Exam Papers Practice

10.2 Fields at Work

Mark Schemes

Course	DP IB Physics
Section	10. Fields (HL only)
Topic	10.2 Fields at Work
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **B** because:

- The escape velocity v_{esc} of a spherical mass *M* is given by $\sqrt{\frac{2GM}{r}}$ where *r* is the radius of the mass
- Therefore:
  - The escape velocity from Europa is given by $\sqrt{\frac{2Gm_E}{r_E}}$
  - The escape velocity from Jupiter is given by $\sqrt{\frac{2Gm_J}{r_J}}$
- The ratio $\frac{escape\ velocity\ of\ Europa}{escape\ velocity\ of\ Jupiter}$ is then
  - $\sqrt{\frac{2Gm_E}{r_E}} \div \sqrt{\frac{2Gm_J}{r_J}} = \sqrt{\frac{2Gm_E}{r_E}} \times \sqrt{\frac{r_J}{2Gm_J}}$
- This simplifies to:
  - $\sqrt{\frac{2Gm_E r_J}{r_E \times (2Gm_J)}} = \sqrt{\frac{m_E r_J}{m_J r_E}}$
  - Therefore, the correct answer is **B**

Make sure you can manipulate expressions algebraically, carefully keeping track of subscripts such that the appropriate quantities refer to the correct bodies as stated in the question.

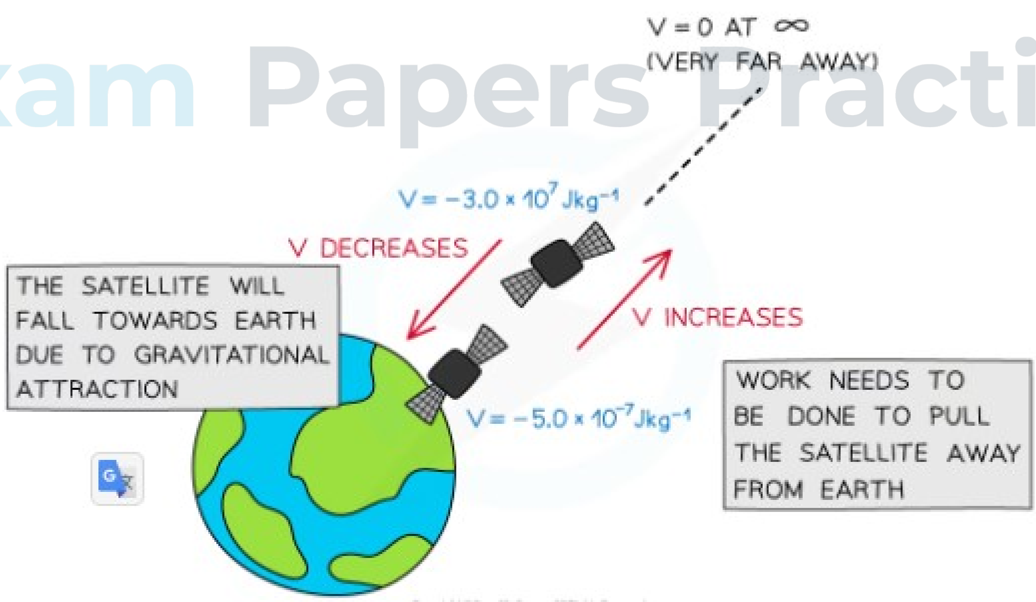
2

The correct answer is **D** because:

- The change in gravitational potential is from –40 MJ kg^{-1} to –10 MJ kg^{-1}
  - Hence, the gravitational potential increases by 30 MJ kg^{-1} and gets closer to 0
  - Therefore, the satellite must be moving further away from the Earth

- Since the satellite is moving further away from Earth, it must be moving antiparallel to a field line
  - This is because field lines point toward the centre of mass of the Earth
  - This eliminates options A and C
- The work done W (or energy transferred) as a satellite of mass m moves across a gravitational potential difference ΔV_g is given by $W = m\Delta V_g$
  - The change in potential $\Delta V_g = 30\ \text{MJ kg}^{-1} = 30 \times 10^6\ \text{J kg}^{-1}$
  - Hence, the change in gravitational potential energy $W = 2000 \times (30 \times 10^6) = 6 \times 10^{10} = 60\ \text{GJ}$
- Therefore, the correct answer is **D**

You should be able to the direction of motion of any test mass in a gravitational field depending on the initial and final values of potential. Since gravitational potential is defined as **zero at infinity**, and negative everywhere else, if the final value of potential is **less negative** than the initial value - as it is in this question - the test mass must be moving **further away** from the source mass (i.e., Earth) towards infinity. Therefore, it is moving **antiparallel** to a gravitational field line, which always points towards the centre of the source mass.



3

The correct answer is **B** because:

- The international space station is in orbit around Earth; it therefore acts as a satellite
- The kinetic energy of a satellite of mass *m* moving with speed *v* is always positive, and is given by $E_K = \frac{1}{2}mv^2$
  - This eliminates options A and C
- The potential energy of a satellite moving in a gravitational field is always negative, and is given by $E_P = -\frac{GMm}{r}$ where *M* is the source mass in the field
- The total energy *E* of an orbiting satellite is negative, and is given by $E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$
  - Therefore, $E_P = E - E_K$
  - Hence, $E > E_P$
- Therefore the correct answer is **B**
  - Because the line corresponding to the total energy *E* is always greater than the line corresponding to the potential energy E_P

Any orbiting satellite has a **total energy** equal to the **sum** of the kinetic and potential energy. Some important facts to remember are:

- The potential energy is **always negative** in a gravitational field
- The total energy is **always negative** for an orbiting body
- The total energy is **always greater** than the potential energy

4

The correct answer is **C** because:

- The total energy of the probe at launch $E_T = E_K + E_P$, where E_K and E_P are the probe's kinetic and gravitational potential energy respectively
  - Therefore, $E_T = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$ where *M* and *R* are the Earth's mass and radius respectively, and *m* is the mass of the probe

- The total energy of the probe at maximum height from the surface of the Earth $E_T = -\frac{GMm}{r}$
  - This is because the kinetic energy is zero at maximum height, as the probe momentarily comes to rest
  - Here, $r = R + h$, where h is the maximum height above the Earth's surface
- Since the velocity $v = \frac{v_{esc}}{2}$ at launch, then the total energy at launch becomes:
  - $E_T = \frac{1}{2}m\left(\frac{v_{esc}}{2}\right)^2 + \left(-\frac{GMm}{R}\right) = \frac{mv_{esc}^{\ 2}}{8} - \frac{GMm}{R}$
- Using $v_{esc} = \sqrt{\frac{2GM}{r}}$
  - $E_T = \frac{m\left(\sqrt{\frac{2GM}{R}}\right)^2}{8} - \frac{GMm}{R} = \frac{2GMm}{8R} - \frac{GMm}{R}$
  - Hence, $E_T = \frac{2GMm}{8R} - \frac{8GMm}{8R} = -\frac{6GMm}{8R} = -\frac{3GMm}{4R}$
- Total energy of the probe is conserved, therefore the total energy is equal to the energy at the probe's maximum height. Therefore:
  - $-\frac{3GMm}{4R} = -\frac{GMm}{r}$
  - $\frac{3}{4R} = \frac{1}{r}$
  - Hence, $r = \frac{4R}{3}$
- Recalling that r is the distance from the Earth's centre of mass, such that $r = R + h$ where h is the height above Earth's surface, then:
  - $R + h = \frac{4R}{3}$
  - Hence, $h = \frac{4R}{3} - R = \frac{R}{3}$
- Therefore, the correct answer is **C**

There are a couple of super important things to remember when tackling questions about launching probes or satellites:

- The total energy E_T is always **conserved**. Hence, you can find an equation for the total energy at launch, which you can equate to **other equations later** in the probe's launch: the form may change, but the quantity does not
- Note the gravitational potential energy **at launch** is given in terms of the planet's radius *R*, whereas later in the probe's journey it is given in terms of a general distance *r*. This is because the launch begins at the Earth's surface, therefore the potential energy must be the value of the potential energy at this point
- Know the equation for the escape velocity, $v_{esc} = \sqrt{\frac{2GM}{r}}$. This is given in your data booklet, but being familiar with its form is very handy!

5

The correct answer is **C** because:

- The escape velocity from any spherical mass, like the Sun, is given by $v_{esc} = \sqrt{\frac{2GM}{r}}$ where *M* is the mass of the Sun and *r* is its radius
- Hence, if the escape velocity is exactly equal to the speed of light *c*, then:
  - $c = \sqrt{\frac{2GM}{r}}$ which gives $c^2 = \frac{2GM}{r}$
- Therefore, the radius at which the escape velocity is exactly equal to the speed of light is given by $r = \frac{2GM}{c^2}$
- Hence,
  - $r = \frac{2 \times (6.67 \times 10^{-11}) \times (2 \times 10^{30})}{(3 \times 10^8)^2}$
- Approximating *G*, *M* and *c* to 1 significant figure allows an estimation of this radius to be computed as follows:
  - $r \approx \frac{2 \times (7 \times 10^{-11}) \times (2 \times 10^{30})}{(3 \times 10^8)^2} = \frac{28 \times 10^{19}}{9 \times 10^{16}}$
  - Hence, $r \approx 3 \times 10^3$ which is approximately 3 km
- Therefore, the correct answer is **C**

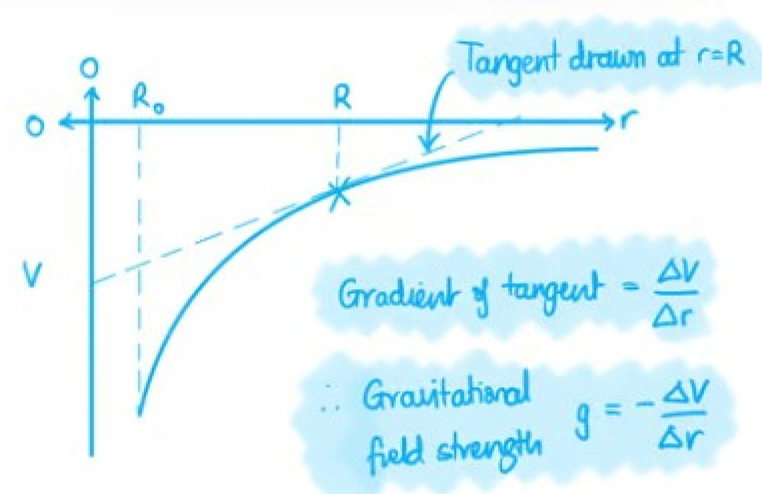
The special radius at which any spherical mass M has an escape velocity equal to the speed of light c is known as the **Schwarzschild Radius**. You do not need to know this for your examination, however; understanding how to use the equation for escape velocity, and estimating quantities by rounding to an appropriate number of significant figures, are useful techniques to practise.

6

The correct answer is **D** because:

- The gravitational field strength $g = -\frac{\Delta V_g}{\Delta r}$
  - The quantity $\frac{\Delta V_g}{\Delta r}$ represents the gradient of a potential-distance graph, like the one in this question
  - Therefore, the gravitational field strength at the distance $r = R$ is determined by calculating the gradient of the potential at that point and multiplying it by –1
- Therefore, the correct answer is **D**

This question catches a lot of candidates out because many are in the (good!) habit of calculating the gradient of a graph **and then stopping there**. Though subtle, you must remember that the field strength is **proportional** to the gradient of a potential, but is **exactly equal** to the **negative** of the gradient. This is sketched out below:



A bit of (non-rigorous!) calculus also shows this nicely:

$$V = -\frac{GM}{r} = -GMr^{-1}$$

$$\frac{dV}{dr} = GMr^{-2} = \frac{GM}{r^2} = -g$$

$$\text{So,} \quad g \propto \frac{dV}{dr}$$

$$g = -\frac{dV}{dr}$$

7

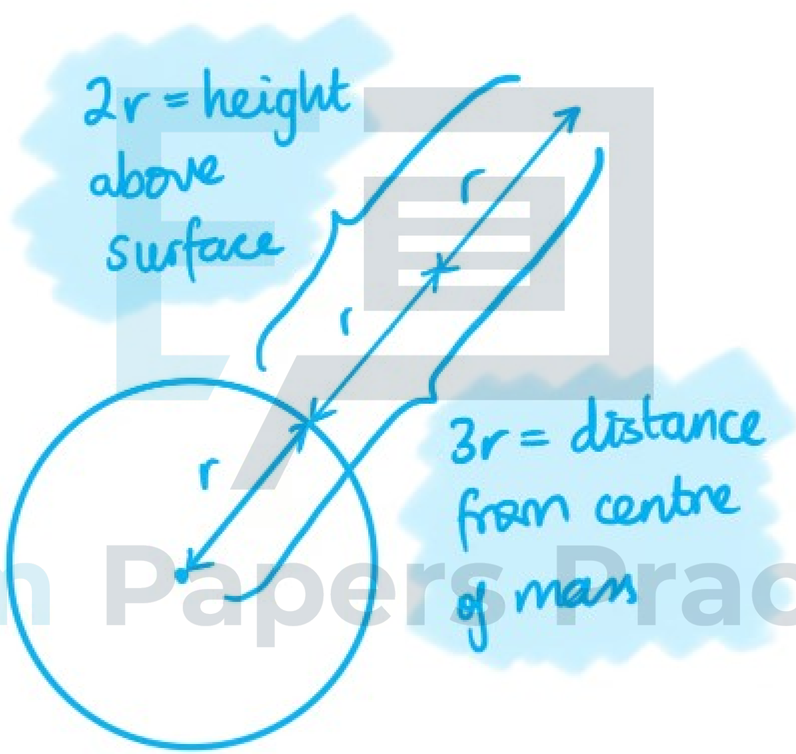
The correct answer is **C** because:

- The magnitude of the gravitational field strength is given by $g = \frac{GM}{r^2}$ for a spherical source of mass M at a distance r from its centre of mass
  - Hence, if the radius of Earth is given as r, then this equation gives the magnitude of the gravitational field strength at its surface
- At a height of $2r$ from the Earth's surface, the distance from Earth's centre is therefore $r + 2r = 3r$
  - Hence, the gravitational field strength at this height $g' = \frac{GM}{(3r)^2} = \frac{GM}{9r^2} = \frac{g}{9}$
- The magnitude of the gravitational potential is given by $V = -\frac{GM}{r}$ for a spherical source of mass M at a distance r from its centre of mass
  - Again, the height from the Earth's surface is $2r$

- Therefore, from its centre of mass the total distance is $r + 2r = 3r$
- The gravitational potential becomes $V' = -\frac{GM}{(3r)} = \frac{V}{3}$

- Therefore the correct answer is **C**

Drawing a quick sketch - as should be the case for any question without one! - will help you visualise the correct distances to use in equations for the Fields At Work topic. Check this out below:

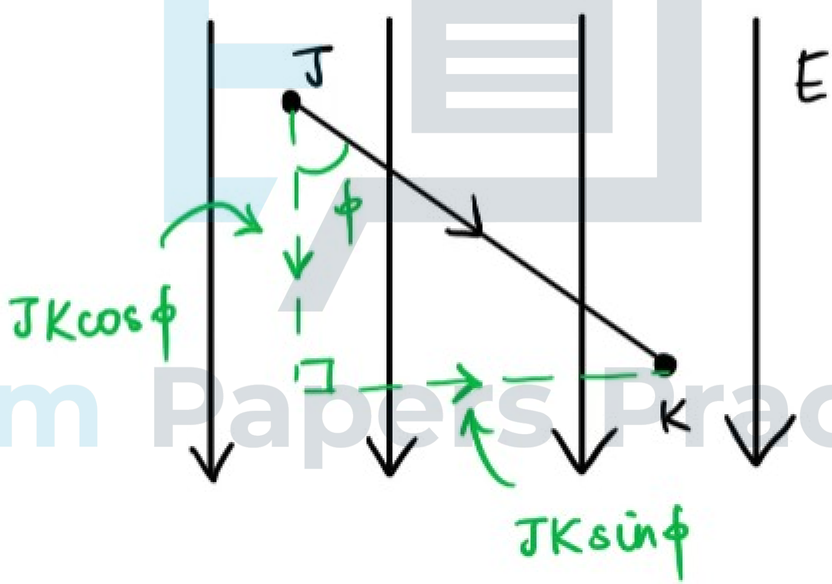


As you can see, the correct distance to use in equations for gravitational field strength and potential at the surface of a planet is *r*, since this is determined from the object's centre of mass. This question, though seemingly simple at first, requires a bit of thought - and as you can see, a sketch to help! It's a commonly used exam trick: so be on the lookout for similar questions which give you distances from the surface of a planet or spherical mass.

8

The correct answer is **A** because:

- The charge moving from J to K in the electric field is equivalent to moving along two components one parallel and one perpendicular to the field:
  - The vertical distance travelled in the electric field, parallel to field lines, is $JK \cos \Phi$
  - The horizontal distance travelled, perpendicular to field lines, is $JK \sin \Phi$
  - This is shown in the image below:



- The work done W (or energy transferred) is a product of force F and distance travelled d parallel to that force
  - The force F exerted on q by the electric field is given by $F = qE$
  - The distance travelled parallel to the force (i.e. parallel to the field lines) is $JK \cos \Phi$
- The energy transferred is the change in the charge's electric potential energy, and is given by:
  - $W = Fd = (qE) \times (JK \cos \Phi) = EqJK \cos \Phi$
  - Therefore, the correct answer is **A**

B is incorrect as	the quantity $JK\sin\Phi$ is the distance travelled perpendicular to field lines – i.e., along an **equipotential.** There is no work done along an equipotential (because there is no change in potential), therefore no energy is transferred and this cannot be an expression for the charge's change in electric potential energy
C is incorrect as	the quantity $\tan\Phi$ is a ratio of the two vertical and horizontal distances, of no physical importance. It is included purely as a 'distractor'!
D is incorrect as	the quantity $EqJK$ is the product of force exerted by the electric field Eq and the distance JK. Since this distance is not parallel to the force exerted, it does not correctly determine the change in the charge's potential energy

9

The correct answer is **D** because:

- Since the particles reach the negatively charged plate at the same time, they must have the same acceleration
- Acceleration is the ratio of resultant force F to mass m
  - The question allows us to neglect gravitational effects
  - Hence, the only (and hence, resultant) force acting on each charge is that from the electric field: $F_1 = Eq_1$ on charge q_1 and $F_2 = Eq_2$ on charge q_2
- Since the acceleration is the same, then the ratio of F to m is the same for both charges:
  - $a = \frac{Eq_1}{m_1}$ for charge q_1 of mass m_1
  - $a = \frac{Eq_2}{m_2}$ for charge q_2 of mass m_2

- Therefore, equating expressions gives:
  - $\frac{Eq_1}{m_1} = \frac{Eq_2}{m_2}$
  - $\frac{q_1}{m_1} = \frac{q_2}{m_2}$
- Hence, the correct answer is **D**

Note that the **ratio** of charge-to-mass must be equal, **not** the individual charges and masses. This is a surprising result: one which only arrives after careful consideration that acceleration **must be the same** for both charges if they travel the same distance in the same amount of time.

Recall, $s = ut + \frac{1}{2}at^2$, so if both charges start from rest, and the distance travelled s and time taken t for each charge is equal, then their acceleration must be equal too. Crucially, this acceleration is given by the ratio of **resultant force** in the electric field to each particle's **mass**: which is actually a ratio of charge to mass in this uniform electric field.

10

The correct answer is **B** because:

- Between two plates, the electric field strength is uniform: $E = \frac{V}{d}$
- Gradient of the electric potential: $E = -\frac{\Delta V_e}{\Delta r}$
- Plate X is charged to –180 V and plate Y is charged to +180 V
  - Therefore, the change in potential from plate X to plate Y, ΔV_e = final potential – initial potential
  - So, $\Delta V_e = (180) - (-180) = 360$ V
- The distance between this change in electric potential Δr is 2.0 m
  - Therefore the gradient $\frac{\Delta V_e}{\Delta r} = \frac{360}{2} = 180 \text{ V m}^{-1}$
  - This eliminates options **C** and **D**

- The electric field is always directed from positive charge to negative charge
  - Therefore, the electric field must be directed to the **left**
- Hence, the correct answer is **B**

Determine the direction of an electric field, first of all, wherever possible. This will provide a starting point from which to consider things like the directions and shape of equipotentials. For this question, which may seem complicated at first glance, you should remember the crucial fact that the electric field strength is proportional to the **gradient** of a potential. Gradients are very simply calculated: the change in electric potential ΔV_e with distance Δr.