DP IB Maths: AA SL

1.4 Proof & Reasoning

Contents

1.4.1 Proof

Language of Proof

What is proof?

- Proof is a series of logical steps which show a result is **true** for all specified numbers
  - 'Seeing' that a result works for a few numbers is not enough to **show** that it will work for all numbers
  - Proof allows us to show (usually algebraically) that the result will work for **all values**
- You must be familiar with the notation and language of proof
- LHS and RHS are standard abbreviations for left-hand side and right-hand side
- **Integers** are used frequently in the language of proof
  - The set of **integers** is denoted by $\mathbb{Z}$
  - The set of **positive integers** is denoted by $\mathbb{Z}^+$

How do we prove a statement is true for all values?

- Most of the time you will need to use algebra to show that the left-hand side (LHS) is the same as the right-hand side (RHS)
  - You **must not** move terms from one side to the other
  - Start with one side (usually the LHS) and manipulate it to show that it is the same as the other
- A **mathematical identity** is a statement that is true for all values of x (or θ in trigonometry)
  - The symbol $\equiv$ is used to identify an identity
  - If you see this symbol then you can use proof methods to show it is true
- You can complete your proof by stating that RHS = LHS or writing QED

Worked example

Prove that $(2x-2)(x-3)+2(x-1)=2(x-2)(x-1)$.

Work with LHS first:

Expand brackets:

FOIL

LHS: $(2x-2)(x-3)+2(x-1)$

$2x^2-6x-2x+6+2x-2$

Simplify, take care with negatives:

$2x^2-6x+4$

Factorise the 2:

$2(x^2-3x+2)$

Factorise remaining quadratic:

$2(x-2)(x-1)$ = RHS as required.

$$(2x-2)(x-3)+2(x-1)=2(x-2)(x-1)$$

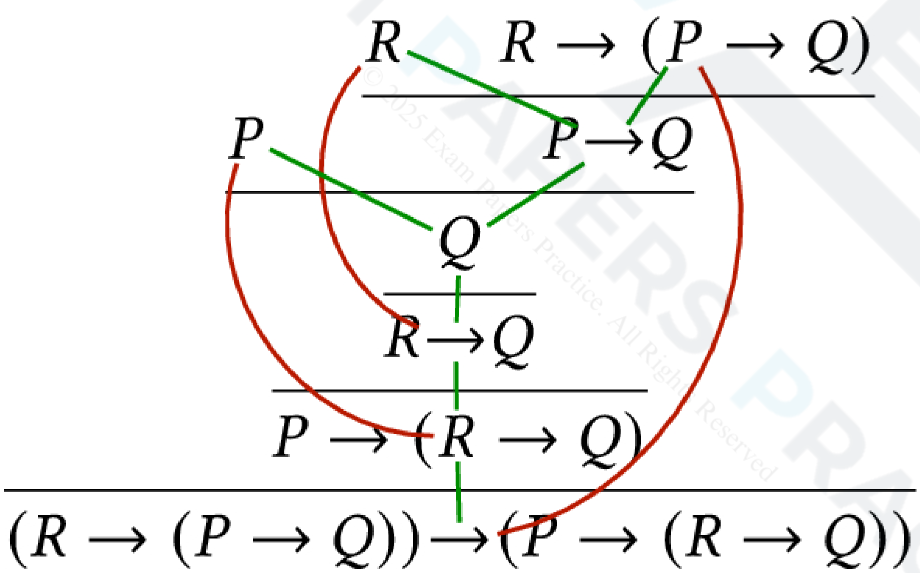
Proof by Deduction

What is proof by deduction?

- A mathematical and logical argument that shows that a result is true

How do we do proof by deduction?

- A proof by deduction question will often involve showing that a result is true for all integers, consecutive integers or even or odd numbers
  - You can begin by letting an integer be n
    - Use conventions for even ($2n$) and odd ($2n-1$) numbers
- You will need to be familiar with sets of numbers $(\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})$
  - $\mathbb{N}$ – the set of **natural numbers**
  - $\mathbb{Z}$ – the set of **integers**
  - $\mathbb{Q}$ – the set of **quotients (rational numbers)**
  - $\mathbb{R}$ – the set of **real numbers**



Worked example

Prove that the sum of any two consecutive odd numbers is always even.

Let $2n - 1$ be an odd number

($2n$) must be even

Let two consecutive odd integers be:

$$2n-1, \quad 2n+1$$

($2n+1$) next odd number

Then their sum is:

$$2n - 1 + 2n + 1 \equiv 4n$$

$$= 2(2n)$$

Any multiple of 2 must be even.