# 11.2 Wave-Particle Duality


# Turning Points in Physics

# AQA A Level Revision Notes

# A Level Physics AQA

## 12.2 Wave-Particle Duality

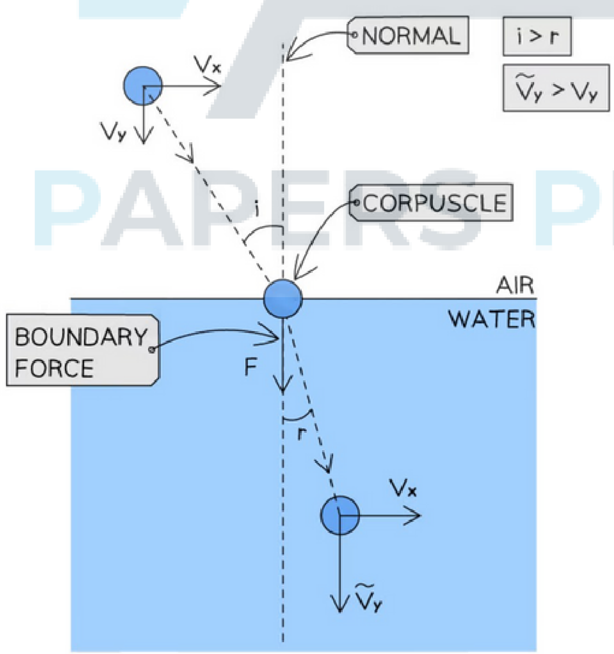
CONTENTS

12.2.1 Theories of Light

## Newton's Corpuscular Theory of Light

- Newton proposed that light was made of small particle-like bodies called **corpuscles**, emitted by luminous objects
  - One prediction of this theory was that objects emitting light were losing mass slowly
  - This theory was able to explain **reflection**, **refraction** and **dispersion**, but not **diffraction**
- To explain reflection:
  - The corpuscles simply hit the reflective surface and experienced an equal and opposite repulsive force from the surface, following Newton's third law
  - This is because corpuscular theory treated corpuscles like solid, elastic spheres
- To explain refraction:
  - Corpuscular theory assumed there was a force of attraction between light and matter
  - In a single medium, such as air, the force supposedly acted on all sides so there was no resultant force
  - But at a boundary between air and a denser medium, Newton said there was a resultant force on the corpuscles acting perpendicular to the boundary, because there was more matter in the new medium
  - A consequence of this was that light travelled **faster** in a more dense medium

A diagram showing Newton's corpuscular explanation for refraction



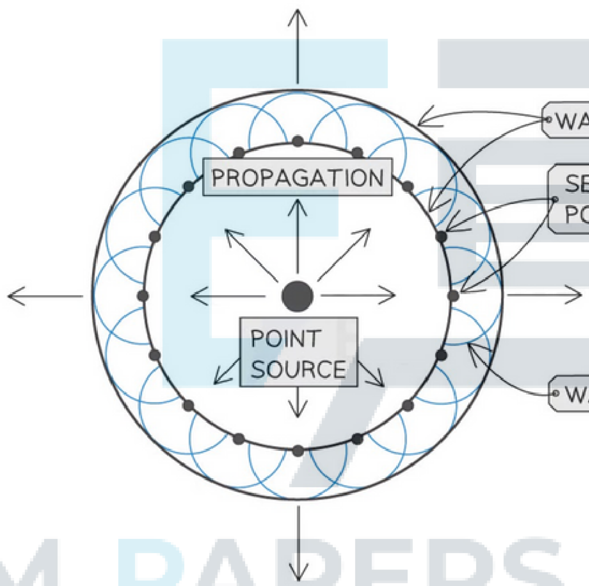

***The boundary force, arising from an attraction to a greater amount of matter, increases the vertical component of velocity, which changes the angle of the corpuscle's path – this means the magnitude of the velocity is greater in water***

# Comparing Corpuscular & Wave Theories

## Huygens' Wave Theory of Light

- Huygens was a Dutch scientist who proposed that light was a wave
  - All other known waves travelled through a medium so he suggested the Universe was filled with a massless medium known as the "luminiferous aether" (more on this in Special Relativity)
- In his theory, light travelled in **wavefronts**
  - These wavefronts were emitted from a point source
  - Any point of the wavefront then acted as a secondary point source, from which **wavelets** could propagate
  - These wavelets joined together to form a new wavefront, and so on

Diagram showing the propagation of wavefronts




***Any point on a wavefront can act as a secondary point source for wavelets – only a few are shown here***

- To explain reflection:
  - When a wavefront hits a reflective surface, the point of reflection becomes a secondary point source for new wavelets
  - Different parts of the wavefront hit the reflective surface at different times, so the new wavefront forms in a new direction
- To explain refraction:
  - This theory, in contrast to Newton's corpuscular theory, relied on light travelling **slower** in a denser medium
  - Again, different parts of the wavefront hit the boundary to a new medium at different times
  - The part of the wavefront which first reaches the boundary slows before the rest of the wavefront, causing it to **change** direction

## Comparing the two Theories

- Similarities between the two theories:
  - Both explained reflection
  - Both explained refraction
  - Both could explain dispersion
- Differences between the two theories:
  - Corpuscular theory said light was composed of particles with mass, while wave theory said it was a wave travelling through a massless medium
  - Corpuscular theory claimed light travelled **faster** in **denser** media, whereas wave theory claimed light travelled **slower** in **denser** media
  - Corpuscular theory had no explanation for diffraction or interference, however, these were common properties of waves

## Why was Newton's theory more accepted than Huygens'?

- Both theories explained the phenomena of light, but both also had flaws
- Newton was already widely respected thanks to his work on motion and gravity
- There was no way of measuring the speed of light or observing diffraction of light at the time, so corpuscular theory was the accepted theory of light for 150 years
  - This changed when diffraction patterns of light were observed that contradicted this theory

### Exam Tip

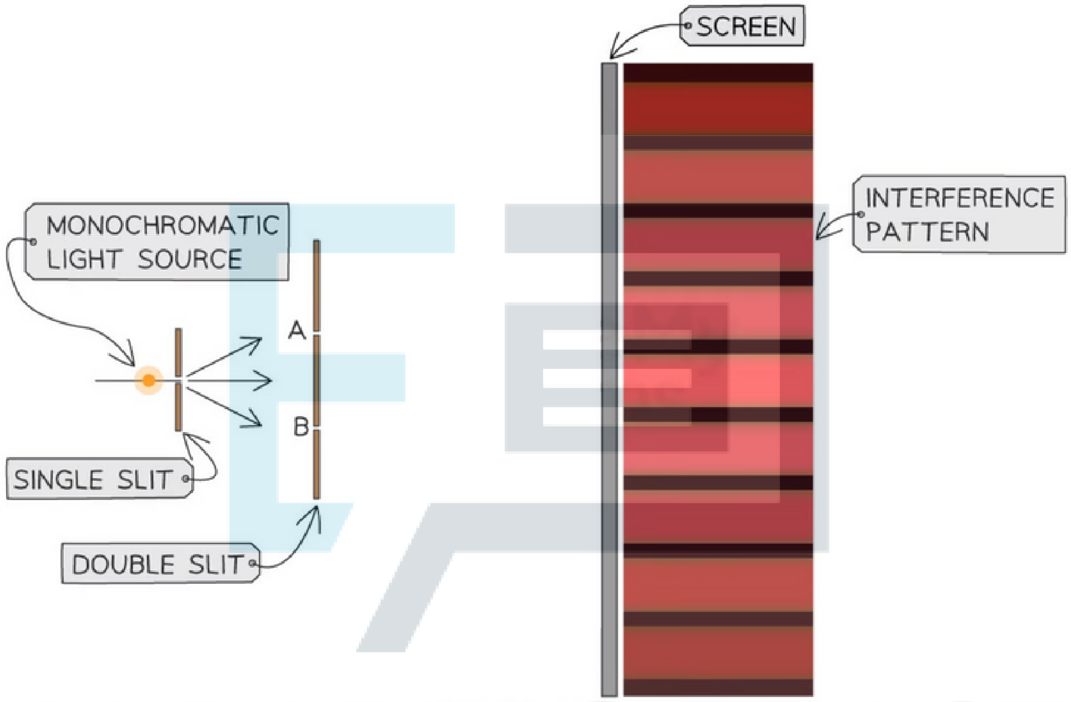
This topic often features a lot of comparison questions - the key things to remember here are the explanations of reflection and refraction, the main differences between the theories and why Huygens' theory wasn't as popular.

12.2.2 Young's Double Slit Interference

## Explanation of Double Slit Interference

- Young's double slit experiment demonstrates how light waves produced an interference pattern
- The experimental setup and results are shown below

### Young's double slit experiment and the resulting interference pattern

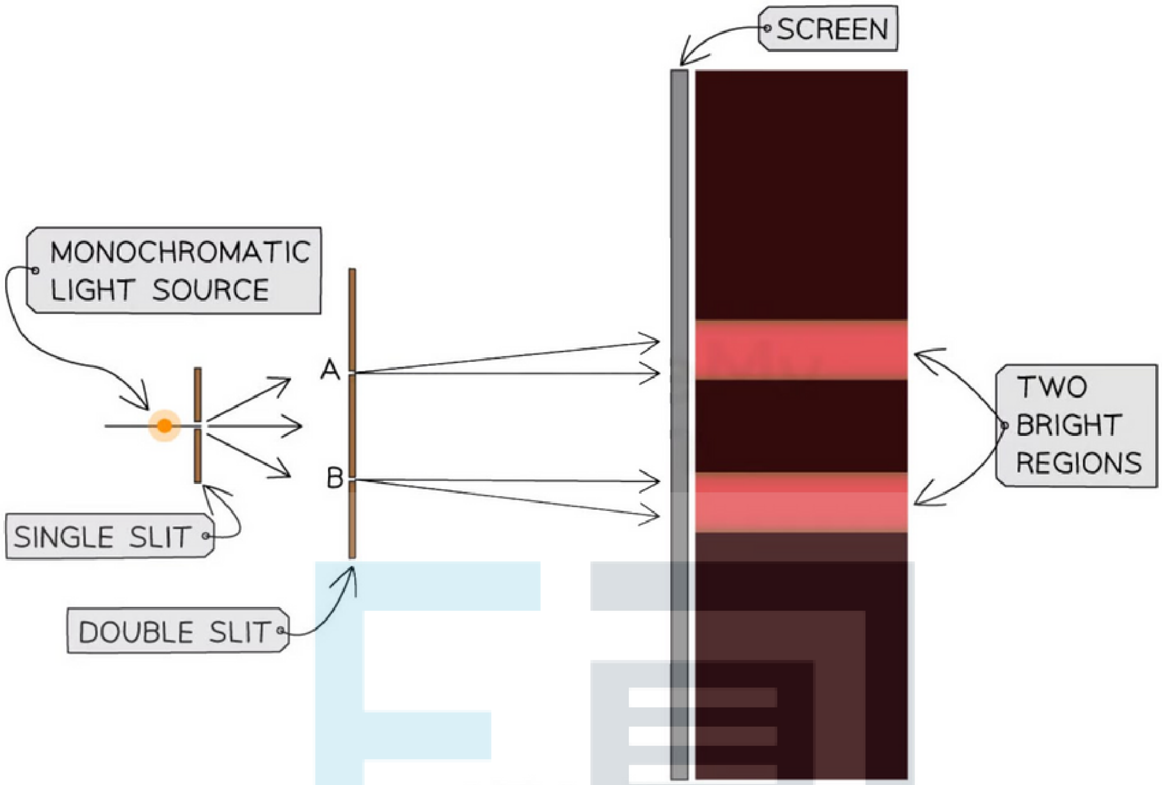





***Young's double-slit experiment arrangement. The screen showed fringes of light – this was an interference pattern***

- A monochromatic source was used to ensure that the two rays were coherent
  - The resulting pattern on the screen showed an **interference** pattern
- This was in disagreement with corpuscular theory, which would have predicted only two bright regions
  - If you fired paintballs through two gaps at a wall, you would expect only two separate regions of the wall to have paint on them

### Results predicted by Newton's corpuscular theory

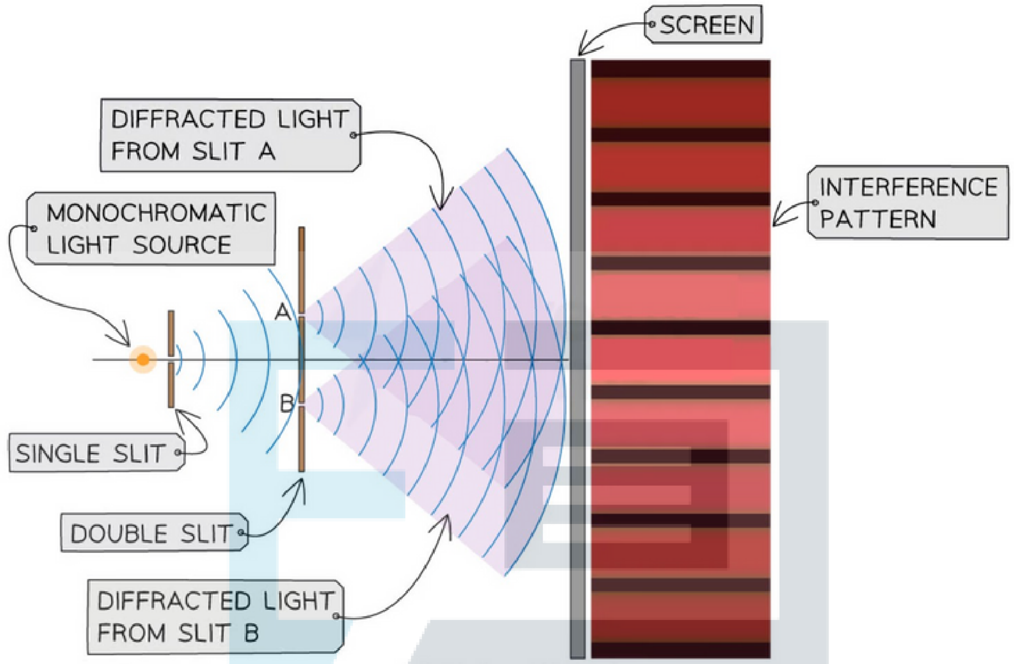



*Particle-like behaviour predicts only two bright regions and cannot account for an interference pattern that Young observed*

## Evidence for Huygens' Wave Theory

- The only explanation for the interference pattern was that light **diffracted** through the thin slits, like a **wave**
    - This was evidence in support of Huygens' wave theory of light

### A diagram showing the wave explanation of Young's interference pattern




***Light behaving as a wave could adequately explain the observed interference pattern. Bright fringes showed where the two coherent sources constructively interfered and dark fringes showed destructive interference.***



- This experiment was definite evidence for the wave theory of light as opposed to the corpuscular theory
    - Corpuscular theory, however, was not immediately rejected
- It took further experiments for the scientific community to consider wave theory as the accepted theory of light
    - The most important of these was Hippolyte Fizeau's experiment to determine the speed of light in water

### Exam Tip

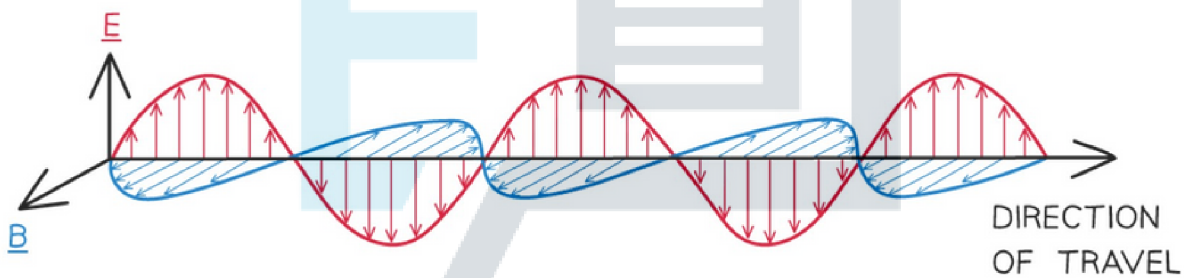
You should already be familiar with Young's double slit interference from the waves section of the A level course. You are not expected to perform calculations in this topic, but you must be able to explain the appearance of the fringes and how they provide evidence in favour of the wave theory of light.

12.2.3 Maxwell's Formula for Electromagnetic Waves

## Nature of Electromagnetic Waves

- James Clerk Maxwell was a Scottish physicist who, in 1864, published a paper relating electric and magnetic fields
  - These included a series of equations which predicted the existence of oscillating electric and magnetic fields which propagated each other, called **electromagnetic waves**
- A charged particle has an electric field
  - An accelerating charge produces an electric field which **alternates** perpendicular to the particle's motion
  - That alternating electric field produces a perpendicular alternating **magnetic** field
  - The alternating magnetic field produces an alternating electric field and so on - this is called self-propagation and is why light does not need a medium to travel

Diagram showing the alternating magnetic and electric fields




***In an electromagnetic wave, the electric (E) field's oscillation generates a perpendicular magnetic (B) field which is also oscillating.***

## Maxwell's Formula for Electromagnetic Waves

- One consequence of Maxwell's equations was a prediction of the speed of electromagnetic waves in a **vacuum**, c :

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

  - Here, $\mu_0$ is the **permeability** of free space, a constant
  - $\varepsilon_0$ is the **permittivity** of free space, also a constant (which has already been encountered in the Electric Fields topic)
  - Both of these values are constant, so the speed of electromagnetic waves in a vacuum is **constant**
- The permeability of free space, $\mu_0$ is a constant that relates magnetic flux density with the current in free space that produces the field
- The permittivity of free space, $\varepsilon_0$ similarly is a constant that relates electric field strength with the charged object in free space producing the electric field

## Worked Example

Maxwell calculated his value of the speed of electromagnetic waves, c, using values for $\varepsilon_0$ and $\mu_0$ which were both well known at the time. In 1855, Weber and Kohlrausch measured the speed of light to be $3.107 \times 10^8$ m s$^{-1}$.

Calculate the difference in Maxwell's value and Weber and Kohlrausch's value of the speed of light.

Give your answer as a percentage of Maxwell's value.

**Answer:**

**Step 1: List the known quantities:**

- Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ F m$^{-1}$
- Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$
- Weber and Kohlrausch's value of the speed of light, $c_{WK} = 3.107 \times 10^8$ m s$^{-1}$

**Step 2: Calculate Maxwell's value of c :**

- Substitute the values for permittivity and permeability into Maxwell's equation for c :

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{(4\pi \times 10^{-7}) \times (8.85 \times 10^{-12})}}$$

$$c = 2.999 \times 10^8 \text{ m s}^{-1}$$

**Step 3: Calculate the difference between this and Weber and Kohlrausch's value:**

- Subtract Maxwell's value from $c_{WK}$ :

$$c_{WK} - c = (3.107 \times 10^8) - (2.999 \times 10^8) = 1.08 \times 10^7 \text{ m s}^{-1}$$

**Step 4: Calculate this as a percentage of c :**

- Divide this difference by c and multiply by 100% for a percentage:

$$\frac{c_{WK} - c}{c} \times 100\% = \frac{1.08 \times 10^7}{2.999 \times 10^8} \times 100\% = 3.60\ \%$$

- The difference in the values is 3.60 % of Maxwell's value

**Exam Tip**

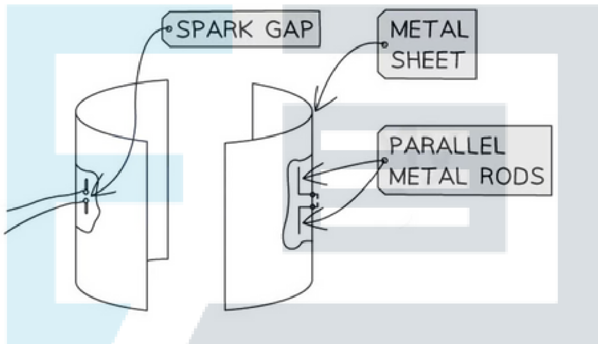
Make sure you understand what the permittivity and permeability of free space relate to - these are easy to mix up with each other. If you are unsure, check your data booklet. The symbols are listed there with their names, so you only need to remember that $\varepsilon$ (**e**psilon) is **e**lectric and $\mu$ (**m**u) is **m**agnetic.

12.2.4 Hertz's Discovery of Radio Waves

## Discovery of Radio Waves

- Heinrich Hertz discovered the existence of radio waves
  - He made a short air gap between wires and put a large potential difference across this gap, so high voltage sparks bridged the gap
  - These sparks generated radio waves, so this was a radio wave **transmitter**
- Hertz detected these radio waves with two pieces of equipment:
  - A circular wire with a small break in the circuit produced sparks across the break when held near the source of radio waves
  - A concave metal sheet with two parallel metal rods at the centre which had oscillating potential difference induced across them by the radio waves' alternating electric field

### Equipment used by Hertz to detect radio waves emitted from a transmitter



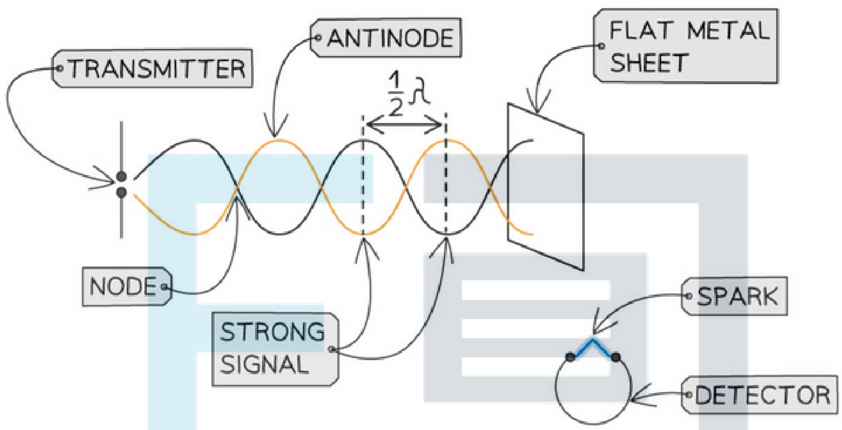

***The high voltage spark gap transmitted radio waves and the detector received these. The detector shown here is the two parallel metal rods surrounded by a concave metal sheet, but Hertz also used an incomplete wire ring which formed sparks across the gap when detecting radio waves.***



- Hertz showed the waves could be **reflected**:
  - He placed a metal screen behind the source and measured a stronger signal with the detector
  - This showed some radio waves were reflected off the screen and back towards the detector
- Hertz showed the waves were able to **penetrate** insulators:
  - When an insulator was placed between the transmitter and detector, there was no difference in the signal detected
- Hertz showed the waves were polarised:
  - When the detector was rotated 90° perpendicular to the path of the radio waves, sparks stopped being produced in the detector
  - This showed the electric fields of the radio waves were only oscillating in a single plane
  - The cause for this was that electrons were only being accelerated in one direction so the radio waves were all polarised in the same plane
- Perhaps most crucially, Hertz measured the **speed** of the radio waves:
  - He reflected radio waves from the transmitter off of a flat metal sheet

- This produced a **standing wave**
- When passing a detector across the region containing the standing wave, a large signal was detected at antinodes, and no signal was detected at nodes
- This allowed Hertz to find the wavelength and he knew the frequency by using the properties of the transmitter circuit
- He used the wave equation ($v = f\lambda$) to determine the speed of the waves
- This speed was very close to the value calculated by Maxwell, showing that radio waves are electromagnetic waves

Standing wave to determine speed of radio waves




***Here, the second type of detector is shown - an incomplete ring of wire which forms sparks. Hertz passed this along the standing wave and located antinodes using the strongest sparks. The distance between antinodes represents half the wavelength of the radio wave.***

## Worked Example

Explain how a radio wave transmitter, a detector and a flat sheet of metal can be used to determine the speed of radio waves, provided their frequency is known.

**Answer:**

**Step 1: Explain the function of the flat sheet of metal:**

- The sheet of metal reflects the radio waves back in the opposite direction

**Step 2: Explain how the incident and reflected waves interact:**

- Constructive and destructive interference occurs between the two coherent waves
- This forms a standing wave

**Step 3: Explain how wavelength is determined:**

- The detector shows a large signal when placed at antinodes in the standing wave
- This is used to find the distance between adjacent antinodes
- This distance is half the wavelength of the wave

**Step 4: Calculate speed from this:**

- Multiply the value for wavelength with frequency to calculate the wave's speed

## Exam Tip

This topic builds on knowledge of standing waves, wave speed and polarisation. Make sure you are confident with those topics to ensure you fully understand this one.

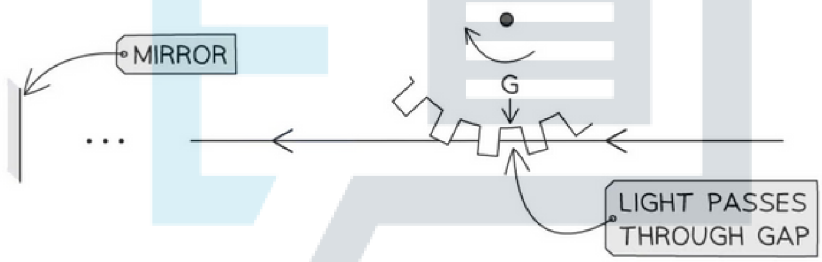
## Fizeau's Determination of the Speed of Light

- Scientists used to believe that light covered distance instantaneously and travelled at infinite speed
  - Some astronomical observations seemed to contradict this, however
  - Following this, Hippolyte Fizeau measured a finite speed for light

# How did Fizeau measure the speed of light?

- Fizeau shone a beam of light at a mirror several kilometres away
- In the path of the light, he placed a "toothed wheel" which was spinning at a very high speed
  - The toothed wheel was positioned so the teeth of the wheel and the gaps between them periodically passed over the beam of light
  - This created regular pulses of light travelling towards the distant mirror

### The path of the light passing through a gap in the toothed wheel




***Light from the source was continuous, but the toothed wheel caused the mirror to receive periodic bursts of light. The light here passes through a particular gap, labelled G, while the toothed wheel rotates.***



- In the example shown in the diagram, the light passes through a particular gap, labelled G
- The light had to travel a distance $d$ from the source to the mirror and then back to the observer
  - The total path length of the light was $2d$ and the speed of light was labelled c
  - The total time for the light to pass through the toothed wheel and return was:

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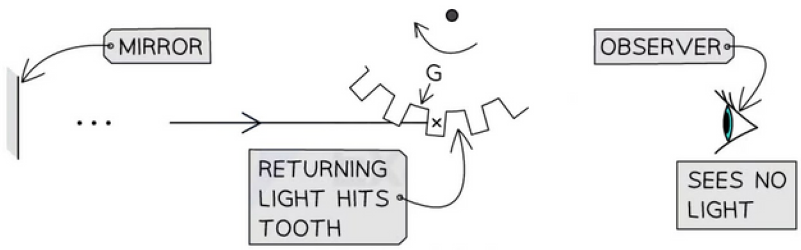
- The speed at which the wheel rotated could be changed - at a certain wheel speed, the light on its returning path would hit the tooth next to G
  - This meant the observer would see no light returning from the mirror

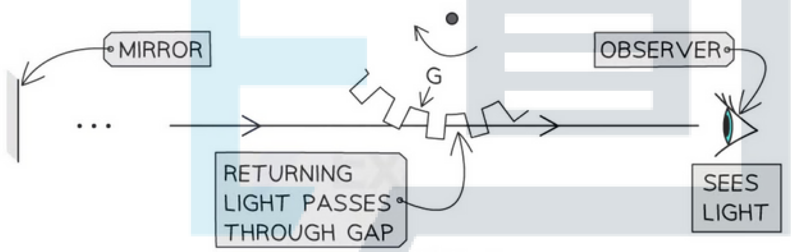
### Diagram showing the returning light blocked by a tooth




***In the previous diagram, the initial beam of light passed through gap G. The wheel then kept rotating. By the time the light has travelled to the mirror and returned, gap G has now been replaced with the tooth next to it and the light is blocked.***

- The wheel's rotational speed is increased slightly
  - Now, by the time the light that passed through G has returned, G has been replaced with the next gap
  - The returning light passes through this gap and the observer sees reflected light through the toothed wheel

Diagram showing the returning light passing through the next gap




***At this rotational speed, the same thing happens for light passing through every gap. The time of the light's path is equal to the time taken for one gap to be replaced by the next gap.***

- The toothed wheel has $n$ gaps and $n$ teeth which both have the same width
  - This means that, if a full revolution has a period of $T$, then the time taken for a gap to be replaced by a tooth is:

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- Recall that time period is the reciprocal of frequency $f$ (of teeth passing a point per second) so we can write this as:

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- When the observer sees the reflected light disappear, the time taken for light to travel to the mirror is the time taken for a gap to be replaced by a tooth, so:

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- Rearranging this allowed Fizeau to calculate the speed of light as:

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## Other Experiments done by Fizeau

- Fizeau also measured the speed of light in water by a similar method and found it to be **slower** than the speed of light in air
  - This was another piece of evidence contradicting Newton's corpuscular theory - recall that this predicted light would travel faster in a denser medium

### Exam Tip

As with many other concepts in Turning Points, this builds on a lot of other ideas. Revising circular motion will help a lot with calculations on this topic.

### Worked Example

Fizeau repeated his experiment, but this time he increased the speed of the toothed wheel until the light appeared again at maximum brightness. Rewrite his equation to calculate the speed of light for this new wheel frequency, $\tilde{f}$.

**Answer:**

**Step 1: Determine which part of the equation needs changing:**

- We no longer need to calculate the time taken for a gap to be replaced by a tooth, but the time taken for a gap to be replaced by another gap
- As the teeth and gaps are of equal width, this should take twice as long as it would for a gap to be replaced by a tooth, so:

$$t = 2 \times \frac{1}{2n\tilde{f}} = \frac{1}{n\tilde{f}}$$

**Step 2: Re-Derive Fizeau's equation for c:**

- Equating this to the time taken for light to return gives:

$$\frac{1}{n\tilde{f}} = \frac{2d}{c}$$

$$c = 2dn\tilde{f}$$

- A quick logic check confirms this
  - For a gap to now replace the initial gap, the teeth must cover twice the distance of before in the same time
  - Therefore the new frequency must be twice the old frequency, so $\tilde{f} = 2f$
  - Substituting this into the above expression gives our original equation

EXAM PAPERS PRACTICE

Exam Tip

A great skill in an exam is finding alternative methods to verify your answers - if you finish with a bit of spare time, go back to yor calculation questions. Try answering them with a different method and see if you get the same answer.

## UV Catastrophe & Black-Body Radiation

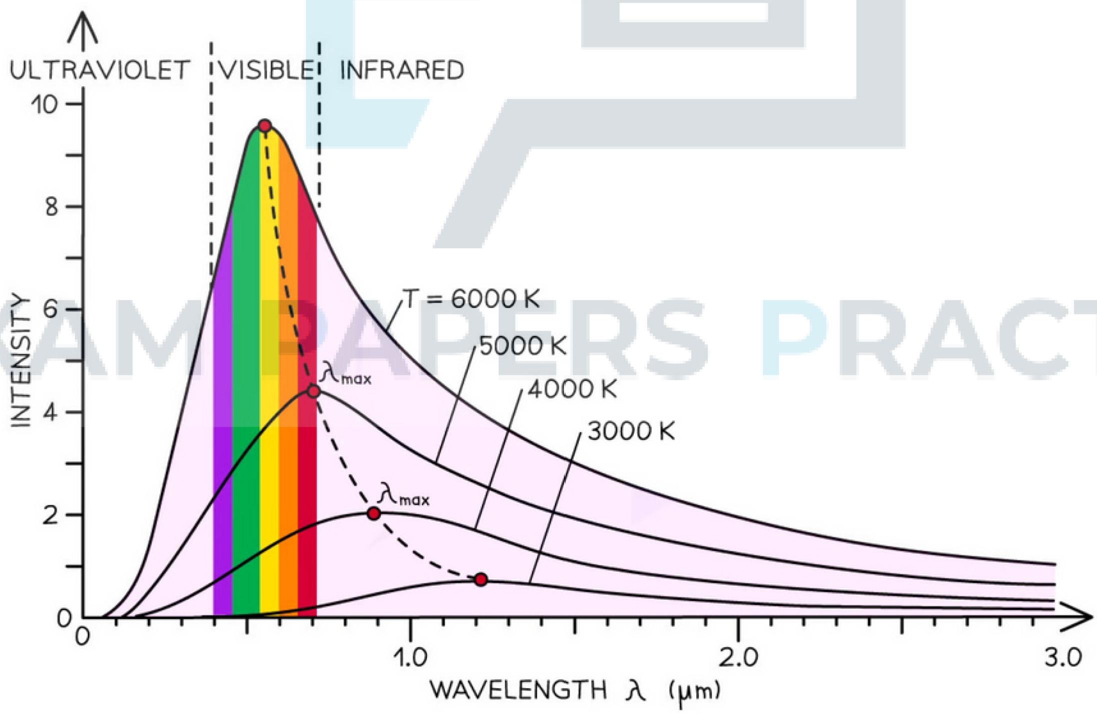
# What is Black-Body Radiation?

- A perfect black body is defined as:

    **A theoretical object that absorbs all of the radiation incident on it and does not reflect or transmit any radiation**

- Since a good absorber is also a good emitter, a perfect black body would be the best possible emitter too
- The spectrum of electromagnetic radiation that would be emitted from this hypothetical object is called the black-body spectrum
    - This changes depending on the temperature of the black-body
    - A common example of this is that a cube of metal at room temperature emits invisible infrared radiation
    - When heated to 3000 K, however, it emits a large amount of visible light and we see it glow red, orange or white

A graph showing the spectrum of radiation emitted by a black-body at different temperatures




***Each curve is for the same black-body at different temperatures. The peak of each line shows the wavelength of radiation emitted with the most intensity.***

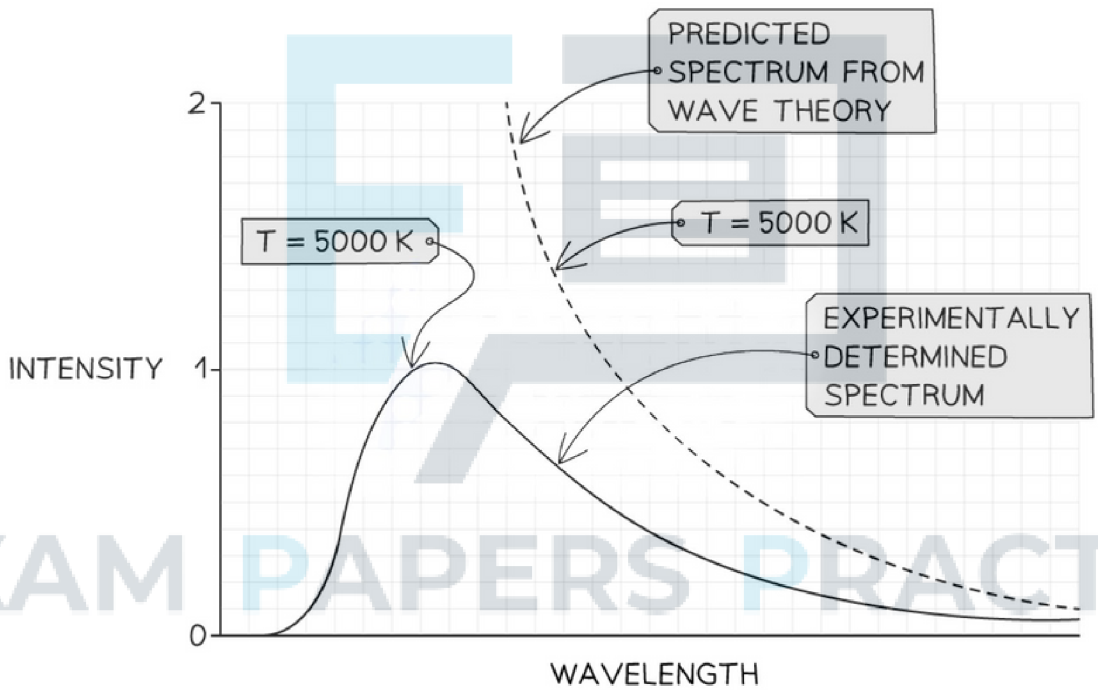
- For cooler objects, the wavelength of radiation emitted at the highest intensity is in the infrared range

- As the object's temperature increases, shorter wavelengths become the most intensely emitted

## What was the Ultra-Violet Catastrophe?

- This dramatically-named event came from a disagreement with experimentally measured black-body spectra and the spectra predicted by classical physics
  - Through experiments with objects very close to being perfect black-bodies, their emission spectra looked much like the diagram above
  - By treating electromagnetic radiation as a **wave**, however, the spectra were theoretically predicted to emit an **infinite** amount of ultra-violet as the temperature of the object increased

Graph showing spectrum from experiment and wave theory's prediction




***A big discrepancy between sound experimental data and the currently accepted theory meant that the theory was incorrect and needed to be adapted or completely replaced***

## Planck's Interpretation

- Max Planck addressed this problem by coming up with mathematical descriptions (the details of which you will not be examined on) for the emission and absorption occurring within a black-body
- In this description, he made his theory fit experimental data
- Oscillators were responsible for emitting electromagnetic radiation, and he assumed the energy emitted was quantised
  - This meant it could only be emitted in **integer** multiples of these packets (or quanta) of energy
- Therefore, the energy emitted by an oscillator of frequency $f$ was given as:

$$E = nhf$$

  - $n$ represents the integer multiple of the packet $hf$
  - $h$ is Planck's constant and has the value of $6.63 \times 10^{-34}$ J s
- This allowed Planck to develop a theory that explained the observed spectra of almost-perfect black-bodies

### Exam Tip

The concept of quantisation also appeared when referring to the charge of an electron - if you are struggling with this concept, revisiting that topic and seeing quantisation in another context may aid your understanding.

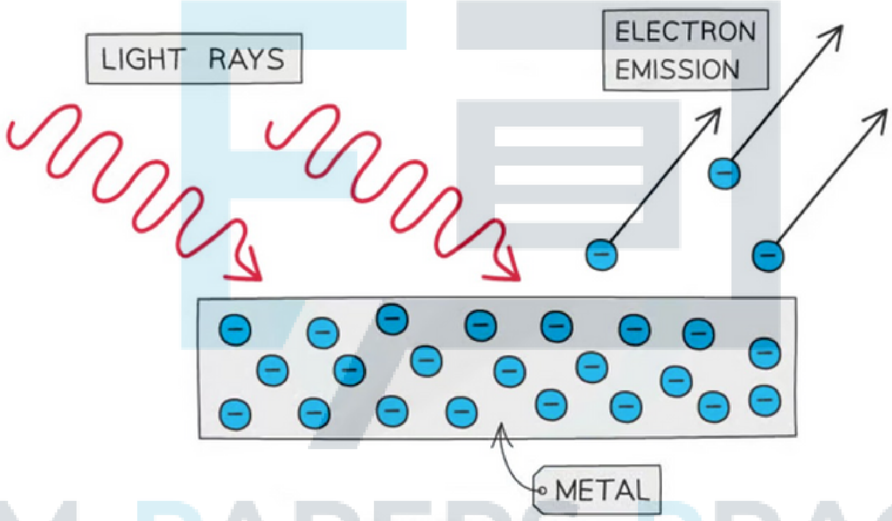
## 12.2.7 The Discovery of Photoelectricity

### Classical Wave Theory & Photoelectricity

## How did the Photoelectric Effect Contradict Wave Theory?

- The details of the photoelectric effect have already been covered in the Particles & Radiation section of this course
- In the photoelectric effect, incident radiation on a metal's surface causes it to emit electrons
  - However, this only happens for radiation above a certain frequency
  - If the radiation is below this threshold frequency, no matter how great the intensity, photoelectrons will not be emitted from the metal's surface

A diagram recapping the process of photoelectric emission




***Light rays with a frequency above the metal's threshold frequency cause immediate emission of electrons from the surface***

- This was in direct contradiction with wave theory, which predicted that an electromagnetic wave transferred energy continuously
  - According to wave theory, if low frequency radiation was aimed at the metal at a high enough intensity, then enough energy would be transferred to remove photoelectrons
- However, as soon as a radiation above the threshold frequency was shone on the metal's surface, even at low intensities, photoelectrons were **immediately** emitted
  - Additionally, the intensity of incident radiation only affected the number of photoelectrons emitted, but not the energy they left with
  - All of this evidence contradicted the idea that electromagnetic radiation transfers energy like a wave does

# Explanation of Photoelectricity

## Einstein's Explanation

- In 1905 Einstein released three ground-breaking papers, one of which was a theoretical explanation of the photoelectric effect - this explanation built on Max Planck's work on black-bodies
- Einstein proposed that electromagnetic radiation was made from discrete quanta, or packets, of energy of the size:

$$E = hf$$

  - Where $h$ is Planck's constant and $f$ is the frequency of the radiation
  - These quanta were called **photons** at a later date
  - These photons were **massless** in Einstein's theoretical description
- This theory was able to explain the experimental results of the photoelectric effect
- To explain why radiation below the threshold frequency didn't cause photoelectric emission:
  - Only one photon was able to transfer its energy to only one electron
  - If $hf$ was not large enough to sufficiently energise an electron, the photons could not combine to energise that electron
- To explain why the energy of emitted photoelectrons increased with the frequency of incident light:
  - Photons transferred all of their energy $hf$ to electrons
  - If this was greater than the energy needed to emit the electrons, the rest of the energy was transferred to the kinetic store of the electrons
  - If $hf$ was larger, more energy was left over for the kinetic store of the electrons
- Once this was all experimentally confirmed 10 years later, Einstein received a Nobel Prize in Physics

## Worked Example

Zinc has a threshold frequency of $6.5 \times 10^{14}$ Hz.

Explain why wave theory predicted that photoelectrons would be emitted if zinc was exposed to electromagnetic radiation of $6.3 \times 10^{14}$ Hz for long enough. Explain why this is not possible, using photon theory.

**Answer:**

**Step 1: Recall the way waves transfer energy:**

- Wave theory predicted that electromagnetic radiation transferred energy continuously, as waves do
- Over time, the electrons would be given enough energy to be removed from the metal

**Step 2: Recall how energy is transferred in photon theory:**

- In photon theory, one photon transfers all of its energy to one electron
- The photons of radiation with a frequency of $6.3 \times 10^{14}$ Hz do not have enough energy to sufficiently energise the electrons

12.2.8 De Broglie's Hypothesis of Wave-Particle Duality

## De Broglie's Hypothesis of Wave-Particle Duality

# What was DeBroglie's hypothesis?

- Louis DeBroglie hypothesised that all particles can behave both like waves and like particles, following Einstein's work with photons
- By equating two equations from Einstein, he derived an equation for the momentum of a photon:

$$E = mc^2 \text{ (more on this in Special Relativity)}$$

$$E = hf \text{ (the energy of a photon)}$$

$$mc^2 = hf$$

$$mc = h\frac{f}{c} = \frac{h}{\lambda}$$

  - Where $h$ is Planck's constant, $c$ is the speed of light, $m$ is mass, $\lambda$ is wavelength and $f$ is frequency
- $mc$ is the momentum, $p$, of a photon - DeBroglie extended this idea to particles with mass to obtain the relation you should recall from Particles & Radiation:

$$p = \frac{h}{\lambda}$$

# Finding the Wavelength of Accelerated Particles

- This idea can be applied to accelerated electrons to find their wavelength
  - Finding their momentum directly is difficult, but recall from The Discovery of the Electron that the work done on an electron by an electric field ($eV$) is equal to its kinetic energy - this can be used to find the electron's speed:

$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

- This can be substituted into the momentum term in DeBroglie's hypothesis to then find wavelength:

$$p = mv = m\sqrt{\frac{2eV}{m}} = \sqrt{m^2}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{m^2 2eV}{m}} = \sqrt{2meV}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

  - The wavelength of the electron depends on the work done on it by the electric field, $eV$

- From this equation, as $eV$ increases, $\lambda$ decreases
- When the electron is accelerated to a **higher speed**, its DeBroglie **wavelength decreases**

## Worked Example

An electron is accelerated through an electric field and is found to have a DeBroglie wavelength of $\lambda$. The potential difference across the electric field then increases by a factor of 25. Write the new wavelength of the electron in terms of $\lambda$.

**Answer:**

**Step 1: Write out the equation for an accelerated particle's wavelength from your data and formulae sheet:**

- The wavelength of an accelerated particle is:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

**Step 2: Label the new wavelength and substitute the new potential difference:**

- We call label the new wavelength $\tilde{\lambda}$ and substitute the new potential difference, $25V$:

$$\tilde{\lambda} = \frac{h}{\sqrt{2me \times 25V}}$$

- Now we will manipulate this expression until we can pull out the original expression for $\lambda$:

$$\tilde{\lambda} = \frac{h}{\sqrt{25 \times 2meV}} = \frac{h}{\sqrt{25} \times \sqrt{2meV}}$$

$$\tilde{\lambda} = \frac{1}{\sqrt{25}} \times \frac{h}{\sqrt{2meV}} = \frac{1}{5} \times \frac{h}{\sqrt{2meV}} = \frac{1}{5} \times \lambda$$

- Therefore the new wavelength is:

$$\tilde{\lambda} = \frac{\lambda}{5}$$

  - This checks out with common sense - the particle is moving faster under a stronger potential difference so, as was mentioned above, its new wavelength should be smaller

### Exam Tip

This equation requires some confidence in algebra involving square roots. Remembering that you can combine square roots when multiplying or combining will help a great deal:

$$\sqrt{m} \times \sqrt{n} = \sqrt{m \times n}$$

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{\frac{m}{n}}.$$

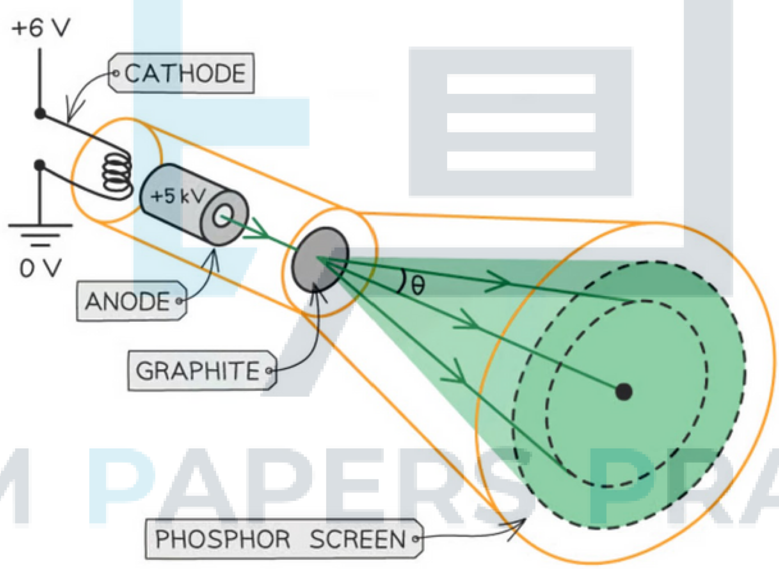
12.2.9 Electron Diffraction

## Electron Diffraction

# Do Electrons Diffract?

- To investigate whether electrons did exhibit wave-like properties, an electron **diffraction** tube was produced
- The electrons were accelerated in an electron gun with a high potential difference, such as 5000 V, and were then directed through a thin film of graphite
  - The gaps between the carbon atoms were sufficiently small to cause diffraction, using the predicted DeBroglie wavelength of electrons
  - Diffraction is a property of waves when passing through a small gap - if the electrons were seen to diffract then this was proof of their wave-like properties
- The electrons were indeed seen to diffract from the gaps between carbon atoms and produce a circular pattern on a fluorescent screen made from phosphor

Diagram of the electron diffraction experiment

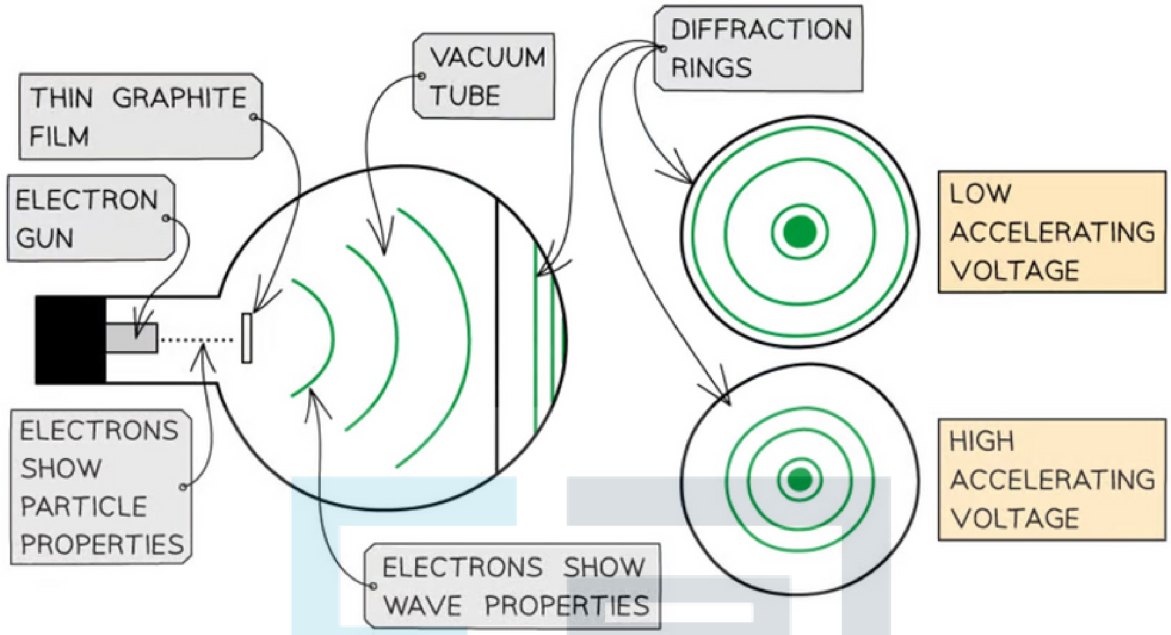



***A low voltage causes thermionic emission of electrons at the cathode, then these are accelerated towards anode directed through the graphite. The electrons diffract through the gaps between atoms in the graphite, forming a circular diffraction pattern made visible by the phosphor screen.***

# The Effect of Potential Difference on the Diffraction Pattern

- Recall that DeBroglie's hypothesis predicted that, if more work was done on the electrons, their wavelength would be shorter
  - Recall from the diffraction grating subtopic of the Waves topic, that a shorter wavelength leads to a smaller diffraction angle $\theta$ for a given gap width
- Therefore, if DeBroglie was right, increasing the potential difference would cause the diffraction rings to move closer to the centre of the phosphor screen

Diagram showing the effect of increasing potential difference on diffraction




***The left of this diagram shows the equipment from another angle. The right of this diagram shows that increasing the voltage caused the diffraction rings to move closer to the centre of the screen, following DeBroglie's equation.***

- This was the first experimental proof of electrons exhibiting wave-like properties
  - The wavelength of electrons was significantly smaller than high energy forms of electromagnetic radiation, such as X-rays



### Exam Tip

This topic builds on knowledge acquired from studying diffraction gratings. As is becoming a common theme with Turning Points, go back and make sure your understanding of diffraction is solid before getting stuck in with this topic.

## Estimate of Anode Voltage

- Changing the voltage across the anode (it is the voltage that causes the electrons to accelerate), scientists can manipulate the wavelength of the electron
- A microscope's **resolving power** (its ability to differentiate between two nearby points on an object) depends on the wavelength of the radiation being used
    - A shorter wavelength means the microscope can be used to see finer details in an object
    - Electrons had wavelengths much shorter than X-rays
- The size of an atom is roughly $10^{-10}$ m
    - By using DeBroglie's equation for an accelerated electron, researchers could find the accelerating voltage needed to make the electron's wavelength the same size as an atom

## Worked Example

An atom of carbon has a diameter of $1.5 \times 10^{-10}$ m. Determine the anode potential difference needed in an electron diffraction tube to produce electrons with a wavelength equal to the diameter of a carbon atom.

**Answer:**

**Step 1: Write down the equation for wavelength of accelerated electrons from the data and formulae booklet:**

- The wavelength of an accelerated electron is:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

**Step 2: List the known quantities:**

- Mass of an electron, $m = 9.11 \times 10^{-31}$ kg
- Magnitude of charge on an electron, $e = 1.60 \times 10^{-19}$ C
- Planck's constant, $h = 6.63 \times 10^{-34}$ J s
- Required wavelength of the electron, $\lambda = 1.5 \times 10^{-10}$ m

**Step 3: Rearrange the wavelength equation to make anode potential difference the subject:**

- Separate the $V$ term from the square root:

$$\lambda = \frac{h}{\sqrt{2me}\sqrt{V}}$$

$$\sqrt{V} = \frac{h}{\sqrt{2me} \times \lambda}$$

- Square the whole equation:

$$V = \frac{h^2}{2me\lambda^2}$$

**Step 4: Substitute the known quantities:**

$$V = \frac{(6.63 \times 10^{-34})^2}{2 \times (9.11 \times 10^{-31}) \times (1.60 \times 10^{-19}) \times (1.5 \times 10^{-10})^2}$$

$$V = 67.0 \text{ V}$$

### Exam Tip

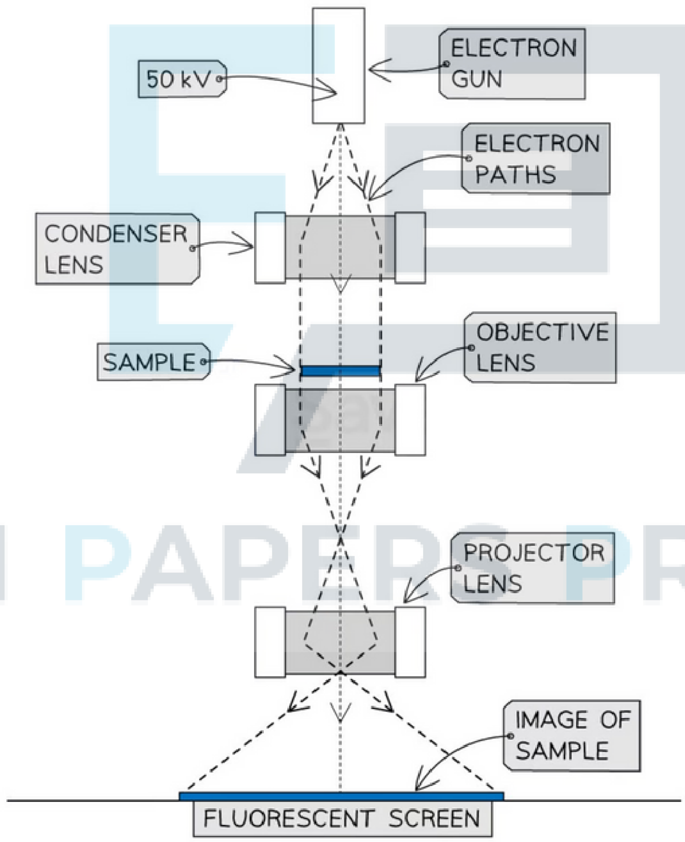
In a high pressure situation like an exam, it's easy to make mistakes on your calculator when substituting values in standard form. Use brackets carefully and retype your equation to ensure you get the same value when checking over your answers.

## Transmission Electron Microscope (TEM)

# How does a TEM work?

- In the 1930s, experimental scientists realised that the much shorter wavelengths that electron waves offered could be used to construct microscopes with a higher resolving power
- The first to be constructed were **transmission electron microscopes** (TEMs)
  - Where light microscopes had used convex optical lenses, these microscopes focused beams of electrons using **magnetic lenses**
  - The electrons passed through a sample and formed an image on a fluorescent screen

A diagram showing the path of electrons through magnetic lenses




***A cross-sectional diagram of a TEM. The dotted lines represent the paths of electrons – those travelling along the microscope's axis (the middle vertical line) are not deflected. Each magnetic lens has a different purpose.***

- The electron gun emits electrons through thermionic emission
  - These are then accelerated to high speeds (and therefore short wavelengths) by a large potential difference
- The function of the condenser lens:

- The condenser lens' magnetic field deflects the electrons into a wide beam travelling parallel to the axis of the microscope
  - This parallel beam is **uniformly** incident on the sample
- The function of the objective lens:
  - This lens forms an **image** of the sample
  - It deflects the outer electrons in the beam towards the central axis, much like a convex optical lens does for light
  - Electrons travelling along the microscope's axis are not deflected, again similarly to light in a convex lens
- The function of the projector lens:
  - This lens causes the beams from the objective lens to spread out, magnifying the image created by the objective lens
  - This magnified image is directed onto a fluorescent screen, emitting light where electrons are incident

## Drawbacks of the TEM

- The level of detail available in an image depends on the resolving power
  - In an electron microscope, electrons need to be travelling as fast as possible to have the shortest wavelength and therefore highest resolving power
- In the TEM the electrons must pass **through** the sample
  - This reduces the speed of electrons, increasing wavelength and reducing resolving power so electron waves are unable to resolve as much detail as their short wavelength would allow
- Additionally, not all electrons emitted by thermionic emission have the same speed, and not all electrons are slowed by the sample to the same degree
- This means electrons in the beam have a **range** of speeds
  - Electrons travelling at different velocities through a magnetic field are deflected by different amounts
  - This means electrons passing through a single point in the sample are projected onto a range of locations on the fluorescent screen instead, forming a blurrier image

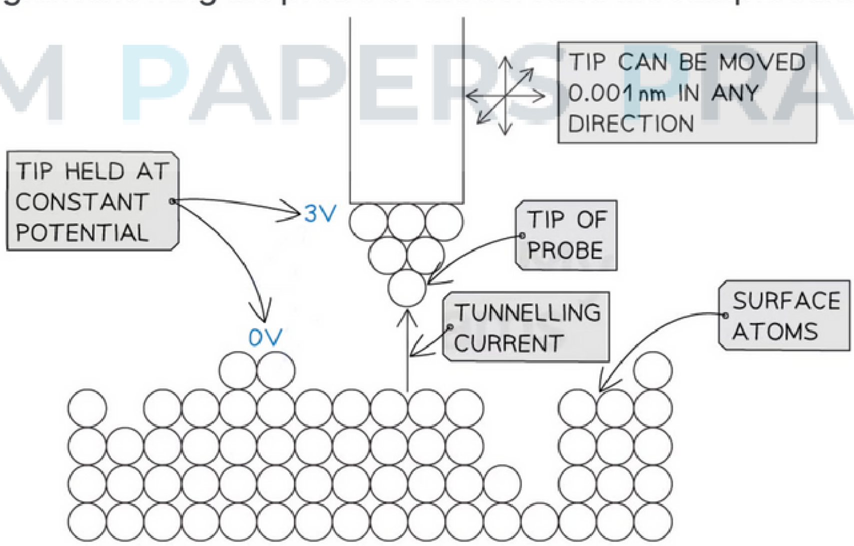
## 12.2.11 Scanning Tunnelling Microscope

# Scanning Tunnelling Microscope (STM)

How Does an STM Work?

- By the 1980s, many advances had been made in research technology and in physics on the scale of the atom
- The **scanning tunnelling microscope** (STM) was created
  - This microscope is able to resolve objects just 0.001 nm apart
  - This means it can produce images showing individual rows of **atoms**
- A probe with a very fine tip (only a few atoms across) is held a few nanometres above the surface of an object
  - The probe is moved by pieces of equipment called piezoelectric transducers
  - These are able to move the tip in any direction by tiny increments of 0.001 nm
- The probe is held at a constant potential difference with the surface of the sample
  - Because the tip is so close to the surface of the sample, some electrons are able to jump, or **tunnel**, to the tip
  - This produces a **tunnelling current** which is recorded
- To be able to scan this surface to produce an image of it:
  - When the tip reaches a raised atom, the distance from the tip to the surface decreases, and more electrons tunnel so the tunnelling current increases
  - Likewise, when the tip reaches a dip in the surface, the tunnelling current decreases because the gap is larger and fewer electrons tunnel
  - These changes in tunnelling current are used to produce a map of the surface of a sample

Diagram showing the probe of the STM and the sample surface




***Due to the potential difference across the gap and its short distance, some electrons tunnel across the gap to the tip of the probe, generating a current. The size of this current depends on the width of the gap it must cross.***

## Constant Height or Constant Current?

- STMs operate in two modes: constant height or constant current
- In constant height mode:
  - The tip does not move vertically up or down while scanning areas
  - This means that changes in the surface increase or decrease the gap size
  - The tunnelling current therefore varies
  - This is used to produce an image of the surface
- In constant current mode:
  - When a change in tunnelling current is detected, the tip moves up or down to keep the current constant
  - This means the gap size is always the same
  - The vertical motion of the probe is used to map an image of the surface

## Quantum Tunnelling

- Quantum tunnelling is a result of the wave-like behaviour of particles such as electrons
- The gap between the surface and the tip acts as a barrier for electrons
- The amplitude of the matter-wave of the electrons is decreased by this barrier, but on the other side of the gap, this amplitude is non-zero
  - This effect only occurs if the barrier is weak enough (i.e. the distance is small enough)
  - This is like thin surfaces not being opaque to visible light because some of the wave can pass through
- This results in a current passing from sample to probe
  - This current is very sensitive to changes in gap distance, which allows the STM to have a great resolving power



### Exam Tip

Don't worry, details of quantum tunnelling are not examinable, but a general understanding of why it happens is necessary. The word "quantum" can be intimidating, but it just refers to processes that are a result of the quantisation of things like energy.