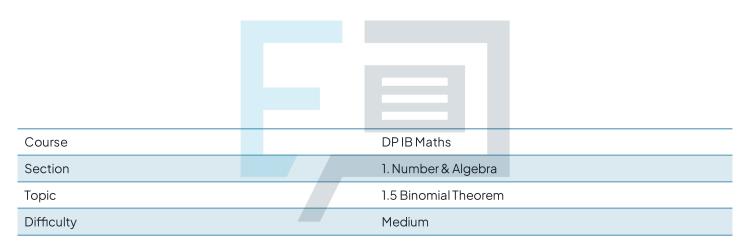


1.5 Binomial Theorem

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Exam Papers Practice

To be used by all students preparing for DP IB Maths AA SL Students of other boards may also find this useful



$$n = 8, \quad A = 2, \quad b = -\infty$$
Substitute values into the formula for the binomial theorem:

$$(a+b)^{n} = a^{n} + \dots + {}^{n}C_{r}a^{n-r}b^{r} + \dots + b^{n}$$
where ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$$(2-x)^{8} = \sum_{r=0}^{k} {}^{8}C_{r}(2)^{8-r}(-x)^{r} \qquad Coefficient of$$

$$(2-x)^{8} = \sum_{r=0}^{k} {}^{8}C_{r}(2)^{8-r}(-x)^{3}$$
Non-calculator, so work out ${}^{n}C_{r}$ separately:
 ${}^{8}C_{3} = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{(3 \times 2)(5 \times 4 \times 3 \times 2)}$

$$= \frac{8 \times 7 \times 6}{5} = 56$$
So the term when $r=3$ is $56 \times 2^{5} \times (-\infty)^{3}$

$$= 56 \times -32x^{3}$$

$$= -1792 x^{3}$$
Coefficient of $x^{3} = -1792$

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a = 3, b = x, n = 4Substitute values into the formula for $(a+b)^n$ $(a+b)^n = a^n + {}^nC_1 a^{n-1}b + ... + {}^nC_r a^{n-r}b^r + ... + b^n$ where ${}^nC_r = \frac{n!}{r!(n-r)!}$ Question asks for ascending powers of x, so start with the term in x. $(3+x)^4 = 3^4 + {}^4C_1(3)^{4-1}(x)^1 + 4c_2(3)^{4-2}(x)^2 + ...$ constant term in x term in x² $\approx 81 + \frac{4!}{3!} \times 3^3 \times x + \frac{4!}{2!2!} \times 3^2 \times x^2$ $\approx 81 + 4 \times 27 \times t 6 \times 9 x^2$ $\approx 81 + 108 \times t 54 x^2$ **Xan (3+x)⁴ = 81 + 108 + 54 x⁴**



$$a = a, b = -x, n = 4$$

Substitute values into the formula for $(a+b)^n$
 $(a+b)^n = \sum_{r=0}^n {}^nC_r a^{n-r}b^r$
 $(a-x)^4 = \sum_{r=0}^4 {}^4c_r a^{4-r} (-x)^r \cdot given the coefficient of the term in x3, so evaluate the term when $r=2$.
Term in x³ = $4c_2(a)^{4-2}(-x)^2 = 96x^2$
 $6a^2(-x)^2 = 96x^2$
 $a^2 = 16$
 $a = \pm 4$
It is given in the $a = 4$ and $a = 4$ and $a = 4$$

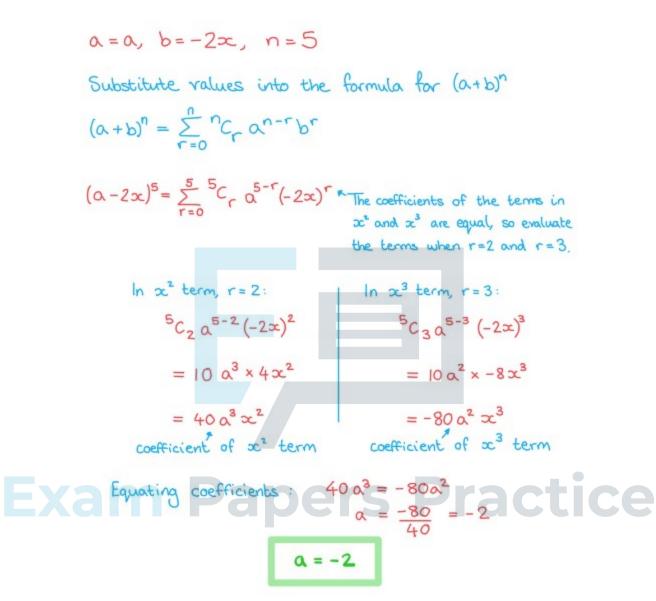


$$a = 9 \quad b = -2x \quad n = 5$$

Substitute values into the formula for $(a+b)^n$
 $(a+b)^n = a^n + {}^nC_1 a^{n-1}b + ... + {}^nC_r a^{n-r}b^r + ... + b^n$
Question asks for ascending powers of x; so start with
the constant term, a^n .
 $(9-2x)^5 = 9^5 + 5c_1 (9)^{5-1}(-2x) + 5c_2 (9)^{5-2}(-2x)^2 + ...$
 $\approx 59049 + 5 \times 6561 \times -2x + 10 \times 729 \times 4x^2$
 $\approx 59049 - 65610x + 29160x^2$
 $(9-2x)^5 \approx 59049 - 65610x + 29160x^2$

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$$a = 3 \qquad b = px \qquad n = 6$$

$$(3 + px)^{6} = \sum_{r=0}^{6} {}^{6}C_{r}(3)^{6-r}(px)^{r} \qquad \text{evaluate the terms when } r=2$$

$$and r = 4.$$

$$\ln x^{2} \text{ term, } r=2:$$

$$\int_{0}^{6}C_{2}(3)^{6-2}(px)^{2} \qquad \ln x^{4} \text{ term, } r=4:$$

$$\int_{0}^{6}C_{2}(3)^{6-2}(px)^{2} \qquad = 15 \times 9 \times p^{4}x^{4}$$

$$= 15 \times 81 \times p^{2}x^{2} \qquad = 135 p^{4}x^{4}$$

$$coefficient of x^{3} \qquad coefficient of x^{4}$$

$$coefficient of x^{4} = 4(coefficient of x^{2})$$

$$135 p^{4} = 4(1215p^{2})$$

$$135 p^{4} = 4860p^{2}$$

$$p^{2} = \frac{4860}{135} = 36$$

$$p = 6 \text{ or } -6$$



(a) A binomial expansion has n+1 terms. $n = 5 \implies n+1 = 6$ (4ax - 3)⁵ has 6 terms

(b)
$$a = 4ax$$
, $b = -3$, $n = 5$
 $(4ax - 3)^5 = \sum_{r=0}^{5} 5c_r (4ax)^{5-r} (-3)^r$
we have been given the coefficient
of the term in x^4 , so evaluate the
term when $r = 1$.
Term in $x^4 = -61440x^4$
 $\Rightarrow 5c_1(4ax)^{5-1}(-3)^1 = -61440x^4$
XAM P5 $x - 3x 4^4 x a^4 x x^4$ **P61440** x^4
XAM P5 $x - 3x 4^4 x a^4 x x^4$ **P61440** x^4
 $-3840a^4 = -61440$
 $a^4 = -61440$ = 16
 $a = 4\sqrt{16} = 12$
It is given in the question that a is a positive constant.



Question 8 (a)
$$a = x^{3}$$
, $b = \frac{4}{x}$, $n = 4$
 $(a + b)^{n} = \sum_{r=0}^{n} c_{r} a^{n-r} b^{r}$
 $(x^{3} + \frac{4}{x})^{4} = \sum_{r=0}^{4} c_{r} (x^{3})^{4-r} (\frac{4}{2c})^{r}$ The numerator has
 $(x^{3} + \frac{4}{x})^{4} = \sum_{r=0}^{4} c_{r} (\frac{4^{r}x^{3(4-r)}}{x^{r}})^{r=0}$ as the first term
 $= \sum_{r=0}^{4} c_{r} (\frac{4^{r}x^{3(4-r)}}{x^{r}})^{r=0}$ as the first term
 $(x^{3} + \frac{4}{x})^{4} = 4c_{0}(\frac{4^{0}x^{3(4-0)}}{x^{0}}) + 4c_{1}(\frac{4^{1}x^{3(4-1)}}{x^{1}})$
 $= \frac{1 \times 1x^{12}}{1} + 4 \times \frac{4x^{q}}{x} + 6 \times \frac{16x^{6}}{x^{2}}$
 $\approx x^{12} + 16x^{8} + 96x^{4}$
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(b)
$$(x^3 + \frac{4}{x})^4 = \sum_{r=0}^{4} {}^{4}C_r (x^3)^{4-r} (\frac{4}{3c})^r$$

= $\sum_{r=0}^{4} {}^{4}C_r (\frac{4^r x^{3(4-r)}}{x^r})$

The constant term is the term in ∞° , so we need r such that 3(4-r)-r=0

$$3(4-r) - r = 0$$

$$12 - 4r = 0$$

$$r = 3$$

$$r = 3 \text{ gives } 4C_3\left(\frac{4^3 x^{3(4-3)}}{x^3}\right) = 4 \times 4^3 \times 1$$

$$= 256$$

Constant term = 256

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first expand (ax+3), rearranging this to (3+ax) makes it easier to spot the correct term to use. $(3 + \alpha x)^5 = \sum_{r=0}^{5} {}^{5}C_r (3)^{s-r} (\alpha x)^r$ The question gives the term in x^3 , so here we are looking for the term in oct. Evaluate for r=4. The term when r=4: $(3+\alpha x)^{5} = \dots + {}^{5}C_{4}(3)^{5-4}(\alpha x)^{4} + \dots$ $= \dots + 15a^4x^4 + \dots$ The coefficient of x^7 in the expansion $x^3(ax+3)^5 = 1215$, therefore : $x^{3}(15a^{4})x^{4} = 1215x^{7}$ $15a^4 = 1215$ $a^4 = 81$ $a = \pm 4 \sqrt{81}$ **Papers** Practice A = 3 or -3