



1.4 Simple Proof & Reasoning

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1.4.1 Proof

Language of Proof

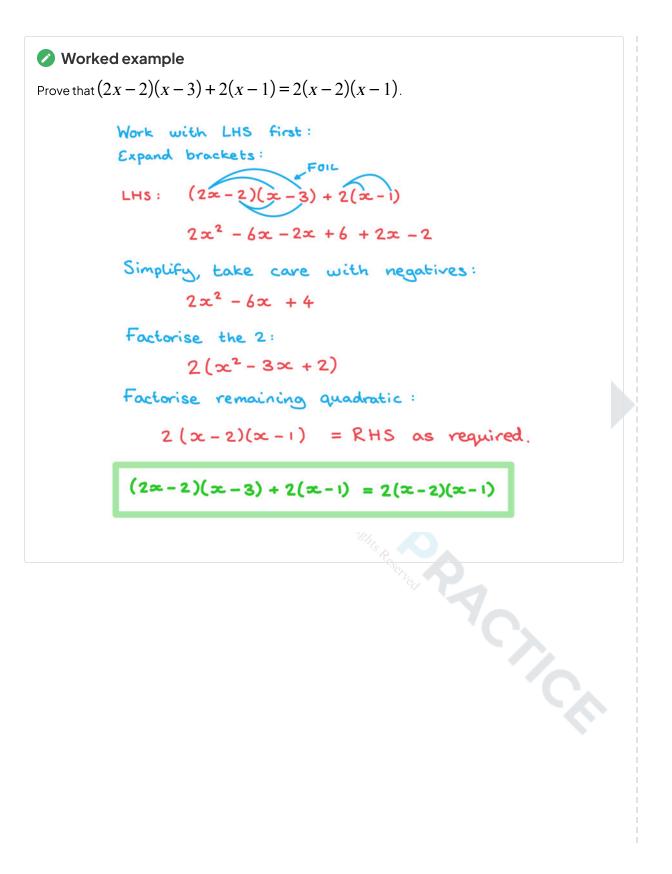
What is proof?

- Proof is a series of logical steps which show a result is **true** for all specified numbers
 - 'Seeing' that a result works for a few numbers is not enough to **show** that it will work for all numbers
 - Proof allows us to show (usually algebraically) that the result will work for **all values**
- You must be familiar with the notation and language of proof
- LHS and RHS are standard abbreviations for left-hand side and right-hand side
- Integers are used frequently in the language of proof
 - The set of **integers** is denoted by $\mathbb Z$
 - The set of **positive integers** is denoted by \mathbb{Z}^+

How do we prove a statement is true for all values?

- Most of the time you will need to use algebra to show that the left-hand side (LHS) is the same as the right-hand side (RHS)
 - You **must not** move terms from one side to the other
 - Start with one side (usually the LHS) and manipulate it to show that it is the same as the other
- A mathematical identity is a statement that is true for all values of x (or θ in trigonometry)
 - The symbol \equiv is used to identify an identity
 - If you see this symbol then you can use proof methods to show it is true
- You can complete your proof by stating that RHS = LHS or writing QED







Proof by Deduction

What is proof by deduction?

• A mathematical and logical argument that shows that a result is true

How do we do proof by deduction?

- A proof by deduction question will often involve showing that a result is true for all integers, consecutive integers or even or odd numbers
 - You can begin by letting an integer be n
 - Use conventions for even (2n) and odd (2n 1) numbers
- You will need to be familiar with sets of numbers $(\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})$
 - N the set of **natural numbers**
 - \mathbb{Z} the set of integers
 - Q the set of quotients (rational numbers)
 - \mathbb{R} the set of **real numbers**

What is proof by exhaustion?

- Proof by exhaustion is a way to show that the desired result works for every allowed value
 This is a good method when there are only a limited number of cases to check
- Using proof by exhaustion means testing every allowed value not just showing a few examples
 - The allowed values could be specific values
 - They could also be split into cases such as even and odd

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Worked example Prove that the sum of any two consecutive odd numbers is always even. Let 2n-1 be an odd number must be even Let two consecutive odd integers be: 2n-1, 2n+1 K next odd number Then their sum is: $2n - 1 + 2n + 1 \equiv 4n$ = 2(2n)multiple of 2 must be even. Any S Plactice S. Reserved



Disproof by Counter Example

What is disproof by counter-example?

- Disproving a result involves finding a value that does **not** work in the result
- That value is called a **counter-example**

How do I disprove a result?

- You only need to find **one** value that does not work
- Look out for the set of numbers for which the statement is made, it will often be just integers or natural numbers
- Numbers that have unusual results are often involved
 - It is often a good idea to try the values 0 and 1 first as they often behave in different ways to other numbers
 - The number 2 also behaves differently to other even numbers
 - It is the only even prime number
 - It is the only number that satisfies $n + n = n^n$
 - If it is the set of real numbers consider how rational and irrational numbers behave differently
 - Think about how positive and negative numbers behave differently
 - Particularly when working with inequalities



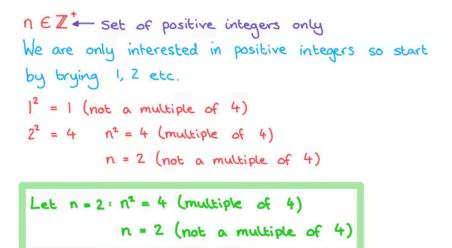
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Worked example

For each of the following statements, show that they are false by giving a counterexample:

a) Given $n \in \mathbb{Z}^+$, if n^2 is a multiple of 4, then n is also a multiple of 4.



b) Given $x \in \mathbb{Z}$ then 3x is always greater than 2x.



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x \in \mathbb{Z} - Set of integers only
We are interested in both positive and negative
integers and zero so consider how each of
these groups behave:
Positive integers, e.g. let x = 1:
2\infty = 2
3\infty = 3 : 3\infty > 2\infty
Zero:
Let x = 0
2x = 0
3x = 0 : 3x = 2x (this is enough to disprove
                        the result)
Negative integers, e.g. let x = -1:
2\infty = -2
3x = -3 : 2x > 3x (Any negative integer can
                       disprove the result)
  Let x = 0
 2x = 0
  3x=0 : 3x \neq 2x
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