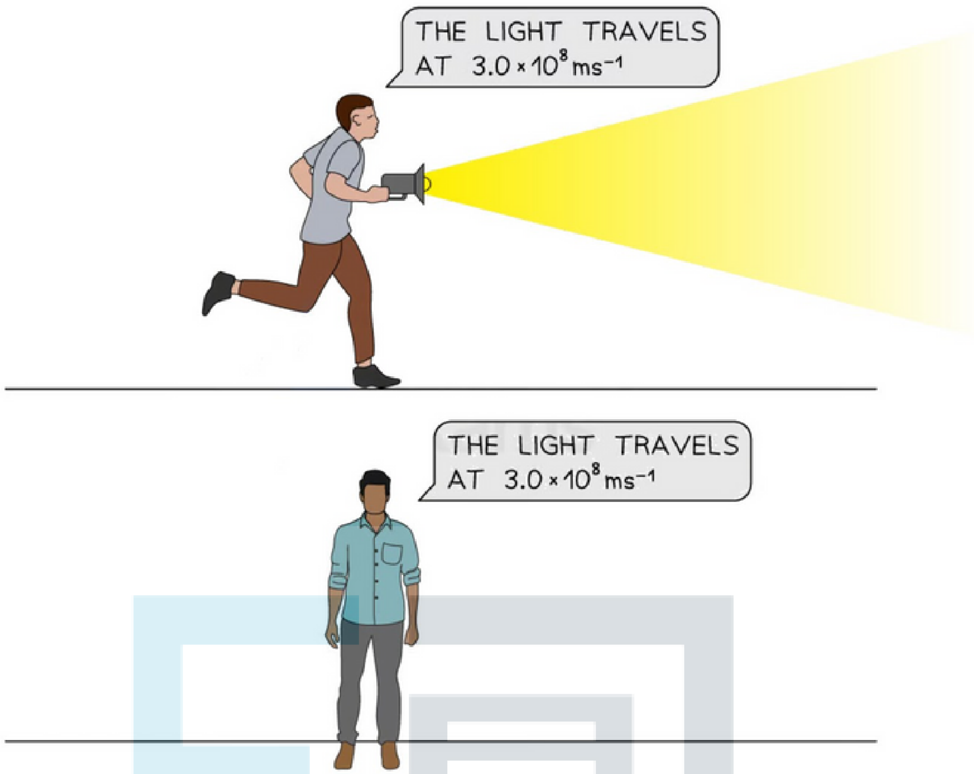
The Second Postulate of Relativity

- Einstein's second postulate of special relativity states:

The speed of light, c, in a vacuum, is the same in all inertial frames of reference

- Two different observers will always measure the speed of light to be the same value, c in their reference frame
  - It makes no difference whether they are travelling or not. If it did, you would know whether you are moving, which counteracts the first postulate
- For example, a runner holding a flashlight in front of them will measure the speed of the light as **c**
  - However, someone stationary observing the runner will **also** see the speed of light as c and **not** c + the velocity of the runner
- This only works for the **speed of light**, not any other speed

Diagram demonstrating Einstein's postulate of special relativity



Both a moving and stationary observer would measure the same speed of light from the torch

Exam Tip

You must remember these two postulates, as they play an important part conceptually and mathematically in further equations in special relativity.

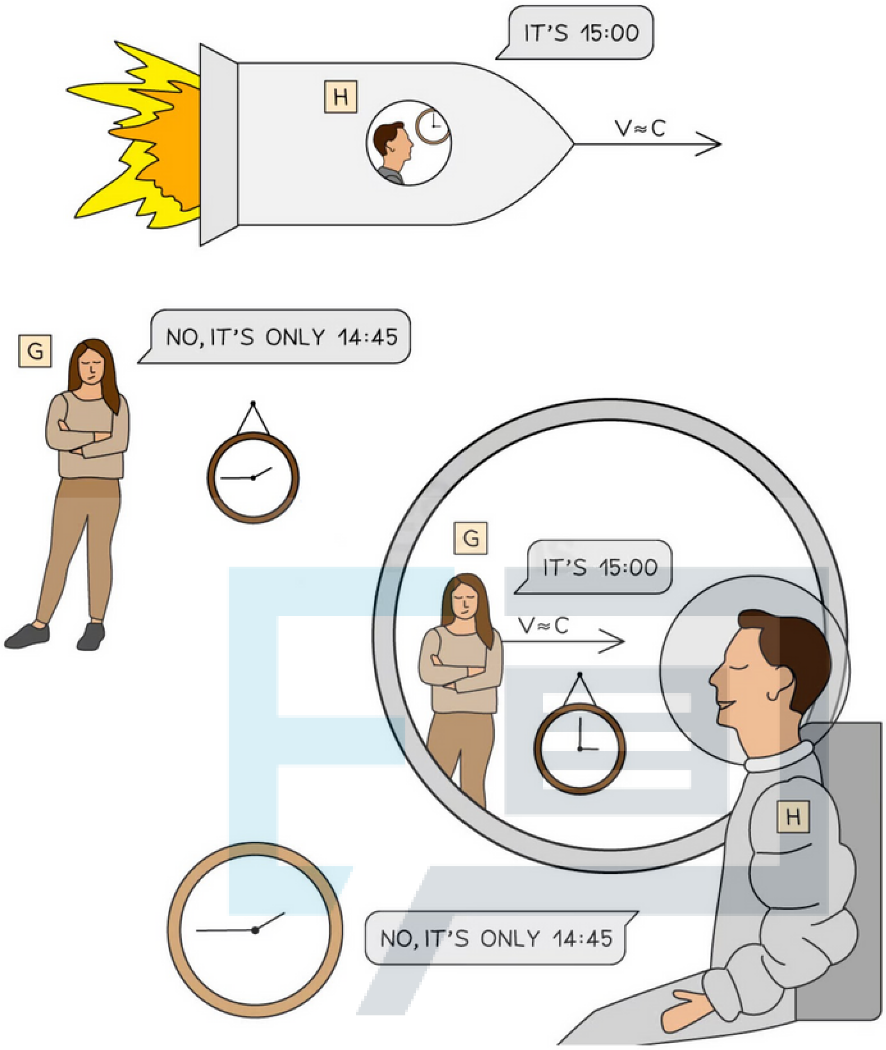
12.3.5 Time Dilation

Time Dilation

- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **slowed down**
  - This is best demonstrated by **clocks**

- Observer H is in a rocket moving close to the speed of light
  - They will see their clock ticking at a regular pace, say, it is reading 15:00
- Observer G at rest on Earth, with remarkable eyesight, will measure the clock as ticking **slower**
  - They will observe that time has **slowed down** in the spaceship from **their** reference frame i.e. they may see the time as 14:45 instead of 15:00

- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H will measure observer G's clock as ticking **slower** i.e. they see time **slow down** on Earth from **their** reference frame



A stationary observer in their reference frame views clocks as running slower in the moving reference frame. We're back to disagreeing

The Gamma Term

- Recall from Einstein's Postulates that moving between frames of reference means the speed needs to be changed through simple addition or subtraction
  - When travelling at speeds close to the speed of light, this no longer applies, because the speed of light is independent of the frame of reference
- This transformation from one reference frame to another is corrected by multiplying by a scaling factor called the **gamma factor**, γ

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

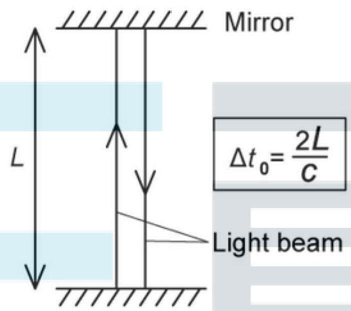
- As v is always less than c (the speed of light), this means γ is always **greater than 1**
- This is especially important for time dilation and length contraction

- We will use time dilation to explain where this gamma term comes from

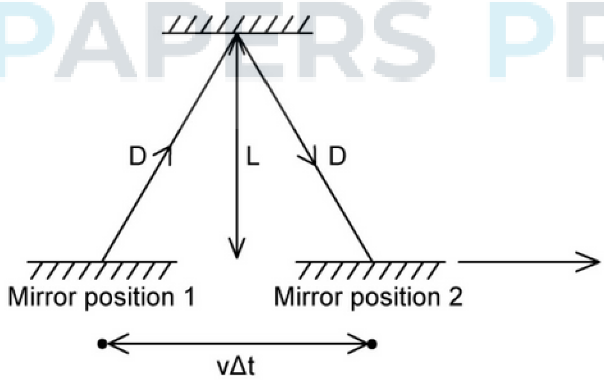
Time Dilation Equation

- Consider a light clock. This consists of two mirrors facing each other with a beam of light travelling up and down between them
- An observer is at **rest relative** to the clock on a train platform and watches the light reflect between the mirrors
- The distance between the mirrors is *L* and the light travels at a speed c
- Therefore, the time interval for the light to travel from the top mirror back down to the bottom is:

$$\Delta t_0 = \frac{2L}{c}$$



- Another light clock is on a moving train, **relative** to the initial observer on the platform, travelling at constant velocity *v*
  - This velocity *v* is close to the speed of light
- The stationary observer on the platform sees the light clock on the train and watches the reflection of the rays between the mirrors
- It appears that the light rays travel to the right at an angle to the direction of motion



- The observer on the train platform sees the mirror at:
  - Position 1, when the light leaves the bottom mirror
  - Position 2, when the light returns to it
- The distance between the mirrors, *L, is the same,* but the light now travels a longer path, *D*
- The distance travelled by the light ray is now 2*D,* and the time observed between the reflections is now

$$\Delta t = \frac{2D}{c}$$

- The apparent distance horizontally travelled by the mirror is $v\Delta t$ where v is the speed of the train
- Notice that this is part of a right-angled triangle, so using Pythagoras' theorem we can see that:

$$D^2 = L^2 + \left(\frac{v\,\Delta t}{2}\right)^2$$

- Where:

$$D = \frac{c\Delta t}{2}$$

- We want to find Δt, the time taken for the light ray to travel up and down in the reference frame of the stationary observer on the train platform, who is moving **relative** to the light clock on the train
  - Remember, although it is the train that is moving, in the reference frame of an observer on the train it is the observer on the **platform** that is moving

$$\Delta t = \frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Remember the gamma factor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
- Therefore:

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - Δt = the time interval measured from an observer **moving relative** to the time interval being measured (s)
  - Δt_0 = the **proper** time interval (s)

- As $\gamma > 1$, this means that the $\Delta t > \Delta t_0$
  - In other words, a clock observed from a reference frame moving **relative** to it will be measured to tick slower than a clock that is at rest in its frame of reference

- The observer on the **platform** will view the **train** clock as moving **slower**
- The observer on the **train** will view the **platform** as moving **slower**

- You may be wondering why it's the time that slows down for the light beam, and not the light beam just speeding up to hit each mirror at the same frequency

- This is due to Einstein's second postulate
- Both Observers G and H **must** measure the speed of light to be c, so it doesn't slow down or speed up according to either reference frame

- It is important to note that the time has been measured at the same **position**
    - In other words, the time interval is the position at which the light leaves the first mirror and at which it returns to the second mirror in the reference frame of the mirror

Worked Example

Alex's spacecraft is on a journey to a star travelling at 0.7 c. Emma is on a space station on Earth at rest. According to Emma, the distance from the base station to the star is 14.2 ly.

Show that Alex measures the time taken for her to travel from the base station to the star to be about 14.5 years.

Answer:

Step 1: List the known quantities:

- Distance of space station according to Emma = 14.2 ly
- Speed of Alex's spacecraft, $v = 0.7c$

Step 2: Analyse the situation

- We are trying to find the time that Alex measures for **her** travel i.e. the time she would measure on her own clock in the spaceship which **she is stationary relative to**
- This is the proper time, Δt_0

Step 3: Calculate the time taken according to Emma, Δt

- 1 ly (light year) is the **distance**, *s*, light (at speed c) travels in a year
- Therefore it takes the light 14.2 years (time) to travel the distance at speed c

$$s = speed \times time = 14.2c \text{ m}$$

- Therefore, the time taken according to Emma is:

$$\Delta t = \frac{s}{v} = \frac{14.2c}{0.7c} = \frac{14.2}{0.7} = 20.29 \text{ years}$$

Step 4: Substitute values into the time dilation equation

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma}$$

$$\Delta t_0 = \frac{20.29}{\frac{1}{\sqrt{1 - \frac{(0.7c)^2}{c^2}}}} = \frac{20.29}{\frac{1}{\sqrt{1 - (0.7)^2}}} = 14.5 \text{ years}$$

Step 5: Check whether your answer makes sense

- Since Emma (who is stationary) is viewing Alex's clock (which is moving) she would measure a **longer** time for Alex to reach the star than Alex will
- As Emma records 20.29 years, but Alex only records 14.5 years, this time makes sense

Exam Tip

A nice way to remember this is 'moving clocks run slower'. The caveat is what is considered 'moving' **depends on the reference frame.**

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

The notion of 'proper time' is incredibly important here, as it depends on the reference frame the time interval is being measured from.

The maths for the derivation is only using $speed = \frac{distance}{time}$ and Pythagoras' theorem.

12.3.6 Muon Lifetime Experiment

Muon Lifetime Experiment

- Muon decay experiments provide **experimental evidence** for time dilation and length contraction
- **Muons** are unstable, subatomic particles that are around 200 times heavier than an electron and are produced in the upper atmosphere as a result of pion decays produced by cosmic rays
- Muons travel at 0.98c and have a half-life of 1.6 μs (or mean lifetime of 2.2 μs)
  - The distance they travel in one half-life is around 470 m
- A considerable number of muons can be detected on the Earth's surface, which is about 10 km from the distance they are created
  - Therefore, according to Newtonian Physics, very few muons are expected to reach the surface as this is about 21 half-lives!
- The detection of the muons is a product of time dilation (or length contraction, depending on the viewpoint of the observer)

Muon Decay From Time Dilation

- According to the muon's reference frame, time is dilated so its half-life is **longer**
- We can see this from the time dilation equation

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - The gamma factor, $\gamma = \frac{1}{\sqrt{1 - (0.98)^2}} = 5$
  - Δt = the half-life measured by an observer on Earth
  - Δt_0 = the proper time for the half-life measured in the muon's inertial frame
- Therefore, in the reference frame of an observer on Earth, the muons have a lifetime of

$$\Delta t = 5 \times 1.6 = 8 \ \mu s$$

- The time to travel 10 km at 0.98c is 33 μs or 4.1 half-lives, so a significant number of muons remain undecayed at the surface

Muon Decay From Length Contraction

- According to the muon's reference frame, length is contracted so the distance they need to travel is **shorter**
- We can see this from the length contraction equation

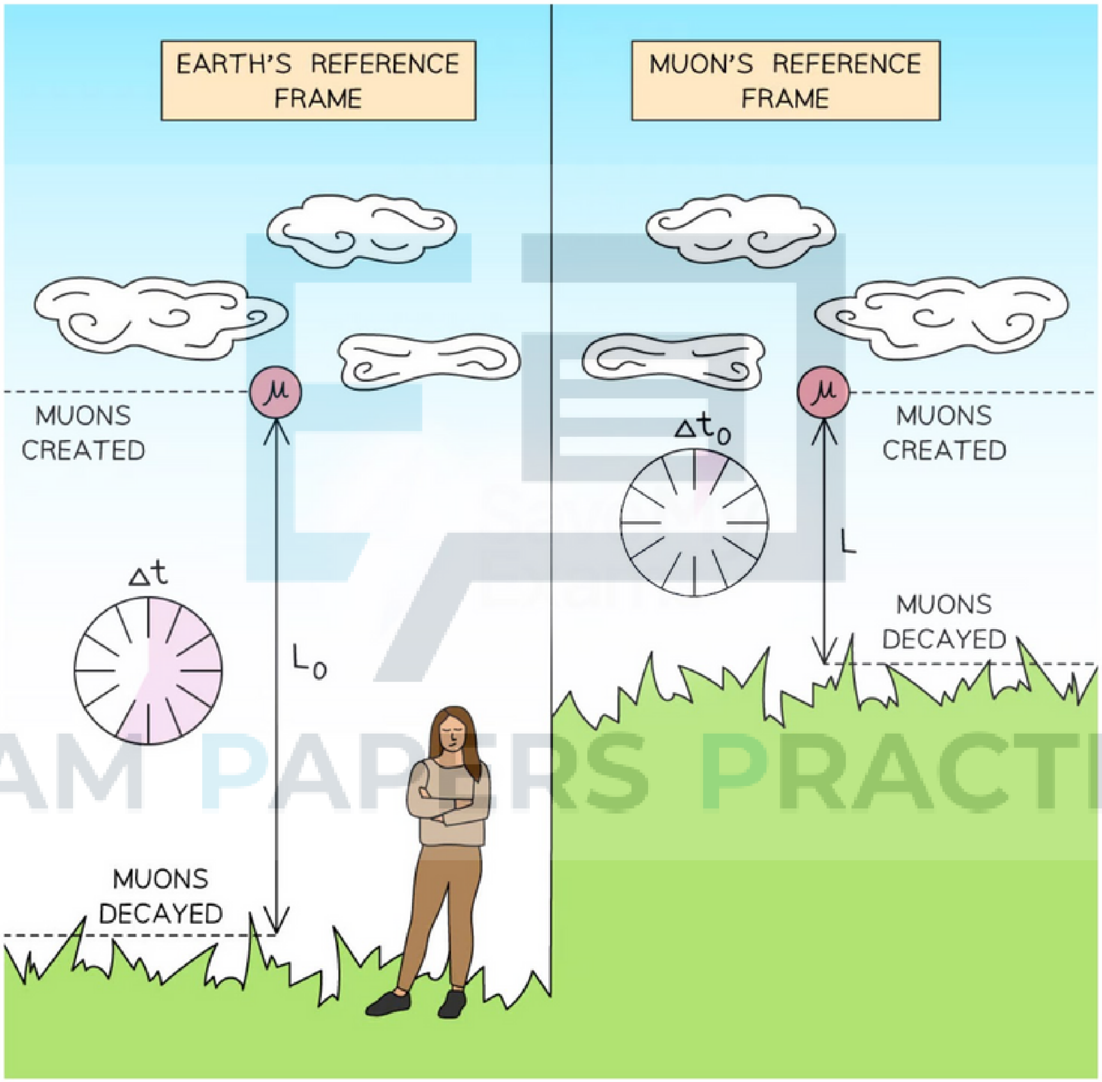
$$L = \frac{L_0}{\gamma}$$

- Where:
  - L_0 = the proper length for the distance measured in the muon's inertial frame
  - L = the distance measured by an observer on Earth

- Therefore, in the reference frame of the muons, they only have to travel a distance:

$$L = \frac{10\,000}{5} = 2000 \text{ m}$$

- To travel this distance takes a time of $\frac{2000}{0.98c}$ = 6.8 μs which is about 4.3 half-lives again, so a significant number of muons remain undecayed at the surface



Muon decay from the Earth's and a muon's reference frame

Worked Example

Muons are created at a height of 4250 m above the Earth's surface. The muons move vertically downward at a speed of 0.980c relative to the Earth's surface. The gamma factor for this speed is 5.00. The half-life of a muon in its rest frame is 1.6 μs.

(a)
Estimate the fraction of the original number of muons that will reach the Earth's surface before decaying, from the Earth's frame of reference, according to:

(i)
Newtonian mechanics

(ii)
Special relativity

(b)
Demonstrate how an observer moving with the same velocity as the muons, accounts for the answer to (a)(ii).

Answer:

(a) (i) Newtonian mechanics

Step 1: List the known quantities

- Height of muon creation above Earth's surface, h = 4250 m
- Speed of muons, v = 0.980c
- Lifetime of muon, t = 1.6 μs = 1.6×10^{-6} s

Step 2: Calculate the time to travel for the muon

$$time = \frac{distance}{speed} = \frac{h}{v}$$

$$t = \frac{4250}{0.98 \times (3.0 \times 10^8)} = 1.45 \times 10^{-5}\ \text{s}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{1.6 \times 10^{-6}} = 9 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^9} \times 100\ \% = 0.2\ \%$$

(a) (ii) Special relativity

Step 1: List the known quantities

- Time for the muon to travel, $\Delta t_0 = 1.45 \times 10^{-5}$ s

Step 2: Calculate the time travelled in the muons rest frame

$$\Delta t = \gamma \Delta t_0$$

$$\Delta t = 5 \times (1.6 \times 10^{-6}) = 8 \times 10^{-6}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{8 \times 10^{-6}} = 1.8 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^{1.8}} \times 100\,\% = 29\,\%$$

(b)

Step 1: Analyse the situation

- An observer moving with the same velocity as the muons will measure the **distance** to the surface to be **shorter** by a factor of $\left(\frac{9}{1.8}\right)$ = 5 **OR** length contraction occurs

Step 2: Calculate the distance travelled in the muon's rest frame

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{4250}{5} = 850 \text{ m}$$

Step 3: Calculate the time to travel

$$\text{time taken} = \frac{distance}{speed} = \frac{850}{0.98 \times (3 \times 10^8)} = 2.9 \times 10^{-6}$$

Step 4: Calculate the number of half-lives

$$\frac{2.9 \times 10^{-6}}{1.6 \times 10^{-6}} = 1.8 \text{ half-lives (same as (a)(ii))}$$

Exam Tip

Remember that it is the observer on **Earth** that viewed the muons' lifetime or half-life as **longer** (time dilation), whilst it is the **muons'** reference frame that views the distance needed to travel as **shorter** (length contraction).

Always do a sense check with your answer, you must always end up with a longer time or shorter distance for the muons to be observed on the Earth's surface.

Any exam questions on this topic will only use the following equations:

- Time dilation
- Length contraction
- $speed = \frac{distance}{time}$

Calculating half-lives through $\frac{1}{2^{number\ of\ half-lives}}$ is a common way to calculate the number of muons remaining:

- After 1 half-life, $\frac{1}{2}$ the original muons remain
- After 2 half-lives, $\frac{1}{4}$ or $\frac{1}{2^2}$ of the original muons remain
- After 3 half-lives, $\frac{1}{8}$ or $\frac{1}{2^3}$ of the original muons remain, and so on

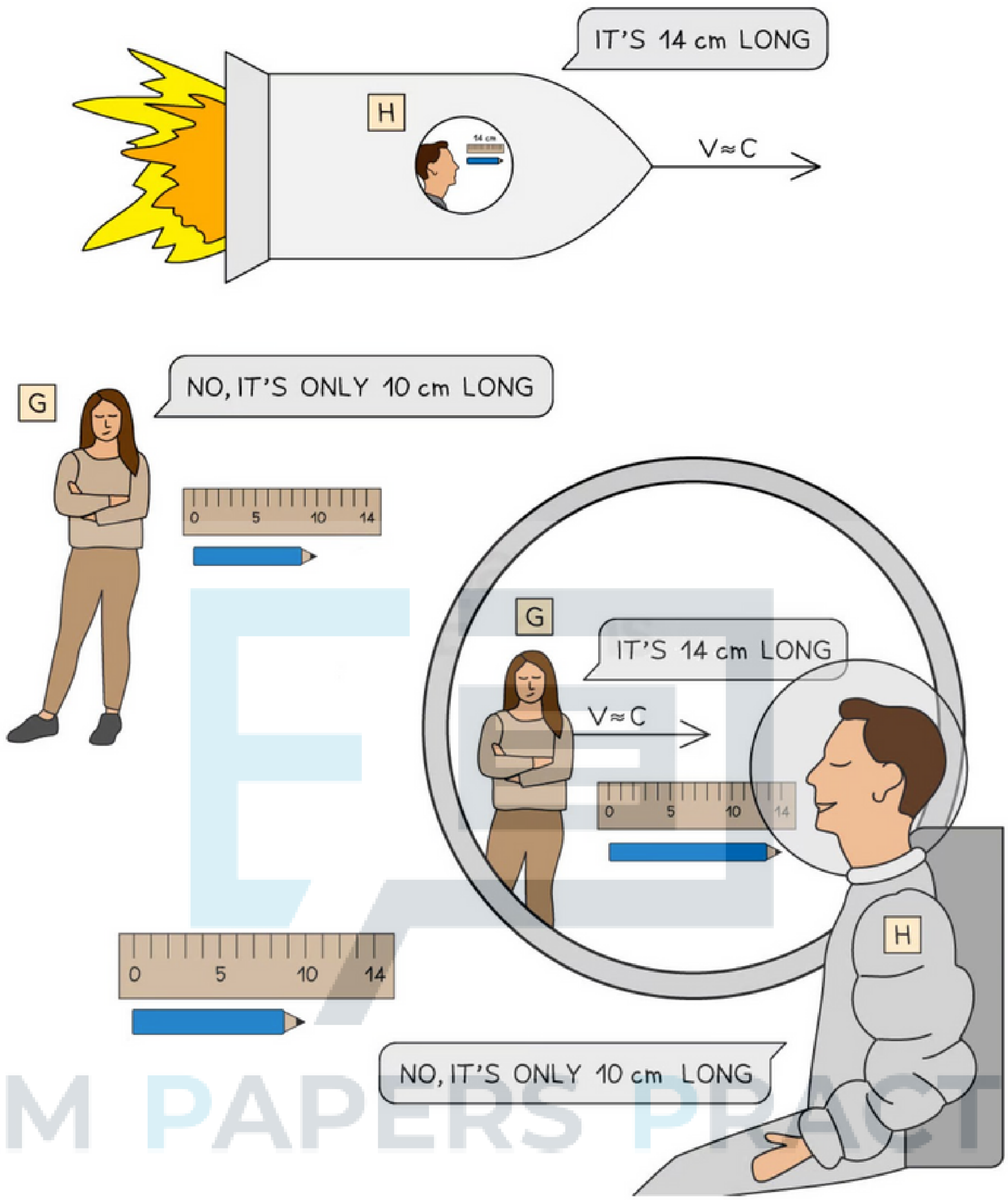
12.3.7 Length Contraction

Length Contraction

- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **become shorter**
  - This is best demonstrated using **rulers**

- Observer H, in their rocket moving close to the speed of light, measures the length of their pencil to be 14 cm
- Observer G, at rest on Earth, would measure (with remarkable eyesight) the length of the pencil to be **shorter**
  - They will see lengths **contracted** in the spaceship from **their** reference frame, e.g. the length may appear to be 10 cm instead of 14 cm

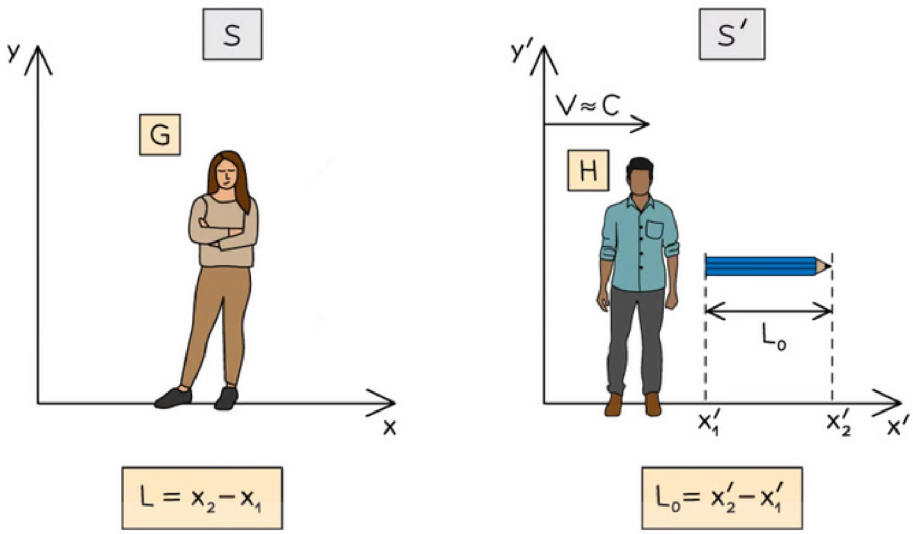
- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H would measure the length of observer G's pencil as **shorter** i.e observer H, on the rocket, sees lengths **contracted** on Earth from **their** reference frame



A stationary observer in their own reference frame views lengths as shorter in the moving reference frame

Length Contraction Equation

- The length of an object is the difference in the position of its ends
- Consider the observers G and H measuring the length of a pencil, which is stationary in reference frame S' (for observer H)



Observer G measures the length of the pencil to be different to observer H

- As observer H is in the moving frame S', they measure the length of the ruler as:

$$\Delta x' = x_2' - x_1' = L_0$$

- This is the **proper length, L_0** as the pencil is **not** moving relative to observer H
  - Both the pencil and observer H are, however, moving relative to observer G
- Observer G needs to measure the length of the pencil by measuring the position of its ends at the same time (just like observer H did)
- They measure the length of the ruler to be:

$$\Delta x = x_2 - x_1 = L$$

- This is the **observed length, *L*** as the pencil **is** moving relative to observer G
  - Moving between different reference frames tell us how the *x* and *x'* are related
- We want to find Δx, the length measured in the reference frame of the stationary observer on Earth (G), who is moving **relative** to the observer on the rocket (H)
- Transforming these distances gives:

$$x_1' = \gamma(x_1 - vt)$$

$$x_2' = \gamma(x_2 - vt)$$

- These are then substituted into the equation for the proper length, L_0:

$$x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$$L_0 = \gamma L$$

- Therefore:

$$L = \frac{L_0}{\gamma}$$

- Where:
  - L = the length measured by an observer **moving relative** to the length being measured (m)
  - L_0 = the **proper** length (m)

- As $\gamma > 1$, this means that the $L < L_0$
  - In other words, lengths measured from a reference frame moving **relative** to the object will be measured as shorter than the lengths measured at rest from within their frame of reference

- Similar to time dilation, length contraction is also due to Einstein's second postulate
  - Both observers G and H **must** measure the speed of light to be c
  - Since the time for observer H will run slower, according to observer G (i.e. *t* increases), then for c to stay the same, the length of the object, *L* must **decrease**

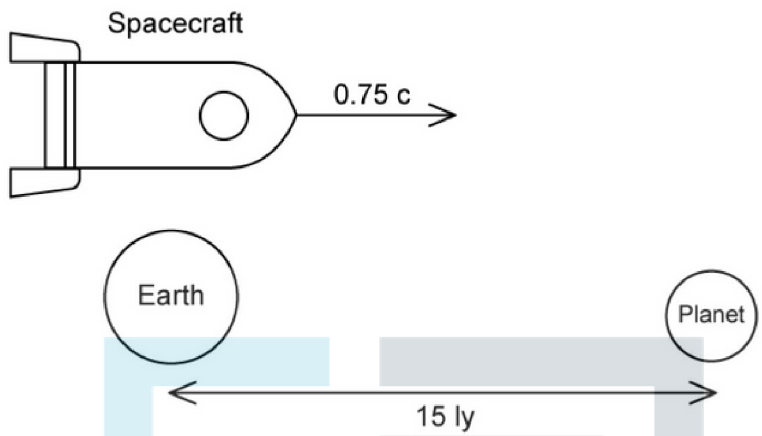
- It is important to note that the length has been measured at the same **time**
  - This length is the difference between the ends of the pencil, with both ends measured at the same time

- The ruler used in both reference frames is **stationary** in their **own** reference frame
  - Otherwise, observer G would see the ruler on observer H's rocket contracting as well and wouldn't measure any difference in length

Worked Example

A spacecraft leaves Earth and moves towards a planet.

The spacecraft moves at a speed of 0.75c relative to the Earth. The planet is a distance of 15 ly away according to the observer on Earth.



The spacecraft passes a space station that is at rest relative to the Earth. The proper length of the space station is 482 m.

Calculate the length of the space station according to the observer in the spacecraft.

Answer:

Step 1: List the known quantities

- Speed of the spacecraft, $v = 0.75c$
- Proper length of the space station, $L_0 = 482$ m

Step 2: Analyse the situation

- We are trying to find the length of the space station in the reference frame of the observer in the **spacecraft**
- In this observer's reference frame, it is the **space station** that is moving away from them at 0.75c
- Therefore, we a measuring a length in the moving reference frame (relative to the spacecraft) - this is the length, ***L***

Step 3: Substitute values into the length contraction equation

$$L = \frac{L_0}{\gamma} = \frac{L_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$L = \frac{482}{\dfrac{1}{\sqrt{1 - \dfrac{(0.75c)^2}{c^2}}}} = \frac{482}{\dfrac{1}{\sqrt{1 - (0.75)^2}}} = 319 \text{ m}$$

Step 4: Check whether your answer makes sense

- As the observer in the spacecraft is stationary, the length of the space station they measure should be **shorter** than the proper length
- As the length recorded from the spacecraft is 319 years, and the proper length is 482 m, this length makes sense

Exam Tip

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

The notion of 'proper length' is incredibly important here, as it depends on the reference frame the length is being measured from.

You will find in some exam questions you can use time dilation **or** length contraction, you will receive marks for either way.

12.3.8 Relativistic Mass

Equivalence of Mass & Energy

- Einstein showed in a1905 paper that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$E = mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Scenarios where mass is converted to energy or energy to mass include:
  - The **fusion** of hydrogen into helium in the centre of the Sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

Relativistic Mass

- An object's proper mass (that is, its mass measured by an observer at rest relative to the object) is also called its **rest mass**, m_0
- An observer in an inertial reference frame moving at speed v **relative** to the object measures the object's mass as m, which is given by:

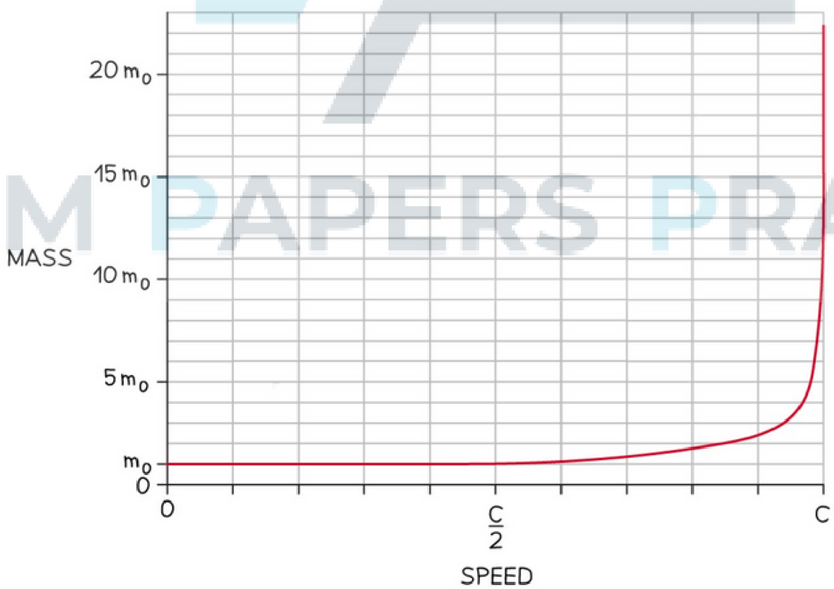
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- This can also be written as:

$$m = \gamma m_0$$

- Because $\gamma > 1$, when an object is moving relative to your frame of reference, you measure its mass as **larger** than it's rest mass
  - The derivation for this relativistic mass is a good deal more complex than the derivation for relativistic time and length and is not examinable so it is omitted from this revision note
- One interesting outcome of this relativistic mass is the **speed limit** imposed on objects
  - As an object's speed nears the speed of light, it's mass gets larger and larger
  - Very close to the speed of light, an object's mass tends towards an **infinite** mass

Graph showing how relativistic mass changes with velocity



The effect of speed, as a fraction of c, on an object's mass, in multiples of rest mass. Mass increases rapidly as the object's speed approaches c.

Relativistic Energy

- Let's reconsider Einstein's famous equation, taking relativistic effects into account:

$$E = (\gamma m_0)c^2$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- The mass of a stationary object is its **rest mass**, m_0, so it has a **rest energy** of $E = m_0c^2$

Worked Example

A particle in a particle accelerator has a rest mass of M_0 and is accelerated to a maximum speed of 0.998c.

Calculate the total energy of the particle at its maximum speed, as a multiple of its proper energy, from the reference frame of an observer who is stationary relative to the accelerator.

Answer:

Step 1: List the known quantities:

- Maximum speed, $v = 0.998c$

Step 2: Write down the relevant equations:

- Rest energy, $E_0 = m_0c^2$
- Relativistic energy, $E = \gamma m_0c^2$
- Write the equation with the gamma term out in full

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Step 3: Substitute the known quantities:

- The relativistic energy in terms of rest mass M_0 is

$$E = \frac{M_0 c^2}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}} = \frac{M_0 c^2}{\sqrt{1 - 0.998^2}} = 15.8 M_0 c^2$$

Step 4: Give the answer as a multiple of proper energy:

- Proper energy is another way of saying rest energy, which is M_0c^2
- The particle's energy is **15.8** times larger than the rest energy, according to the observer

Exam Tip

When you see a speed and a mass, it's tempting to go into auto-pilot and stick everything into the equation $E_k = ½mv^2$, but this would gain you no marks in this worked example! The trick is to look at how large the speed is, if it is close to the speed of light, the relativistic effects are significant and you need to account for them. The context of the question can also provide clues, such as mentioning frames of reference or rest mass.

12.3.9 Relativistic Energy

Relativistic Energy

Relativistic Kinetic Energy

- We know that an object in motion relative to an observer has energy $E = (\gamma m_0)c^2$ and its rest energy is $E_0 = m_0c^2$
  - The kinetic store is the energy store associated with motion
  - If an object in motion has a total energy E which is greater than its rest mass E_0, the additional energy must be in the kinetic store

$$E_k = mc^2 - m_0c^2$$

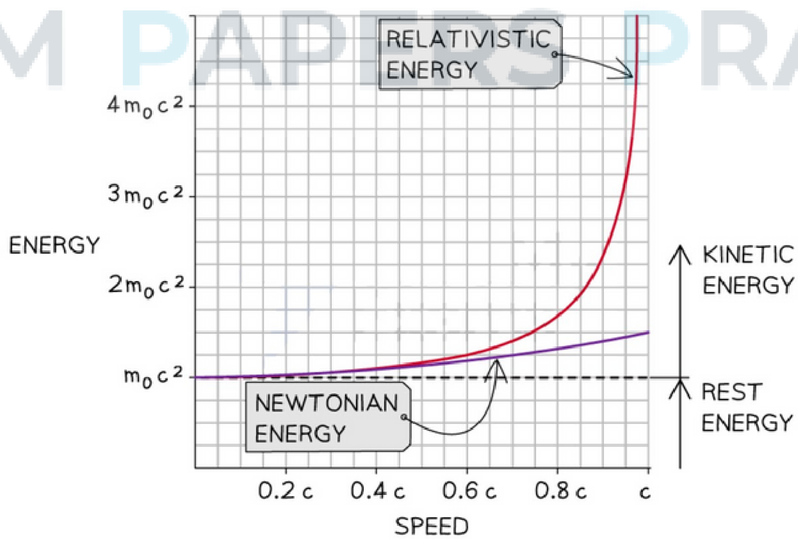
- Where:
  - E_k is energy in the kinetic store
  - m is relativistic mass
  - m_0 is the object's rest mass
  - c is the speed of light
- This can be rearranged for an expression for the total energy, E, of a particle:

$$E = mc^2$$

$$E = m_0c^2 + E_k$$

- The total energy of an object increases at a rapidly increasing rate as it's speed approaches the speed of light

Graph showing how total energy increases as an object's speed approaches the speed of light



- As with mass on the previous page, the asymptote on this graph shows that an infinite amount of energy is needed to make an object with mass reach the speed of light

Worked Example

A spacecraft with a proper mass of 450 kg accelerates to a speed of 0.7c. Calculate the difference in its relativistic kinetic energy and its kinetic energy calculated using Newtonian physics.

Answer:

Step 1: List the known quantities:

- Rest mass, $m_0 = 450$ kg
- Speed, $v = 0.7c$
- Speed of light, $c = 3.0 \times 10^8$ ms^{-1}

Step 2: List the relevant equations:

- Newtonian kinetic energy, $E_{kn} = ½m_0v^2$
- Relativistic kinetic energy, $E_{kr} = \gamma m_0c^2 - m_0c^2$

Step 3: Calculate the Newtonian kinetic energy:

- Substitute the known quantities:

$$E_{kn} = \frac{1}{2} \times 450 \times (0.7 \times (3.0 \times 10^8))^2 = 9.9 \times 10^{18} \text{ J}$$

Step 4: Calculate the gamma term:

- Substitute the speed into the gamma term:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.7c)^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.7^2}} = 1.40$$

Step 5: Substitute the gamma term into the relativistic kinetic energy equation:

- Substitute the rest mass, speed of light and speed as well:

$$E_{kr} = \gamma m_0c^2 - m_0c^2 = m_0c^2(\gamma - 1)$$

$$E_{kr} = 450 \times (3.0 \times 10^8)^2 \times (1.40 - 1) = 1.6 \times 10^{19} \text{ J}$$

Step 6: Calculate the difference between the two kinetic energies:

- Subtract the smaller Newtonian kinetic energy from the larger relativistic energy

$$\Delta E_k = (1.6 \times 10^{19}) - (9.9 \times 10^{18}) = 6.1 \times 10^{18} \text{ J}$$

- As you can see, the relativistic kinetic energy is almost **twice** as large as the Newtonian kinetic energy!

12.3.10 Bertozzi's Experiment

Bertozzi's Experiment

How do you Measure Kinetic Energy?

- When accelerating a particle with charge q from rest, the work done by an electric field with potential difference V is equal to the energy gained by the particle's kinetic store, E_k:

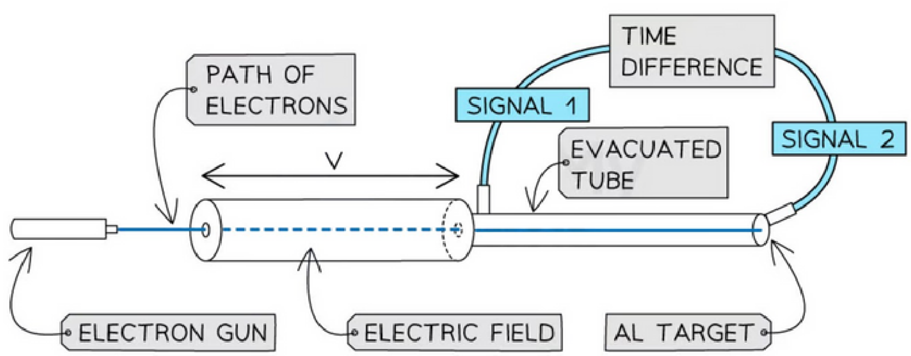
$$qV = E_k$$

- This gives a way of **measuring** the kinetic energy of a particle of known charge, such as an electron

What was Bertozzi's Experiment?

- Bertozzi accelerated electrons to speeds close to the speed of light and measured their kinetic energies
  - The aim of the experiment was to plot the squared speeds of electrons against their kinetic energies and compare the results to Newtonian and relativistic predictions
- A strong electric field was used to transfer energy to the kinetic store of the electrons
  - This meant that, for an electric field of potential difference V, each electron had a kinetic energy of eV, where e is the magnitude of charge on an electron
  - These electrons were fired at an aluminium target
- The speed of the electrons was simply calculated using distance divided by time
  - Once the electrons left the electric field, they travelled at constant speed, as no forces acted
  - A signal after the electric field and a signal attached to the aluminium target were used to measure a time interval using an oscilloscope
  - The distance was measured between the two signals and speed was calculated

A diagram outlining the experimental setup of Bertozzi's electron experiment

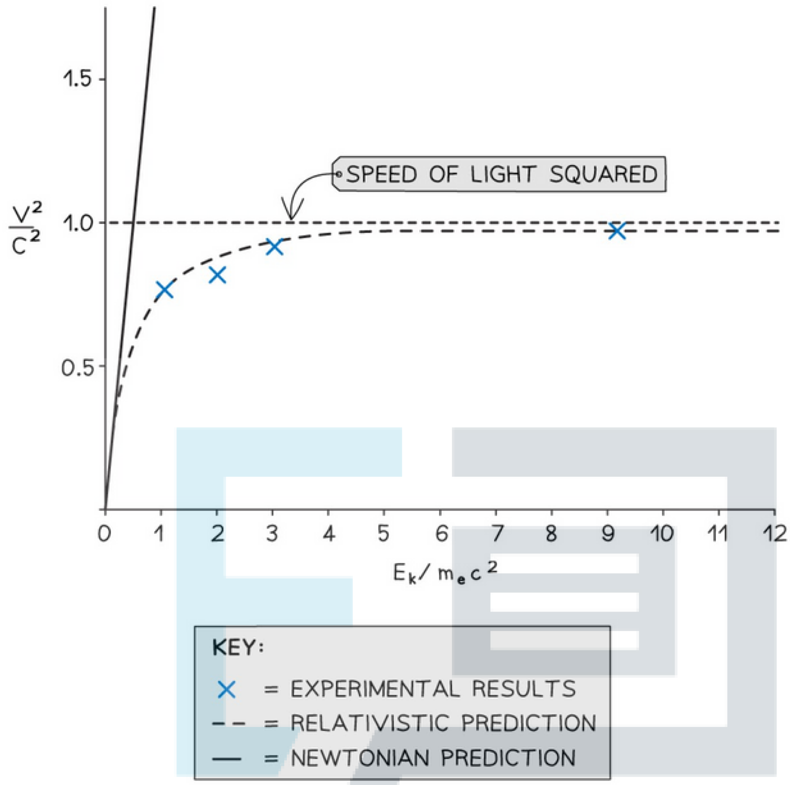


Electrons are accelerated by an electric field and trigger a signal once they have left the field and trigger another when they hit the aluminium target, allowing speed to be calculated

The Results

- With several values of speed-squared and kinetic energy, Bertozzi produced a graph showing the following results

A graph showing speed-squared against kinetic energies



This graph shows the kinetic energy of the electrons, with units of m_ec^2, plotted against their squared speeds, with units of c^2. The Bertozzi results agree closely with the relativistic predictions of speeds and show electrons cannot exceed the speed of light.

- This experiment's data agreed with the relativistic predictions far more than the Newtonian predictions

Verifying the Kinetic Energy of the Electrons

- To avoid any doubts about whether the electrons actually did have a kinetic energy equal to eV, Bertozzi and his team measured the kinetic energy of the electrons **directly** as they hit the aluminium target
- They did this by measuring the temperature change of the target and using the equation $\Delta E = mc\Delta\theta$
  - This energy change is equal to the total kinetic energy lost by the incoming electrons
  - The total charge transferred to the target was used to calculate the number of electrons which hit the target
- With total energy transferred to the target and the number of electrons known, the team could calculate the kinetic energy per electron
  - These values were in agreement with the energy transferred to the electrons by the electric field, showing the results were valid and the electrons behaved relativistically

at speeds close to c

Worked Example

In a repeat of the Bertozzi experiment, electrons are fired through an evacuated tube at a 0.0250 kg aluminium plate. After the experiment has run for a time, the target has accumulated –6.20 μC of charge and has increased in temperature by 1.80 °C.

Calculate the kinetic energy per electron in units of MeV.

Specific heat capacity of aluminium = 902 J kg^{-1} K^{-1}

Answer:

Step 1: List the known quantities

- Mass of aluminium, m = 0.0250 kg
- Specific heat capacity of aluminium, c = 902 J kg^{-1} K^{-1}
- Change in temperature, $\Delta\theta$ = 1.80 °C
- Charge transferred to aluminium plate, Q = –6.20 μC
- Charge of an electron, e = -1.60×10^{-19} C

Step 2: Write out the relevant equation

- Thermal energy transferred to aluminium, $\Delta E = mc\Delta\theta$

Step 3: Calculate the energy transferred to the aluminium's thermal store

- The thermal energy gained by the plate is:

$$\Delta E = mc\Delta\theta = 0.0250 \times 902 \times 1.80 = 40.59 \text{ J}$$

Step 4: Calculate the number of electrons incident on the aluminium plate

- Each electron transfers a charge of e to the plate
- The total charge transferred to the plate divided by e will give the number of electrons, n, incident on the plate:

$$n = \frac{Q}{e} = \frac{-6.20 \times 10^{-6}}{-1.60 \times 10^{-19}} = 3.875 \times 10^{13}$$

Step 5: Calculate the kinetic energy per electron

- The total thermal energy gained by the plate is equal to the total kinetic energy, $E_{k,tot}$, of all the electrons hitting it
- $E_{k,tot}$ divided by the number of electrons gives the kinetic energy per electron, E_k:

$$E_k = \frac{E_{k,tot}}{n} = \frac{\Delta E}{n} = \frac{40.59}{3.875 \times 10^{13}} = 1.0475 \times 10^{-12} \text{ J}$$

Step 6: Convert the kinetic energy per electron into mega-electronvolts

- Recall that 1 eV is equal to 1.60×10^{-19} J:

$$E_k = \frac{1.0475 \times 10^{-12}}{1.60 \times 10^{-19}} = 6.547 \times 10^6 \text{ eV}$$

- Finally, the answer must be to 3 significant figures, as all the figures in the question were given to this number:

$$E_k = 6.55 \text{ MeV}$$

Exam Tip

You may be asked on either method of determining the kinetic energy of each electron so expect to see the thermal energy equation and work done by an electric field in this topic.