Exam Papers Practice

1.3 Sequences & Series

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Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.3 Sequences & Series
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

a) For a geometric sequence the common ratio, r, is given by

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3} \ldots$$

$u_2 = 44$ $\quad$ $u_3 = 55$

sub u_2 and u_3 into r formula

$$r = \frac{55}{44}$$

$$\boxed{r = \frac{5}{4}}$$

b) $r = \frac{u_2}{u_1}$

$$\frac{5}{4} = \frac{44}{u_1}$$

rearrange for u_1

$$u_1 = \frac{44}{\left(\frac{5}{4}\right)}$$

$$\boxed{u_1 = 35.2}$$

c) For a geometric sequence the sum of the first n terms is given by

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$ (in formula booklet)

sub u_1, r and $n = 5$

$$S_5 = \frac{35.2\left(\left(\frac{5}{4}\right)^5 - 1\right)}{\left(\frac{5}{4}\right) - 1}$$

$$S_5 = 288.8875$$

$$S_5 = 289 \text{ (3sf)}$$

Question 2

a) For an arithmetic sequence the sum of the first n terms is given by

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$S_{16} = 920 \qquad u_1 = 27.5$

sub in S_{16}, u_1 and $n = 16$

$920 = \frac{16}{2}(2(27.5) + (16-1)d)$

$920 = 8(55 + 15d)$

$115 = 55 + 15d$

$60 = 15d$

$\boxed{d = 4}$

b) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$S_{16} = 920 \qquad d = 11$

sub in S_{16}, d and $n = 16$

$920 = \frac{16}{2}(2u_1 + (16-1)(11))$

$920 = 8(2u_1 + 165)$

$115 = 2u_1 + 165$

$-50 = 2u_1$

$\boxed{u_1 = -25}$

Question 3

a) For a geometric sequence the sum of the first n terms is given by

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$ (in formula booklet)

$S_5 = 461.12$ $\quad u_1 = 200$

sub in S_5, u_1 and $n = 5$ and solve for r using your GDC

$$461.12 = \frac{200(r^5 - 1)}{r - 1}$$

$$r = \frac{3}{5}$$

Alternative GDC methods

- Plot $y = 461.12$ and $y = \frac{200(x^5 - 1)}{x - 1}$ and find intersection.
- Use the algebraic solver.

b) $S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$S_5 = 461.12 \qquad r = -2$

sub in S_5, r and $n = 5$ and solve for u_1 using your GDC

$$461.12 = \frac{u_1((-2)^5 - 1)}{(-2) - 1}$$

$$u_1 = 41.92 \text{ (2dp)}$$

Alternative GDC methods

- Plot $y = 461.12$ and $y = \frac{x((-2)^5 - 1)}{(-2) - 1}$ and find intersection.
- Use the algebraic solver.

a) For an arithmetic sequence the common difference, d, is given by

$d = u_2 - u_1 = u_3 - u_2 = u_4 - u_3 \ldots$

$a_2 = 12$ $\quad$ $a_3 = 30$

sub in u_2 and u_3 into d formula

$d = 30 - 12$

$\boxed{d = 18}$

Use $d = 18$ to find a_1 and a_4

$18 = 12 - a_1$ $\quad$ $18 = a_4 - 30$

$\boxed{a_1 = -6}$ $\quad$ $\boxed{a_4 = 48}$

b) For a geometric sequence the common ratio, r, is given by

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3} \dots$$

$b_2 = 12$ $\quad$ $b_3 = 30$

sub in b_2 and b_3 into r formula

$$r = \frac{30}{12}$$

$$r = 2.5$$

use $r = 2.5$ to find b_1 and b_4

$$2.5 = \frac{12}{b_1} \qquad 2.5 = \frac{b_4}{30}$$

$$b_1 = 4.8 \qquad b_4 = 75$$

c) The nth term formula for an arithmetic sequence is given by

$u_n = u_1 + (n-1)d$ (in formula booklet)

$c_1 = 80$ $c_4 = 10$

sub c_1 and c_4 into the nth term formula to find d.

$10 = 80 + (4-1)d$

$3d = 10 - 80$

$3d = -70$

$d = -\frac{70}{3}$

Use the nth term formula to find c_2 and c_3

$c_2 = 80 + (2-1)(-\frac{70}{3})$ $c_3 = 80 + (3-1)(-\frac{70}{3})$

$c_2 = \frac{170}{3}$ $c_3 = \frac{100}{3}$

d) The nth term formula for a geometric sequence is given by

$u_n = u_1 r^{n-1}$ (in formula booklet)

$d_1 = 80$ $d_4 = 10$

sub d_1 and d_4 into the nth term formula to find r

$10 = 80 r^{4-1}$

$r^3 = \frac{10}{80}$

$r = \left(\frac{1}{8}\right)^{1/3}$

$r = \frac{1}{2}$

Use the nth term formula to find d_2 and d_3

$d_2 = 80\left(\frac{1}{2}\right)^{2-1}$ $d_3 = 80\left(\frac{1}{2}\right)^{3-1}$

$d_2 = 40$ $d_3 = 20$

Question 5

a) i) $u_n = u_1 + (n-1)d$ (in formula booklet)

$u_4 = 20$ $\quad$ $u_{10} = 44$

$20 = u_1 + (4-1)d$ $\quad$ $44 = u_1 + (10-1)d$

$20 = u_1 + 3d$ ① $\quad$ $44 = u_1 + 9d$ ②

② − ①

$$\begin{array}{r} 44 = u_1 + 9d \\ -20 = u_1 + 3d \\ \hline 24 = \quad 6d \end{array}$$

$d = 4$

ii) sub $d = 4$ into ① to find u_1

$20 = u_1 + 3(4)$

$20 = u_1 + 12$

$20 - 12 = u_1$

$u_1 = 8$

b) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$n = 20$ $\quad u_1 = 8$ $\quad d = 4$

sub n, u_1 and d into S_n formula

$S_{20} = \frac{20}{2}(2(8) + (20-1)(4))$

$S_{20} = 10(16 + 76)$

$S_{20} = 10(92)$

$S_{20} = 920$ students

Question 6 a) Identify the arithmetic sequence

The 10th week will be u_{10} in the sequence.

$u_n = u_1 + (n-1)d$ (in formula booklet)

$u_1 = 4$ $\quad d = 1.5$

sub in u_1 and d into the formula to find u_{10}.

$u_{10} = 4 + (10-1)(1.5)$

$u_{10} = 4 + 9(1.5)$

$u_{10} = 4 + 13.5$

$u_{10} = 17.5$ km

b) $u_n = u_1 + (n-1)d$ (in formula booklet)

$u_1 = 4$ $\quad d = 1.5$ $\quad u_n = 26.5$

sub in u_1, d and u_n into the formula to find n

$26.5 = 4 + (n-1)(1.5)$

$22.5 = (n-1)(1.5)$

$\frac{22.5}{1.5} = n-1$

$15 = n-1$

$n = 16$

∴ Marie runs 26.5 km in the 16th week.

Alternative GDC methods

- Plot $y = 26.5$ and $y = 4 + (x-1)(1.5)$ and find intersection.
- Use the algebraic solver.

c) $S_n = \frac{n}{2}(2u_1 + (n-1)d$ (in formula booklet)

$u_1 = 4 \quad d = 1.5 \quad S_n = 220$

sub in u_1, d and S_n into the formula

$220 = \frac{n}{2}(2(4) + (n-1)(1.5))$

$440 = n(8 + (n-1)(1.5))$

Put the equation into the algebraic solver in your GDC.

$n = 15.0968$

$\therefore S_{15} < 220$

$S_{16} > 220$

Marie will complete a total of 220 km during the 16th week.

Alternative GDC method

- Plot $y = 220$ and $y = \frac{x}{2}(8 + (x-1)(1.5))$ and find intersection.

Question 7

a) i) $u_n = u_1 + (n-1)$ (in formula booklet)

$u_8 = 18$ $d = 2$

sub in u_8 and d into the formula to find u_1

$18 = u_1 + (8-1)(2)$

$18 = u_1 + 14$

$u_1 = 4$

ii) sub in u_1 and d into the formula to find u_{17}

$u_{17} = 4 + (17-1)(2)$

$u_{17} = 4 + 32$

$u_{17} = 36$

b) Geometric sequence

$u_3 = 4$ $\quad u_5 = 36$

$u_n = u_1 r^{n-1}$ (in formula booklet)

sub in u_3 and u_5 into formula

$4 = u_1 r^{3-1}$ $\quad 36 = u_1 r^{5-1}$

$4 = u_1 r^2$ ① $\quad 36 = u_1 r^4$ ②

② ÷ ①

$\frac{36}{4} = \frac{u_1 r^4}{u_1 r^2}$ i) $r = \pm 3$

$9 = r^2$

sub r into ① to find u_1

$4 = u_1 (\pm 3)^2$

$4 = u_1 (9)$ ii) $u_1 = \frac{4}{9}$

NB $(+3)^2 = (-3)^2$
$9 = 9$

Question 8 a) i) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_3 = 160$ $\quad r = \frac{1}{4}$

sub in u_3 and r into formula to find u_1

$160 = u_1 \left(\frac{1}{4}\right)^{3-1}$

$u_1 = \frac{160}{\left(\frac{1}{4}\right)^2}$

$u_1 = 2560$

ii) sub in u_1 and r into formula to find u_6

$u_6 = 2560 \left(\frac{1}{4}\right)^{6-1}$

$u_6 = 2560 \left(\frac{1}{4}\right)^5$

$u_6 = 2.5$

b) The sum of an infinite geometric sequence.

$S_\infty = \frac{u_1}{1-r}$, $|r| < 1$ (in formula booklet)

$u_1 = 2560$ $\quad r = \frac{1}{4}$

$S_\infty = \frac{2560}{1 - \frac{1}{4}} = \frac{10\,240}{3} = 3413.333...$

$S_\infty = \frac{10\,240}{3}$ or 3410 (3sf)

c) i) Arithmetic sequence

$u_7 = 2560$ $u_9 = 160$

$u_n = u_1 + (n-1)d$ (in formula booklet)

sub u_7 and u_9 into formula

$2560 = u_1 + (7-1)d$
$2560 = u_1 + 6d$ ①

$160 = u_1 + (9-1)d$
$160 = u_1 + 8d$ ②

① − ②

$2560 = u_1 + 6d$
$-\ 160 = u_1 + 8d$
$2400 = -2d$

$d = -1200$

ii) sub d into ① to find u_1

$2560 = u_1 + 6(-1200)$
$2560 = u_1 - 7200$
$u_1 = 2560 + 7200$

$u_1 = 9760$

Alternative GDC methods

- Plot ① and ② and find intersection.
- Use the simultaneous equation solver.

Question 9

a) Using sigma notation

$a_1 + a_2 + a_3 + \ldots + a_{12} = \sum_{k=1}^{12} a_k$

$\sum_{k=1}^{12} (32 - 7k)$

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$a_1 = 25 \qquad d = -7 \qquad n = 12$

sub in a_1, d and n

$S_{12} = \frac{12}{2}[2(25) + (12-1)(-7))$

$S_{12} = -162$

Alternative GDC method using sigma notation.

b) Using sigma notation

$a_4 + a_5 + a_6 + \ldots + a_{15} = \sum_{k=4}^{15} a_k$

$$\sum_{k=4}^{15} (32 - 7k)$$

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$a_1 = 4 \qquad d = -7 \qquad n = 12$

sub in a_1, d and n

$S_{12} = \frac{12}{2}[2(4) + (12-1)(-7))$

$S_{12} = -414$

Alternative GDC method using sigma notation.

Question 10

a) Using sigma notation

$g_1 + g_2 + g_3 + \ldots + g_{10} = \sum_{k=1}^{10} g_k$

$\sum_{k=1}^{10} (4 \times 3^{k-1})$

$S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$g_1 = 4 \qquad r = 3 \qquad n = 10$

sub in g_1, r and n

$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$

$S_{10} = 118\,096$

$S_{10} = 118\,000$ (3sf)

Alternative GDC method using sigma notation.

b) Using sigma notation

$g_8 + g_9 + g_{10} + \ldots + g_{18} = \sum_{k=8}^{18} g_k$

$\sum_{k=8}^{18} (4 \times 3^{k-1})$

$S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$g_1 = 8748 \quad r = 3 \quad n = 11$

sub in g_1, r and n

$S_{11} = \frac{8748(3^{11} - 1)}{3 - 1}$

$S_{11} = 774\ 836\ 604$

$S_{11} = 775\ 000\ 000$ (3sf)

Alternative GDC method using sigma notation.

Question 11

a) Identify the geometric sequence.

The common ratio, r, will be equal to the percentage of the remaining population every year (as a decimal).

Population decrease is 2% (0.02) every year.
Therefore the remaining population every year is

$100\% - 2\% = 98\%$

$1 - 0.02 = 0.98$

Hence $r = 0.98$

$u_1 = 68\,000$ $\quad r = 0.98$

Be sure to select the correct value for n.

u_1: 2021, u_2: 2022, u_3: 2023 ... u_{10}: 2030

$u_n = u_1 r^{n-1}$ (in formula booklet)

$u_{10} = 68000\,(0.98)^{10-1}$

$u_{10} = 56\,694.84782$

The expected population of kiwis in 2030 is 56 700.

b) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 68\,000$ $r = 0.98$ $u_n < 50\,000$

sub in u_1, r and u_n into the formula

$50\,000 > 68000(0.98)^{n-1}$

solve the equation for n using your GDC swapping the inequality (>) to an equal sign (=)

$50\,000 = 68000(0.98)^{n-1}$

$n = 16.22$

$\therefore u_{16} > 50\,000$ $u_{17} < 50\,000$

u_{16}: 2036 u_{17}: 2037

The population of kiwis will fall below 50 000 in 2036.

Question 12

a) Identify the geometric sequence

$u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 240$ $r = 1.125$ $n = 5$

sub in u_1, r and n

$u_5 = 240(1.125)^4$

$u_5 = 384$ km (3sf)

b) $S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$u_1 = 240$ $r = 1.125$ $n = 10$

sub in u_1, r and n

$S_{10} = \frac{240(1.125^{10} - 1)}{1.125 - 1}$

$S_{10} = 4310$ km (3sf)

Question 13

a) i) $u_n = u_1 r^{n-1}$ (in formula booklet)

$u_1 = 0.5 \quad r = 3 \quad n = 4$

sub in u_1, r and n

$u_4 = 0.5(3)^{4-1}$

$u_4 = 13.5$

ii) $S_n = \frac{u_1(r^n - 1)}{r - 1}$ (in formula booklet)

$u_1 = 0.5 \quad r = 3 \quad n = 5$

sub in u_1, r and n

$S_5 = \frac{0.5(3^5 - 1)}{3 - 1}$

$S_5 = 60.5$

b) $u_n = u_1 + (n-1)d$ (in formula booklet)

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (in formula booklet)

$u_4 = 13.5$ $S_5 = 60.5$

$13.5 = u_1 + 3d$ ① $60.5 = \frac{5}{2}(2u_1 + 4d)$ ②

Input ① and ② into your GDC to solve for u_1 and d.

$u_1 = 9.3$ $d = 1.4$

Alternative GDC methods

- Plot ① and ② and find intersection.
- Input ① and ② into the simultaneous equation solver.

Question 14

a) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 5000 \quad r = 2.24\% \quad k = 4 \quad n = 8$

Sub PV, r, k and n into formula.

$FV = 5000\left(1+\frac{2.24}{100(4)}\right)^{(4)(8)}$

$FV \approx \$5976.31$ (2dp)

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 10\,000 \quad PV = 5000 \quad k = 2 \quad n = 10$

Sub FV, PV, k and n into formula and solve for r using your GDC.

$10\,000 = 5000\left(1+\frac{r}{100(2)}\right)^{(2)(10)}$

$r \approx 7.05\%$

Question 15

a) i) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 15000 \quad r = 4.78\% \quad k = 12$

Sub PV, r and k into formula.

$$FV = 15000\left(1+\frac{4.78}{100(12)}\right)^{12n}$$

ii) Sub $n = 3$ into expression for amount after 3 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(3)}$

$$FV \approx \$17\,307.94 \quad (2dp)$$

Sub $n = 5$ into expression for amount after 5 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(5)}$

$$FV \approx \$19\,040.64 \quad (2dp)$$

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 1.5(15\,000) = 22\,500$

$PV = 15\,000 \quad r = 4.78\% \quad k = 12$

Sub FV, PV, r and k into formula and solve for n using your GDC.

$22\,500 = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{12n}$

$n = 8.5$

$\therefore$ 8 years and 6 months

Robert will be 48 years and 6 months old.

Question 16

Sub $u_1 = 30(r-1)$ into ①.

$15.3 = \frac{30(1-r)(1-r^2)}{(1-r)}$ ↓ cancel

$15.3 = 30(1-r^2)$ ↓ rearrange

$r^2 = 1 - \frac{15.3}{30}$ ↓ $\sqrt{\ }$

$r = \sqrt{1 - \frac{15.3}{30}}$

$r = 0.7$