Exam Papers Practice

1.4 Proof & Reasoning

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Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.4 Proof & Reasoning
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

EXPAND BRACKETS ON LHS

$(4x-1)(2x+3) - (2x+1)(2x+1)$ FOIL

$(8x^2 - 2x + 12x - 3) - (4x^2 + 2x + 2x + 1)$

$(8x^2 + 10x - 3) - (4x^2 + 4x + 1)$

SIMPLIFY, TAKE CARE WITH NEGATIVES

$8x^2 + 10x - 3 - 4x^2 - 4x - 1 = 4x^2 + 6x - 4$

FACTOR OF 2

$2(2x^2 + 3x - 2)$

FACTORISE REMAINING QUADRATIC

$2(2x-1)(x+2)$ = RHS AS REQUIRED

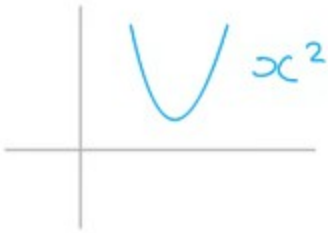
$\therefore$

$$(4x-1)(2x+3) - (2x+1) = 2(2x-1)(x+2)$$

Question 2

PROVE QUADRATIC IS ALWAYS POSITIVE USING DISCRIMINANT $b^2 - 4ac < 0$

$b^2 - 4ac < 0$



$x^2 - 3x + 3 > 0$

$a = 1 \quad b = -3 \quad c = 3$

$(-3)^2 - 4(1)(3) < 0$

$9 - 12 < 0$

$-3 < 0$ ✓✓

$\therefore$

$x^2 - 3x + 3 > 0$ FOR ALL VALUES OF x

Question 3

EXPAND BRACKETS ON LHS

$(a-b)(a-b) - (a+b)(a+b)$

$(a^2 - 2ab + b^2) - (a^2 + 2ab + b^2)$

SIMPLIFY $a^2 - 2ab + b^2 - a^2 - 2ab - b^2$

$-4ab$ = RHS AS REQUIRED

$\therefore$

$(a-b)^2 - (a+b)^2 = -4ab$

Question 4

LET THREE CONSECUTIVE INTEGERS BE

$$n-1,\ n,\ n+1$$

THEN

$$n-1+n+n+1$$

$$\equiv 3n$$

WHICH IS A MULTIPLE OF 3

$\therefore$

THE SUM OF 3 CONSECUTIVE INTEGERS IS A MULTIPE OF 3

Question 5

GIVEN THAT

$$x^2 + 2 \geqslant 2$$

THEN

$$x^2 \geqslant 0$$

ALL SQUARE NUMBERS ARE ALWAYS POSITIVE

$\therefore$ $x^2 + 2 \geqslant 2$ FOR ALL VALUES OF x

Question 6

LET AN EVEN NUMBER BE $2n$

THEN

$$(2n)^2 = 4n^2$$

$$\equiv 4(n^2)$$

WHICH IS A MULTIPLE OF 4

$\therefore$ THE SQUARE OF AN EVEN NUMBER IS ALWAYS A MULTIPLE OF 4

Question 7

a) $(n+1)(n+2)$

b) n^3+3n^2+2n

$n(n^2+3n+2)$

$n(n+1)(n+2)$

c) GIVEN THAT n IS EVEN

CONSECUTIVE INTEGERS ALTERNATE BETWEEN ODD AND EVEN

$n+1$ MUST BE ODD

$n+2$ MUST BE EVEN

d)

GIVEN THAT

$$n^3+3n^2+2n \equiv n(n+1)(n+2)$$

THEN

IF n IS ODD

$n+1$ = EVEN

$n+2$ = ODD

IF n IS EVEN

$n+1$ = ODD

$n+2$ = EVEN

AT LEAST ONE TERM OF $n(n+1)(n+2)$ IS ALWAYS EVEN, THEREFORE

n^3+3n^2+2n MUST ALWAYS BE EVEN

Question 8

a) EXPAND BRACKETS ON LHS

$$(3n+2)(3n+2) - (n+2)(n+2)$$

$$(9n^2+12n+4) - (n^2+4n+4)$$

SIMPLIFY $9n^2+12n+4-n^2-4n-4$

$8n^2+8n$ = RHS AS REQUIRED

$\therefore$

$$(3n+2)^2-(n+2)^2 = 8n^2+8n$$

b) USING $(3n+2)^2-(n+2)^2 = 8n^2+8n$ FROM (a)

FACTORISE

$$8n^2+8n = 8(n^2+n)$$

$8(n^2+n)$ IS A MULTIPLE OF 8

$\therefore$ $(3n+2)^2-(n+2)^2$ IS A MULTIPLE OF 8