1.3 Financial Applications

Mark Schemes

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.3 Financial Applications
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) Simple interest formula

$FV = PV\left(1 + \frac{r}{100}n\right)$ (not in formula booklet)

$PV = 5000 \quad n = 9 \quad r = 3.5\%$

Sub PV, r and n into formula.

$FV = 5000\left(1 + \frac{3.5}{100} \cdot 9\right)$

$FV = \$6575$

b) Compound interest formula

$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 4000 \quad r = 2.5\% \quad k = 1 \quad n = 14$

Sub PV, r, k and n into formula.

$FV = 4000\left(1 + \frac{2.5}{100(1)}\right)^{(1)(14)}$

$FV \approx \$5651.90$ (2dp)

c) Bank A < Bank B

$5000\left(1 + \frac{3.5}{100}n\right) < 4000\left(1 + \frac{2.5}{100}\right)^n$

Find n when Bank A = Bank B on your GDC.

$5000\left(1 + \frac{3.5}{100}n\right) = 4000\left(1 + \frac{2.5}{100}\right)^n$

$^*n = 50.0095$

When $n = 50$ it's the start of 2071 and

Bank A > Bank B.

When $n = 51$ it's the start of 2072 and

Bank A < Bank B.

The amount in Bank B will surpass the amount in Bank A during 2071.

* Possible GDC methods to find n:

- Use the algebraic solver.
- Graph Bank A and Bank B and find their intersection.

Question 2

a) Compound interest formula

$FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 5000 \quad r = 2.24\% \quad k = 4 \quad n = 8$

Sub PV, r, k and n into formula.

$FV = 5000\left(1 + \frac{2.24}{100(4)}\right)^{(4)(8)}$

$FV \approx \$5976.31$ (2dp)

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 10\,000 \quad PV = 5000 \quad k = 2 \quad n = 10$

Sub FV, PV, k and n into formula and solve for r using your GDC.

$10\,000 = 5000\left(1+\frac{r}{100(2)}\right)^{(2)(10)}$

$r \approx 7.05\%$

Question 3

a) Depreciation formula

$FV = PV\left(1-\frac{r}{100}\right)^{n}$ (not in formula booklet)

$PV = 4499 \quad r = 9\% \quad n = 5$

Sub PV, r and n into formula

$FV = 4499\left(1-\frac{9}{100}\right)^{5}$

$FV \approx \$1807.52 \quad (2dp)$

b) Depreciation formula

$FV = PV\left(1 - \frac{r}{100}\right)^n$ (not in formula booklet)

$FV = 999 \quad PV = 4499 \quad r = 9\%$

Sub FV, PV and r into formula and solve for n using your GDC.

$999 = 4499\left(1 - \frac{9}{100}\right)^n$

$n = 15.9564$

$\therefore$ 15 years and 11 months

c) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
12	17.2	4499	-410.76	0	12	12	END

Hence the monthly repayment is \$410.76 (2dp).

Question 4

a)i) Depreciation formula

$FV = PV\left(1 - \frac{r}{100}\right)^n$ (not in formula booklet)

$FV = 14792 \quad PV = 20\,000 \quad n = 2$

Sub FV, PV and n into formula and solve for r using your GDC.

$14792 = 20\,000\left(1 - \frac{r}{100}\right)^2$

$r = 14\%$

ii) $PV = 20\,000 \quad r = 14\% \quad n = 5$

Sub PV, r and n into formula.

$FV = 20\,000\left(1 - \frac{14}{100}\right)^5$

$FV \approx \$9408.54 \text{ (2dp)}$

b) Depreciation formula

$FV = PV\left(1 - \frac{r}{100}\right)^n$ (not in formula booklet)

$FV = 4000 \quad PV = 20\,000 \quad r = 14\%$

Sub FV, PV and r into formula and solve for n using your GDC.

$4000 = 20\,000\left(1 - \frac{14}{100}\right)^n$

$n = 10.67$

$\therefore$ 10 years and 8 months

c) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
36	9	20 000	-635.99	0	12	12	END

Hence the monthly repayment is $635.99 (2dp).

Question 5

a) Terms of the loan require a 15% deposit.

∴ Loan amount = 85% of the van's value.

Loan amount = 18 000 × 85%

Loan amount = $15 300

b)i) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
16	12	15 300	-1218.05	0	4	4	END

Hence the monthly repayment is $1218.05 (2dp).

ii) Total amount paid = N × PMT

Total amount paid = 16 × 1218.05

Total amount paid = $19488.74

Question 6

a) i) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$PV = 15000 \quad r = 4.78\% \quad k = 12$

Sub PV, r and k into formula.

$$FV = 15000\left(1+\frac{4.78}{100(12)}\right)^{12n}$$

ii) Sub $n = 3$ into expression for amount after 3 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(3)}$

$FV \approx \$17\,307.94$ (2dp)

Sub $n = 5$ into expression for amount after 5 years.

$FV = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{(12)(5)}$

$FV \approx \$19\,040.64$ (2dp)

b) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 1.5(15\,000) = 22\,500$

$PV = 15\,000 \quad r = 4.78\% \quad k = 12$

Sub FV, PV, r and k into formula and solve for n using your GDC.

$22\,500 = 15\,000\left(1+\frac{4.78}{100(12)}\right)^{12n}$

$n = 8.5$

$\therefore$ 8 years and 6 months

Robert will be 48 years and 6 months old.

c) Compound interest formula

$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$ (in formula booklet)

$FV = 19\,040.64 \quad PV = 15\,000 \quad k = 4 \quad n = 5$

Sub FV, PV, k and n into formula and solve for r using your GDC.

$19\,040.64 = 15\,000\left(1+\frac{r}{100(4)}\right)^{(4)(5)}$

$r = 4.80\%$

Question 7

a) Use the finance / TVM solver on your GDC.

$N = 25 \text{ years} \times 12 \text{ months}$

$N = 300 \text{ periods}$

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
300	5.5	0	-500	321 018.72	12	12	END

Hence the investment's value in 25 years is $321 018.72 (2dp).

b) $N = \dfrac{FV}{P/Y \times PMT}$

$FV = 321\,018.72 \quad P/Y = 12 \quad PMT = 1250$

Sub FV, P/Y and PMT into formula.

$N = \dfrac{321\,018.72}{12 \times 1250}$

$N = 21.4$ years

c) $N = \frac{\text{Amount invested}}{P/Y \times PMT}$

Amount invested = 500×300

Amount invested = 150 000

P/Y = 12 PMT = 1250

Sub the amount invested, P/Y and PMT into formula.

$N = \frac{150\,000}{12 \times 1250}$

N = 10 years

Question 8

a)i) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
187.48	4.18	220 000	-1600	0	12	12	END

N = 187.48

Number of years = $\frac{187.48}{12}$

Number of years = 15.62

∴ 15 years and 8 months

ii) Total amount paid = N × PMT

Total amount paid = 187.48×1600

Total amount paid = $299 970.16 (2dp)

b) Use the finance / TVM solver on your GDC.

N = 10 years × 12 months

N = 120 periods

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
120	4.18	220 000	-2246.26	0	12	12	END

Hence the new monthly repayment is $2246.26 (2dp).

The decision to pay off the loan faster means Lily will end up paying less overall.

Question 9

a) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
21.26	3.5	0	-2500	80 000	1	2	BEGIN

$N \approx 21.3$ years

b) Use the finance / TVM solver on your GDC.

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
20	4.8036	0	-3500	120 000	1	4	BEGIN

$\therefore r \approx 4.80\%$