11.3 Capacitance

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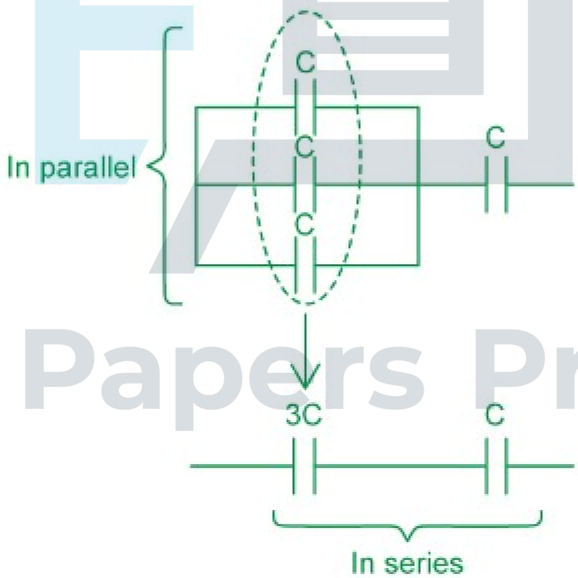
Course	DP IB Physics
Section	11. Electromagnetic Induction (HL only)
Topic	11.3 Capacitance
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **B** because:

- The equation for capacitors in series is given by:
  - $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$
- The equation for capacitors in parallel is given by:
  - $C_{parallel} = C_1 + C_2 + ...$
- Consider option **B**:
  - Total capacitance of the 3 capacitors in parallel is $C + C + C = 3C$
  - Combining this with the 1 capacitor in series is $\frac{1}{\frac{1}{3C} + \frac{1}{C}} = \frac{1}{\frac{4}{3}C} = \frac{3C}{4}$ (0.75 C)



- Therefore, option **B** is correct

A is incorrect as	total capacitance of the 2 capacitors in series is $\frac{1}{\frac{1}{C} + \frac{1}{C}} = \frac{1}{\frac{2}{C}} = \frac{C}{2}$ Adding this to the two capacitors in parallel gives: $\frac{C}{2} + C + C = \frac{5}{2}C$ (2.5 C)

C is incorrect as	total capacitance of the 4 capacitors in parallel is $C+C+C+C = 4C$
D is incorrect as	total capacitance of the 4 capacitors in series is $\frac{1}{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\frac{1}{C}} = \frac{1}{\frac{4}{C}} = \frac{C}{4}$ (0.25 C)

This is one of those MCQs where you may need to try each scenario till you find the correct answer. However, you will be able to eyeball which options are definitely wrong. For example, the capacitors in parallel will add up to 4C, and this is quick to calculate, so this can be eliminated early. It is also important in a non-calculator paper that you are comfortable with calculations with fractions (especially dividing and adding in this case).

When adding fractions, the denominators must be equal and the numerators are then added. Therefore: $\frac{C}{2} + C = \frac{C}{2} + \frac{2C}{2} = \frac{3C}{2}$

The way the total capacitance of capacitors connected in series and parallel is the **opposite** way around to how the total resistance of resistors is calculated.

For resistors:

- $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
- $R_{series} = R_1 + R_2 + \ldots$

For capacitors:

- $C_{parallel} = C_1 + C_2 + \ldots$
- $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

2

The correct answer is **C** because:

- The energy stored in a capacitor is given by:
  - $Energy\ stored\ = \frac{1}{2}CV^2$
- The capacitance C of a parallel plate capacitor is given by:
  - $C\ = \frac{\varepsilon A}{d}$
- Substituting the capacitance equation into the energy stored equation gives:
  - $Energy\ stored = \frac{1}{2}\left(\frac{\varepsilon A}{d}\right)V^2$
- The electric field strength E between two parallel plates is given by:
  - $E\ = \frac{V}{d}$
  - Where rearranging for the potential difference V is $V = Ed$
- Substituting the potential difference V equation into the energy stored equation gives:
  - $Energy\ stored\ = \frac{1}{2}\left(\frac{\varepsilon A}{d}\right)(Ed)^2 = \frac{1}{2}\left(\frac{\varepsilon A}{d}\right)E^2d^2$
- Simplifying this gives the energy stored as:
  - $Energy\ stored\ = \frac{1}{2}\frac{\varepsilon A}{d}\ E^2d^2 = \frac{1}{2}\varepsilon AE^2d$

This question is mainly algebra, so be very careful when simplifying so you don't get caught out with the wrong answer. Make sure to think about other topics that are often merged with the one that is being examined. In this case, capacitance and electric fields are commonly merged together. This means you will sometimes have to used electric field equations such as $E = \frac{V}{d}$ which, remember, is not given on your data booklet!

3

The correct answer is **D** because:

- Capacitance, C is defined by the equation:
  - $C = \frac{Q}{V}$
  - Where Q is the charge on the pixel and V is the potential difference of the pixel
- Rearranging for the potential difference V gives:
  - $V = \frac{Q}{C}$
- The charge Q is the total number of electrons:
  - Each electron has a charge of 1.60×10^{-19} C
  - Total charge, $Q = (1.60 \times 10^{-19}) \times (10^6) = 1.60 \times 10^{-13}$ C
- The capacitance C is given by 6.4 pF:
  - This is equal to 6.4×10^{-12} F
- Substituting these into the potential difference equation gives:
  - $V = \frac{1.60 \times 10^{-13}}{6.4 \times 10^{-12}}$
  - $V = 0.25 \times 10^{-1} = 0.025$ V

Especially in a non-calculator paper, it is important that you're comfortable with combining powers of 10. The general rules are:

- If powers of 10 are **multiplied**, **add** the powers: $10^a \times 10^b = 10^{a+b}$
- If power of 10 are **divided**, **subtract** the powers: $\frac{10^a}{10^b} = 10^{a-b}$

4

The correct answer is **B** because:

- Since the capacitor is connected to a cell of constant emf, no matter what the separation is between the plates, the potential difference V between the plates will always be **constant**
- The capacitance C of parallel plate capacitors is given by:
  - $C = \frac{\varepsilon A}{d}$

- Where ε is the permittivity of the dielectric between the plates, A is the cross-sectional area of the plates and d is the separation of the plates
- Therefore, capacitance C and separation d are **inversely proportional**
  - This means if the separation d **doubles**, then capacitance C **halves**
  - This eliminates row **C**
- The capacitance C of a capacitor is also given by:
  - $C = \frac{Q}{V}$
  - Where Q is the charge on the capacitor plates and V is the potential difference between the plates
- Rearranging for charge Q gives:
  - $Q = CV$
- Therefore, if the capacitance C **halves**, then charge Q also **halves**
  - This eliminates row **D**
- The energy stored E in a capacitor is given by the equation:
  - $E = \frac{1}{2}CV^2$
- Therefore, if the capacitance C **halves**, then then energy stored E also **halves**
  - This is row **B**

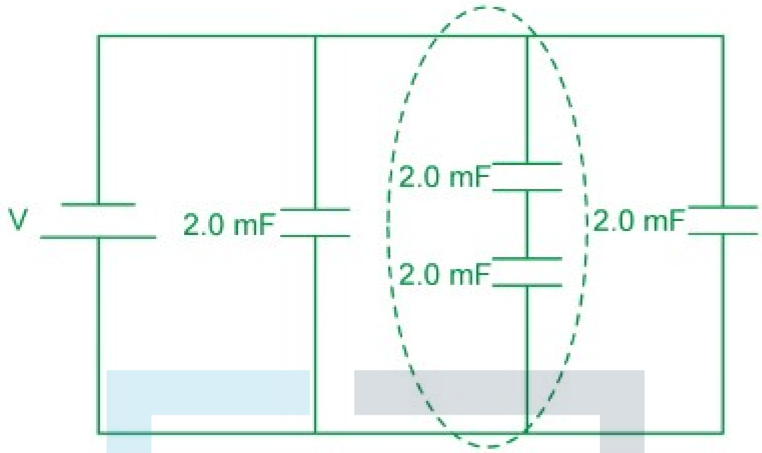
The important thing to notice in any MCQ testing how a variable changes with the separation of the plates is that if the capacitor is connected to a power supply such as a cell, the potential difference across it will always be the **same** as that of the emf **regardless of the separation of the plates**. Therefore, when comparing two variables in the equations, V is always constant.

5

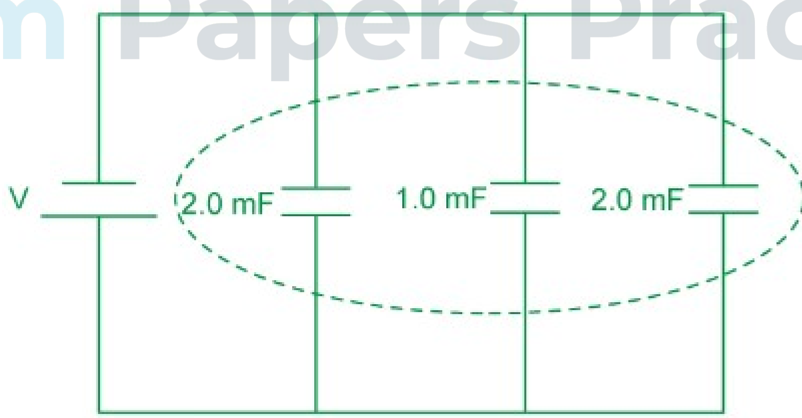
The correct answer is **C** because:

- The equation for capacitors in series is given by:
  - $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

- The equation for capacitors in parallel is given by:
  - $C_{parallel} = C_1 + C_2 + \ldots$
- Combining the two resistors in series gives:



  - $\frac{1}{C_{series}} = \frac{1}{2.0} + \frac{1}{2.0} = 1.0$
  - $C_{series} = 1.0$ mF
- Combining the three resistors now in parallel gives:



- $C_{parallel} = 2.0 + 1.0 + 2.0 = 5.0$ mF
- The energy stored E in the circuit is given by:
  - $E = \frac{1}{2}CV^2$
- Rearranging this for the potential difference V gives:
  - $V = \sqrt{\frac{2E}{C}}$
- Substituting in the values gives:
  - $V = \sqrt{\frac{2 \times 100}{5.0 \times 10^{-3}}} = \sqrt{\frac{200}{5.0 \times 10^{-3}}} = \sqrt{\frac{40}{1.0 \times 10^{-3}}} = \sqrt{4.0 \times 10^4}$
  - $V = 2.0 \times 10^2$ V = 200 V = 0.2 kV

When combining multiple capacitors, you can treat the final value as a single capacitor with that capacitance. In this case, the two capacitors in series combine to give a capacitance of 1 mF, which can then be treated like a capacitor of 1 mF in parallel with the other two. Any sketches can also help you keep track of the ones you have combined.

The way the total capacitance of capacitors connected in series and parallel is the **opposite** way around to how the total resistance of resistors is calculated.

For resistors:

- $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
- $R_{series} = R_1 + R_2 + \ldots$

For capacitors:

- $C_{parallel} = C_1 + C_2 + \ldots$
- $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

6

The correct answer is **A** because:

- The charge Q is given by:
  - $Q = It$
  - Where I is the current and t is the time
- Converting time t into seconds gives:
  - 1 minute = 60 s
  - $t = 2 \times 60 = 120$ s
- Substituting the values into the charge equation gives:
  - $Q = (5.0 \times 10^{-6}) \times 120 = 600 \times 10^{-6}$ C
- The capacitance C is given by:
  - $C = \frac{Q}{V}$
  - Where V is the potential difference across the capacitor plates
- Substituting the values into the capacitance equation gives:
  - $C = \frac{600 \times 10^{-6}}{10} = 600 \times 10^{-7}$ F
  - $C = 60 \times 10^{-6}$ F = 60 μF

B is incorrect as	the charge Q has not been divided by V of 10 V to give the final capacitance C
C is incorrect as	the charge Q has not been divided by V of 10 V to give the final capacitance C and the 2 minutes has not been converted into seconds
D is incorrect as	the 2 minutes has not been converted into seconds

Remember to look out for the correct unit conversions when doing calculations, these trip up many students into choosing the incorrect distractor answer. For the equation $Q = It$, current I must be in **amps** and time t must be in **seconds**.

7

The correct answer is **D** because:

- Each capacitor can be said to have a capacitance C
- The equation for capacitors in series is given by:
  - $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
- The equation for capacitors in parallel is given by:
  - $C_{parallel} = C_1 + C_2 + \ldots$
- The capacitance C is given by:
  - $C = \frac{Q}{V}$
  - Therefore, $Q = CV$
- When connected in parallel, the total capacitance is:
  - $C_{parallel} = C + C + C + C = 4C$
- Therefore, the charge Q in parallel is:
  - $Q = 4CV$
  - 2.4 μC = 4 CV
  - $CV = \frac{2.4}{4}$ = 0.6 μC
- When connected in series, the total capacitance is:
  - $\frac{1}{C_{series}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{4}{C}$
  - $C_{series} = \frac{1}{\frac{4}{C}} = \frac{C}{4}$
- Therefore, the charge q in series is:
  - $q = \frac{CV}{4}$
- This means that the charge q in series is a **quarter** of the charge Q in parallel
- Therefore, substituting in the initial value of the charge gives:
  - $q = \frac{0.6}{4} = \frac{6}{40} = \frac{3}{20}$ μC

A is incorrect as	the capacitors in series and parallel equations have been used the wrong way around and the wrong prefix has been used (mC instead of μC)
B is incorrect as	the capacitors in series and parallel equations have been used the wrong way around
C is incorrect as	the wrong prefix has been used (mC instead of μC)

The way the total capacitance of capacitors connected in series and parallel is the **opposite** way around to how the total resistance of resistors is calculated.

For resistors:

- $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
- $R_{series} = R_1 + R_2 + \ldots$

For capacitors:

- $C_{parallel} = C_1 + C_2 + \ldots$
- $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

8

The correct answer is **B** because:

- The equation for capacitors in series is given by:
  - $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
- Substituting the capacitance of each capacitor gives:
  - $\frac{1}{C_{series}} = \frac{1}{6} + \frac{1}{3} + \frac{1}{4}$
  - $\frac{1}{C_{series}} = \frac{4}{24} + \frac{8}{24} + \frac{6}{24} = \frac{18}{24} = \frac{3}{4}$
  - $C_{series} = \frac{4}{3}$ μF

A is incorrect as	the capacitors in parallel equations has been used instead of the capacitors in series equation
C is incorrect as	$\frac{1}{C_{series}}$ has been calculated instead of just C_{series}
D is incorrect as	this is $\frac{1}{C_{parallel}}$ which is not the correct equation for capacitors in parallel, and the question states that the capacitors are in series

When adding fractions, the denominators must be equal and the numerators are then added. Therefore:

$$\frac{C}{2} + C = \frac{C}{2} + \frac{2C}{2} = \frac{3C}{2}$$

The way the total capacitance of capacitors connected in series and parallel is the **opposite** way around to how the total resistance of resistors is calculated.

For resistors:

- $R_{series} = R_1 + R_2 + \ldots$

For capacitors:

- $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

9

The correct answer is **D** because:

- In a series circuit:
  - The potential difference is **divided** between all the components in the circuit
  - The current is the **same** in all the components in the circuit
- Charge Q is defined by:
  - $Q = It$
  - Where I is the current in the circuit and t is the time

- Therefore, if each capacitor has the same **current** then it must also store the same **charge** on its plates, regardless of its capacitance:
  - This is because the charge stored by a plate of any one capacitor must have come from the plate of its adjacent capacitor
  - This eliminates row **A** and **C**

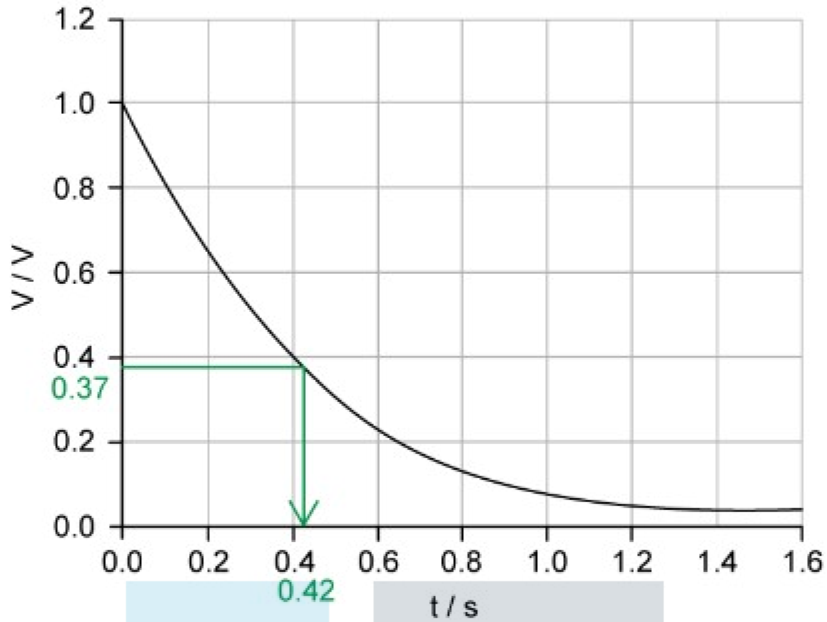
- Capacitance C is defined by:
  - $C = \frac{Q}{V}$
  - Where V is the potential difference between the plates
- Therefore, if Q is constant, then the potential difference V of each capacitor **depends on** the capacitance C of each capacitor:
  - More specifically, C and V are inversely proportional
  - Therefore, row **D** is correct

It is important to not forget all that you have learnt in the electricity topic when it comes to capacitance. At the end of the day, capacitors are another electrical component and belong in circuits. Therefore, it is very useful to still remember Kirchhoff's circuit laws in this topic.

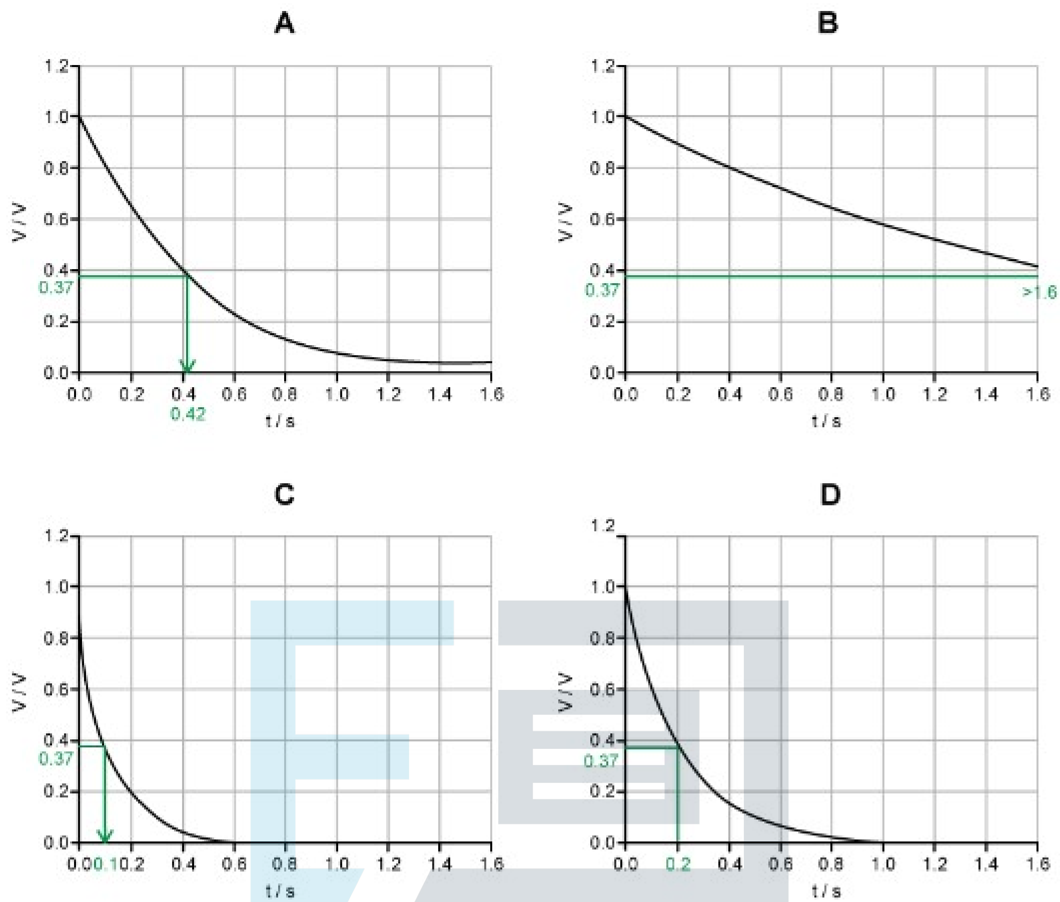
10

The correct answer is **C** because:

- How fast a capacitor discharges is determined by its time constant, τ
- The time constant τ is defined by:
  - $\tau = RC$
  - Where R is the resistance of the resistor and C is the capacitance of the capacitor
- It is also defined as the time taken for the potential difference to fall to $\frac{1}{e}$ (0.37) of its original value
- Therefore, from the graph this is:
  - voltage $V = (1.0 \times 0.37) = 0.37$ V
  - $\tau \approx 0.42$ s



- If the capacitance C changes to $\frac{C}{2}$ and the resistance R changes to $\frac{R}{2}$ then the τ changes to:
  - $\frac{C}{2} \times \frac{R}{2} = \frac{CR}{4}$
- Therefore, the time constant τ becomes $\frac{\tau}{4}$
  - If the time constant of the new combination is a quarter of the previous arrangement, then the capacitor discharges 4 times quicker
- This means the potential difference V now reduces to $\frac{1}{e}$ (0.37) of its original value in a **quarter** of the time
  - This is in $\frac{0.42}{4}$ = 0.105 s ($\frac{0.40}{4}$ = 0.10 s, and $\frac{0.44}{4}$ = 0.11 s)
  - The closest graph with this time constant is graph **C**



It is important to not just remember the equation for the time constant, but also its **definition**. The graph in the question has values on its axes, which indicates that these should be used to help you choose the most accurate graph. Always use a ruler and pencil to make sure you're reading the correct values from the graph axes, you are encouraged to annotate your graphs in the exam.