DP IB Maths: AI HL

1.4 Financial Applications

Contents

- 1.4.1 Compound Interest & Depreciation
- 1.4.2 Amortisation & Annuities

1.4.1 Compound Interest & Depreciation

Compound Interest

What is compound interest?

- Interest is a small percentage paid by a bank or company that is added on to an initial investment
  - Interest can also refer to an amount paid on a loan or debt, however IB compound interest questions will always refer to interest on **investments**
- **Compound interest** is where interest is paid on **both the initial investment** and any interest that has **already been paid**
  - Make sure you know the difference between compound interest and simple interest
    - Simple interest pays interest only on the initial investment
- The interest paid each time will increase as it is a percentage of a higher number
- Compound interest will be paid in instalments in a given timeframe
  - The interest rate, *r*, will be per annum (per year)
    - This could be written *r* % p.a.
  - Look out for phrases such as **compounding annually** (interest paid yearly) **or compounding monthly** (interest paid monthly)
    - If $\alpha\%$ p.a. (per annum) is paid compounding monthly, then $\frac{\alpha}{12}\%$ will be paid each month
    - The formula for compound interest allows for this so you do not have to compensate separately

How is compound interest calculated?

- The formula for calculating compound interest is:

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

  - Where
    - *FV* is the future value
    - *PV* is the present value
    - *n* is the number of years
    - *k* is the number of compounding periods per year
    - *r* % is the nominal annual rate of interest
  - This formula is **given in the formula booklet**, you do not have to remember it
- Be careful with the *k* value
  - Compounding annually means *k* = 1
  - Compounding half-yearly means *k* = 2
  - Compounding quarterly means *k* = 4
  - Compounding monthly means *k* = 12
- Your GDC will have a finance solver app on it which you can use to find the future value
  - This may also be called the TVM (time value of money) solver

- You will have to enter the information from the question into your calculator
- Be aware that many questions will be set up such that you will have to use the formula
    - So for compound interest questions it is better to use the formula from your formula booklet than your GDC

Worked example

Kim invests MYR 2000 (Malaysian Ringgit) in an account that pays a nominal annual interest rate of 2.5% **compounded monthly**. Calculate the amount that Kim will have in her account after 5 years.

Compound interest formula:

$$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$$

- FV ← future value
- PV ← present value
- r ← interest rate
- k ← compounding periods
- n ← number of years

Substitute values in:

$PV = 2000$ (initial investment)

$k = 12$ (compounding monthly)

$r = 2.5\%$

$n = 5$ (number of years)

$$FV = 2000\left(1 + \frac{2.5}{(100)(12)}\right)^{(12 \times 5)}$$

$$= 2266.002\ldots$$

$FV \approx$ MYR 2270 (3sf)

Depreciation

What is depreciation?

- Depreciation is when the **value** of something **falls** over time
- The most common examples of depreciation are the value of cars and technology
- If the depreciation is occurring at a **constant rate** then it is **compound depreciation**

How is compound depreciation calculated?

- The formula for calculating compound depreciation is:

$$FV = PV \times \left(1 - \frac{r}{100}\right)^n$$

  - Where
    - *FV* is the future value
    - *PV* is the present value
    - *n* is the number of years
    - *r*% is the rate of depreciation
  - This formula is **not** given in the formula booklet, however it is almost the same as the formula for compound interest but
    - with a **subtraction** instead of an addition
    - the value of *k* will always be 1
- Your GDC **could** again be used to solve some compound depreciation questions, but watch out for those which are set up such that you will have to use the formula

Worked example

Kyle buys a new car for AUD $14 999. The value of the car depreciates by 15% each year.

a) Find the value of the car after 5 years.

Depreciation formula:

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

FV = future value; PV = present value; r = rate of depreciation; n = number of years

Substitute values in:

$PV = 14999$ (initial cost)

$r = 15\%$

$n = 5$ (number of years)

$$FV = 14999\left(1 - \frac{15}{100}\right)^5$$

$$= 6655.13\ldots$$

FV ≈ AUD $6660 (3sf)

b) Find the number of years and months it will take for the value of the car to be approximately AUD $9999.

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

$FV \approx 9999$

$PV = 14999$

$r = 15\%$

Substitute values in:

$$9999 \approx 14999\left(1 - \frac{15}{100}\right)^n$$

Use GDC to solve:

$n = 2.495\ldots$

2 years ↑ ↑ 0.495^{th} of a year

Convert to years and months:

2 years + $0.495\ldots \times 12$ months

≈ 2 years and 6 months

1.4.2 Amortisation & Annuities

Amortisation

What is amortisation?

- Amortisation is the process of repaying a loan over a fixed period of time
  - Most commonly questions will be about mortgages (loans taken out to buy a home) or loans taken out for a large purchase
- Interest will be paid on the original amount
  - Each repayment that is made will partly repay the original loan and partly pay the interest on the loan
  - As payments are made the amount owed will decrease and so the interest paid will decrease
    - As you continue to repay a loan more of the repayment goes on the loan and less on the interest

How can the GDC be used to make calculations involving loans?

- Your GDC should be used to solve questions involving loans
  - Use the **finance solver mode** (sometimes called the TVM (time value of money) solver)
    - *N* will be the number of **repayment periods** (remember to include months and years if necessary)
    - *I (%)* is the interest rate
    - *PV* is the amount that was borrowed at the start – as this has been received it will be entered as a **positive** number
    - *PMT* is the payments made per period – this is repaying the loan so will be a **negative** number
    - *FV* is the future value (this will be zero as the loan will be paid off at the end of the period)
    - *P/Y* is the number of payments per year, usually 12 as payments are made monthly
    - *C/Y* is the **compounding periods** per year
    - *PMT@* is the time of the year or month the payment is made (assume this is the end unless told otherwise)
  - Leave the section that you need to find out blank and fill in all other sections
  - Your GDC will fill in the last part for you
- It is sensible to check your final answer, you can do this by finding the total amount paid back overall and comparing it to the original loan
  - The total amount repaid will be **a little more** than the original loan plus *I* % of the original loan

Worked example

Olivia takes a mortgage of EUR €280 000 to purchase a house at a nominal annual interest rate of 3.2%, **compounded monthly**. She agrees to pay the bank EUR €1500 at the end of every month to amortise the loan. Find

i) the number of years and months it will take Olivia to pay back the loan,

Use the finance / TVM solver on your GDC:

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
	3.2	280 000	-1500	0	12	12	END

N: GDC will fill this in for you.
PMT: negative because paying this back each month
P/Y: paid monthly
C/Y: compounding monthly
PMT@: paid at the end of each month

$N = 258.61$

Convert to years:

$$\text{Number of years} = \frac{258.61}{12} = 21.55$$

21 years and 7 months

ii) the total amount Olivia will pay to purchase the house.

$$\text{Total amount paid} = N \times PMT$$

$$\text{Total amount paid} = 258.61 \times 1500$$

Total amount paid = €387 915

Annuities

What is an annuity?

- An annuity is a fixed sum of money paid to someone at specified intervals over a fixed period of time
  - Most commonly this will be because of an initial lump sum investment which will be returned at fixed intervals of time with a fixed interest rate
  - Either from personal savings or from receiving an inheritance

How are annuities calculated?

- Your GDC should be used to solve questions involving annuities
  - Use the **finance solver mode** (sometimes called the TVM (time value of money) solver)
    - *N* will be the number of **payment periods** (remember to include months and years if necessary)
    - *I (%)* is the interest rate
    - *PV* is the amount that was invested – as this has been invested it will be entered as a **negative number**
    - *PMT* is the amount paid per period – as this is being received it will be a **positive number**
    - *FV* is the future value (for an annuity this will be **zero** as the balance at the end of the payment period will be zero)
    - *P/Y* is the number of payments per year
    - *C/Y* is the **compounding periods** per year
    - *PMT@* is the time of the year or month the payment is made (usually the start)
  - Leave the section that you need to find out blank and fill in all other sections
  - Your GDC will fill in the last part for you
- Although you are unlikely to need to use it, the formula for calculating an annuity is:

$$FV = A\frac{(1+r)^n - 1}{r}$$

  - Where
    - *FV* is the future value
    - *A* is the amount invested
    - *n* is the number of years
    - *r%* is the interest rate as a decimal (e.g. at 6%, $r = 0.06$)
  - This formula is **not** given in the formula booklet, however your GDC will work out annuities for you so you do not need to remember it

Worked example

Janni invests 2 million DKK (Danish krone) into an annuity for her retirement. The annuity returns 3% compounded annually. Find the monthly payments Janni will receive if she wants the annuity to last for 25 years.

Use the finance / TVM solver on your GDC:

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
300	3	-2 000 000		0	12	1	START

- N: 25 years × 12
- PV: negative because this was invested
- PMT: GDC will fill this in for you.
- P/Y: paid monthly
- C/Y: compounding annualy
- PMT@: paid at the start of each month

PMT = 9418.95

Janni receives DKK 9419 each month