Exam Papers Practice

1.2 Exponentials & Logs

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Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.2 Exponentials & Logs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

(a) ln and e cancel each other out

$\ln e = 1$

(b) Write 16 as a power with the same base number as the log so they cancel out

$2^4 = 16$

$\log_2 2^4 = 4$

$\log_2 16 = 4$

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 25 + \log 4 = \log 100$

$= \log 10^2$ ← The bases are the same so they cancel

$= 2$

$\log 25 + \log 4 = 2$

(d) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log_5 500 - \log_5 4 = \log_5 125$ ← Rewrite 125 as a power of 5

$= \log_5 5^3$ ← The bases are the same so they cancel

$= 3$

$\log_5 500 - \log_5 4 = 3$

Question 2

(a) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\ln 5 = \ln 15 - \ln 3$

$\ln 5 = x - y$

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 45 = \ln 15 + \ln 3$

$\ln 45 = x + y$

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\ln 135 = \ln 45 + \ln 3$

$= (x + y) + y$

$\ln 135 = x + 2y$

Question 3

(a) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 24 = \log 2 + \log 12$

$\log 24 = r + s$

(b) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 4 = \log 2 + \log 2$

$\log 4 = 2r$

$\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 3 = \log 12 - \log 4$

$\log 3 = s - 2r$

(c) $\log_a \frac{x}{y} = \log_a x - \log_a y$ ← Formula booklet

$\log 6 = \log 12 - \log 2$

$= s - r$

$\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log 72 = \log 6 + \log 12$

$= (s - r) + s$

$\log 72 = 2s - r$

Question 4

a) $\frac{(4xy^{-2})(-12x^{-4}y^{12})}{6x^2y}$ expand numerator

$\frac{-48x^{-3}y^{10}}{6x^2y}$ cancelling

$\frac{-\overset{-8}{\cancel{48}}x^{\cancel{-3}}y^{\cancel{10}}y^{9}}{\cancel{6}x^{\cancel{2}}\cancel{y}}$ x^5

$\frac{-8y^9}{x^5}$

b) $(2x^{-1}y^{-2})^{-3}(4x^2y^3)^4$

rewrite as fraction

$$\frac{(4x^2y^3)^4}{(2x^{-1}y^{-2})^3}$$

expand numerator and denominator

$$\frac{256x^8y^{12}}{8x^{-3}y^{-6}}$$

cancelling

$$\frac{\cancel{256}^{32}\,\cancel{x^8}^{x^{11}}\,\cancel{y^{12}}\,y^{18}}{\cancel{8}\,\cancel{x^{-3}}\,\cancel{y^{-6}}}$$

$$32x^{11}y^{18}$$

c) $\sqrt{(9x^6y^{-2}z^4)^3}(3xyz)^{-2}$

rewrite as a fraction and use indice laws

$$\frac{(9x^6y^{-2}z^4)^{\frac{3}{2}}}{(3xyz)^2}$$

expand numerator and denominator

$$\frac{27x^9y^{-3}z^6}{9x^2y^2z^2}$$

cancelling

$$\frac{\cancel{27}^{3}\,\cancel{x^9}^{x^7}\,\cancel{y^{-3}}\,\cancel{z^6}^{z^4}}{\cancel{9}\,\cancel{x^2}\,\cancel{y^2}\,\cancel{z^2}\;\;y^5}$$

$$\frac{3x^7z^4}{y^5}$$

Question 5

$$2 - x\sqrt{3} = \frac{7x}{\sqrt{3}}$$

$$2\sqrt{3} - 3x = 7x$$

$$2\sqrt{3} = 10x$$

$$\frac{2\sqrt{3}}{10} = x$$

$$\frac{\sqrt{3}}{5} = x$$

$$a = 3 \quad b = 5$$

Question 6

(a) $\log_a x^m = m \log_a x$ ← Formula booklet

$$\log_a 64 = \log_a 8^2$$

$$= 2\log_a 8$$

$$= 2 \times 3$$

$$\log_a 64 = 6$$

(b) $a^x = b \iff x = \log_a b$ ← Formula booklet

$$\log_a 8 = 3 \Rightarrow a^3 = 8$$

$$a = \sqrt[3]{8}$$

$$a = 2$$

(c) $a^x = b \iff x = \log_a b$ ← Formula booklet

$\log_{2^2} 8 = x \Rightarrow (2^2)^x = 8$

$2^{2x} = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

Question 7

(a) $\log_b 9 = \log_b 3^2$

$= 2\log_b 3$ ← $\log_a x^m = m\log_a x$ ← Formula booklet

$= 2x$

$\log_b 9 = 2x$

(b) $\log_b 4 = \log_b \sqrt{16}$

$= \log_b 16^{\frac{1}{2}}$

$= \frac{1}{2}\log_b 16$

$= \frac{1}{2}y$

$\log_b 4 = \frac{1}{2}y$

(c) $\log_a xy = \log_a x + \log_a y$ ← Formula booklet

$\log_b 48 = \log_b 3 + \log_b 16$

$\log_b 48 = x + y$

Question 8

(a) Expand the numerator

$$\frac{16 - 16\sqrt{x} + 4x}{8x}$$

Split into 3 separate terms and cancel

$$\frac{\cancel{16}^{2}}{\cancel{8x}_{1}} - \frac{\cancel{16}^{2}\sqrt{x}}{\cancel{8x}_{1}} + \frac{\cancel{4x}^{1}}{\cancel{8x}_{2}}$$

$$\frac{2}{x} - \frac{2\sqrt{x}}{x} + \frac{1}{2}$$

Rewrite powers of x

$$2x^{-1} - 2x^{\frac{1}{2}}x^{-1} + \frac{1}{2}$$

$$2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$$

$$2x^{-1} - 2x^{-\frac{1}{2}} + \frac{1}{2}$$

(b) $8 = 2^3$, $\sqrt{2} = 2^{1/2}$

$8\sqrt{2} = 2^3 \times 2^{1/2}$

$= 2^{7/2}$

$a = \frac{7}{2}$

(c) Expand the numerator

$$\frac{2x^5 - x^{3/2}}{x^2}$$

Simplify the powers of x

$2x^3 - x^{-1/2}$

$a = 3 \quad b = -\frac{1}{2}$

Question 9

Rewrite expression using powers of 4

$(4^2)^x - 3(4^x)(4^1) = 28$

$(4^x)^2 - 12(4^x) = 28$

Let $m = 4^x$

$m^2 - 12m - 28 = 0$

Solve the quadratic by hand or using the GDC

$(m - 14)(m + 2) = 0$

$m = 14$ or $m = -2$ ← m cannot be negative because you can't take a log of a negative

$4^x = 14$

Take ln of both sides

$\ln 4^x = \ln 14$

$x \ln 4 = \ln 14$ ← $\log_a x^m = m \log_a x$ ← Formula booklet

$$x = \frac{\ln 14}{\ln 4}$$

Question 10

Find the largest square number that goes into 425

$\sqrt{425} = \sqrt{25 \times 17}$

$= 5\sqrt{17}$

$a = 5 \quad b = 17$

Question 11

Change each number to a base of 2.

$2^{2x} - 3(2^x)(2^1) = -8$

$(2^x)^2 - 6(2^x) + 8 = 0$

Let $X = 2^x$

$X^2 - 6X + 8 = 0$

$(X - 4)(X - 2) = 0$

$X = 4$ $\qquad$ $X = 2$

$2^x = 4$ $\qquad$ $2^x = 2$

$x = 2$ $\qquad$ $x = 1$