Exam Papers Practice

# 1.2 Uncertainties & Errors

## Mark Schemes

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| Course | DP IB Physics |
| Section | 1. Measurement & Uncertainties |
| Topic | 1.2 Uncertainties & Errors |
| Difficulty | Medium |



To be used by all students preparing for DP IB Physics SL
Students of other boards may also find this useful

1

The correct answer is **D** because:

- Random errors vary from measurement to measurement and can have different sizes, therefore random fluctuations can occur
- Systematic errors tend to have the same value and cause the measurements to be out by the same amount
  - Therefore, statement 1 applies to random errors
- Random errors can never be fully eliminated as they can be influenced by environmental conditions that are hard to control
- Systematic errors tend to be due to experimental set-up and apparatus, so they can be eliminated in a lot of cases
  - Therefore, statement 4 applies to random errors
- Random errors can be reduced by averaging repeated measurements, whereas systematic errors cannot
  - Therefore, statement 5 applies to random errors
- The correct statements are 1, 4 and 5

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| A is incorrect as | Statement 3 is incorrect as random errors cannot be eliminated because they are random<br><br>Statement 6 is incorrect as random errors **can** be reduced through averaging repeated measurements. This is because the errors are random so taking multiple measurements of the same quantity will reduce the error. |

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| **B** is incorrect as | Statement 2 is incorrect, as the error occurs randomly, so it cannot be constantly high or low by **the same amount**.<br><br>Statement 6 is incorrect as random errors **can** be reduced through averaging repeated measurements. This is because the errors are random so taking multiple measurements of the same quantity will eliminate a random error |
| **C** is incorrect as | Statement 2 is incorrect, as the error occurs randomly, so it cannot be constantly high or low by **the same amount**.<br><br>Statement 3 is incorrect as random errors cannot be eliminated because they are random<br><br>Statement 6 is incorrect as random errors **can** be reduced through averaging repeated measurements. This is because the errors are random so taking multiple measurements of the same quantity will eliminate a random error |

Knowing the exact differences between systematic and random error will help in answering questions about practical experiments and in conducting your own experiments.

2

The correct answer is **A** because:

- Random errors affect precision, meaning they cause differences in measurements which causes a spread about the mean
- Repeating the measurements at least three times and taking an average of them is one way to reduce random error
    - This reduces spread about the mean and enables any anomalies to be spotted and discounted from the results

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| **B** is incorrect as | random errors can be the result of either the person taking the measurement but also by the equipment being used or the environment |
| **C** is incorrect as | systematic error affects accuracy and occurs due to the apparatus used or faults in the experimental method so adjusting the apparatus would help reduce systematic error |
| **D** is incorrect as | systematic error affects accuracy, which causes all results to be too high or too low by the same amount each time |



Knowing about random and systematic errors can help in understanding how to complete practical work more accurately and precisely.

3

The correct answer is **D** because:

- The absolute uncertainty of the time needs to be converted into a percentage uncertainty:
  - $\frac{0.3}{3} \times 100 = 10\%$
  - The uncertainty in the time is 10%
- The uncertainty needs to be propagated through the calculation
- In the equation $d = \frac{1}{2}at^2$, the $t$ is squared
- To propagate the error, the percentage uncertainty must be multiplied by 2
  - 10% × 2 = 20% - this is the % uncertainty in $t$
- The uncertainty of $a$ is negligible, so the total percentage error in the value of $d$ is 20%
- The 20% uncertainty of $d$ = 30 m is:
  - 20 % × 30 = 6 m
- So, the absolute uncertainty is ± 6 m

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| **A** is incorrect as | ± 0.6 m is the absolute value of the error in the time 0.3 multiplied by 2. The error must be converted into a percentage error to be propagated through the calculation |
| **B** is incorrect as | The percentage uncertainty in the time is 10%. ± 3 m is an uncertainty of 10% of the 30 m depth. The error has not been propagated in the equation, as it has not considered that the time is **squared** |
| **C** is incorrect as | This is the depth 30 m minus the value calculated for the absolute uncertainty of the depth of 6 m. The uncertainty needs to be written as a ± and not as the lower value of the range that this gives |

Learning the rules to propagate errors into equations makes these questions much easier. Remember also to convert any errors into percentage errors before propagating if two values are multiplied or divided (if they are added or subtracted, add their absolute uncertainties).

4

The correct answer is **A** because:

- The equation relating speed, distance and time is given by:
  - Speed, $v = \frac{\text{distance (s)}}{\text{time (t)}}$
- The value and uncertainty in distance is:
  - $s = 285 - 15 = 270$ mm
- If two values are added together, their **absolute** uncertainties are added together
  - Uncertainty of distance: $\Delta s = \pm(1 + 1) = \pm 2$ mm
- The value and uncertainty in time is:
  - $t = 4.50 - 2.50 = 2.00$ s
- Uncertainties of time are also added:
  - $\Delta t = \pm(0.02 + 0.02) = \pm 0.04$ s
- To work out the fractional uncertainty in speed:
  - $\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t}$
  - $\frac{\Delta v}{v} = \frac{2}{270} + \frac{0.04}{2.00}$
  - Hence, the correct answer is **A**

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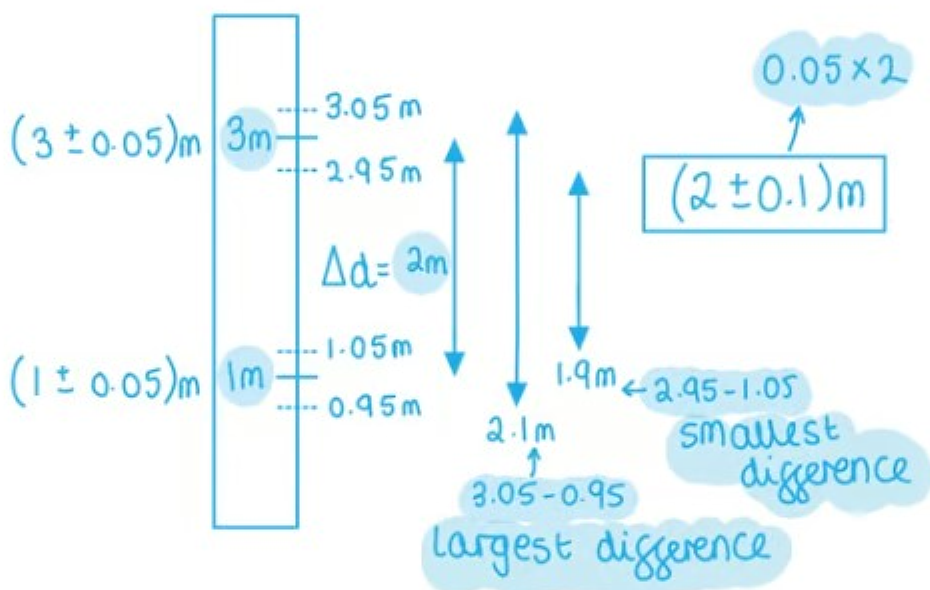
| **B** is incorrect as | The absolute uncertainties for the distance and time need to be added together. For distance this is ± 2 mm and not just<br><br>± 1 mm. For time, this is ± 0.04 s and not just ± 0.02 s |
| **C** is incorrect as | The fractional uncertainty is given by the $\frac{\text{uncertainty}}{\text{total distance or total time}}$ and not by the initial value of the time and distance |
| **D** is incorrect as | The fractional uncertainty is given by the $\frac{\text{uncertainty}}{\text{total distance or total time}}$ and not by the final value of the time and distance |

This question requires a methodical approach in finding the fractional errors of both time and distance. You also need to know how to express fractional errors and how this is different to absolute and percentage.

The uncertainty in the **difference** between two distances is always double the uncertainty of each distance because the amount of error on either end has no doubled. This is demonstrated in the sketch below:



5

The correct answer is **B** because:

- Calculate the percentage uncertainty of both $k$ and $x$:
  - Percentage uncertainty = $\frac{\text{uncertainty}}{\text{value}}$ x 100
  - Percentage uncertainty for $k = \frac{1}{200}$ x 100 = 0.5 %
  - Percentage uncertainty for $x = \frac{0.004}{0.04}$ x 100 = 10%
- Now, propagate the uncertainties into the equation $W = \frac{1}{2}kx^2$
  - For $x^2$, the uncertainty needs to be multiplied by the power, 2
    - 10% × 2 = 20%
  - For $W$ the uncertainty of $k$ needs to be added to the uncertainty for $x^2$
    - 0.5 + 20 = 20.5%
  - So, the percentage uncertainty in $W$ is ± 20.5 %

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| **A** is incorrect as | This is just the percentage uncertainty of $x^2$ and not W. |
| **C** is incorrect as | This is the incorrect propagation of the uncertainty of $k$ and $x$.<br>$k \times x$ is 20 × 0.5 = 10 %<br>Quantities that are multiplied have their uncertainties propagated by addition and not multiplication. |
| **D** is incorrect as | This is the propagation of the uncertainty of $k$ and $x^2$.<br>$k + x$ is 10 + 0.5 = 10.5 % |

Learning the rules for propagating uncertainties will help in completing questions like this much easier. There are:

- Values that are added or subtracted: Add their **absolute** uncertainties
- Values that are multiplied or divided: Add their **percentage** or **fractional** uncertainties
- Values that are to a power: Multiply their **percentage** or **fractional** uncertainty to the power

6

The correct answer is **C** because:

- To obtain the largest value of Z we must divide the largest value of $m_1 - m_2$ by the smallest values of I and *t*.
    - $Z_{large} = \frac{(m_1 - m_2)_{large}}{It_{small}} = \frac{m_{1_{large}} - m_{2_{small}}}{I_{small} \times t_{small}}$
- This gives a value of:
    - $Z_{large} = \frac{(54.39 + 0.01) - (52.06 - 0.01) \times 10^{-3}}{(3 - 1) \times (4800 - 100)}$
    - $Z_{large} = \frac{(54.40 \times 10^{-3}) - (52.05 \times 10^{-3})}{(2) \times (4700)}$
    - $Z_{large} = \frac{(2.35 \times 10^{-3})}{(9400)} = \frac{(2350 \times 10^{-6})}{(9400)} = \frac{(235 \times 10^{-6})}{(940)}$
    - So, $Z_{large} = \frac{235}{940} \times 10^{-6}$ kg C$^{-1}$

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| **A** is incorrect as | The large value of $m_1 - m_2$ is not used only the absolute values are used.<br>$m_1 - m_2$ = 54.39 – 52.06 = 2.33 and this gives the incorrect value for $Z_{large}$ |

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| **B** is incorrect as | The small value of $m_1 - m_2$ is used and not the large value. $m_1 - m_2$ = 54.38 – 52.07 = 2.31 and this gives the incorrect value for $Z_{large}$ |
| **D** is incorrect as | This is the value of $Z$ using all absolute values $\frac{(2.33 \times 10^{-3})}{(3 \times 4800)}$ and not the values ± uncertainty |

If a quantity is equal to a fraction; it can be made larger by increasing the numerator **or** by decreasing the denominator.

This question seems tricky without a calculator but make sure you show your numerical workings on paper with plenty of space. It is possible to do this without a calculator. If you find you are spending too much time doing this, then move on to the next question.

7

The correct answer is **C** because:

- The percentage uncertainty each side of the square is
  - $\frac{0.2}{8} \times 100 = 2.5\%$
- When area is calculated the two sides are multiplied together, so the uncertainties must be added:
  - $2.5\% + 2.5\% = 5\%$
- The absolute uncertainty for the area will be:
  - $64 \times 0.05 = 64 \times \frac{1}{20} = \frac{32}{10} = 3.2$ cm$^2$

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| **A** is incorrect as | The absolute uncertainty for the area of the square is not the same as for the length |
| **B** is incorrect as | The absolute uncertainty for the area needs to be propagated using the rules of combining uncertainties. When two quantities are **multiplied** together their **percentage** uncertainties are added and not just their absolute uncertainties |

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| **D** is incorrect as | The percentage uncertainty for the area of the square has not been propagated. The area uncertainty comes from combining the percentage uncertainty from each length and not just using 2.5% |

You must learn to convert all uncertainties into percentage uncertainties before completing any calculations if two values are multiplied or divided.

8

The correct answer is **A** because:

- Average speed = $\frac{\text{distance}}{\text{time}} = \frac{30.0}{1.50}$ = 20.0 m s$^{-1}$ (3 s.f.)
- Calculate the percentage uncertainties for distance and time
  - % uncertainty = $\frac{\text{uncertainty}}{\text{value}}$ × 100
  - Distance *s*: % uncertainty = $\frac{0.1}{30}$ × 100 = 0.33%
  - Time *t*: % uncertainty = $\frac{0.05}{1.5}$ × 100 = 3.33%
- To work out uncertainty in data which is multiplied or divided we add the percentage uncertainties
  - 0.33 + 3.33 = 3.66%
- The absolute uncertainty of the speed will be 3.66% of 20 m s$^{-1}$
  - 20 × 0.0366 = 0.732 m s$^{-1}$
- So, the final answer is the average speed = 20.0 ± 0.732 m s$^{-1}$

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| **B** is incorrect as | The total percentage uncertainty of the speed from the combined uncertainty of the distance and time is 3.66%. This has been used as the absolute uncertainty for the speed of ± 0.0366 m s$^{-1}$ |
| **C** is incorrect as | Only the uncertainty for the distance 0.33% has been propagated through the equation. This should be combined with the uncertainty for the time before the absolute uncertainty for the speed is calculated |

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| **D** is incorrect as | The percentage uncertainties for distance and time have been divided and not added together. Quantities that are divided or multiplied should have their uncertainties added together |

You must learn to convert all uncertainties into percentage uncertainties before completing any calculations.

For calculations such as $\frac{0.1}{30}$ × 100, multiply by the 100 first with the numerator i.e. $\frac{0.1 \text{ x } 100}{30} = \frac{10}{30}$ = 0.33, as this is much easier.

For calculations such as 20 × 0.0366, first, × 10 then × 2. 0.0366 × 10 = 0.366 × 2 = 0.732.

9

The correct answer is **C** because:

- The absolute uncertainty in any reading is always quoted to 1 significant figure
  - This eliminates options **B** and **D**
- Measuring the temperature difference requires two single readings
  - Therefore, the temperature rise $\Delta T = T_2 - T_1$
- $T_2$ is approximately (88.0 ± 0.5) degrees
  - The absolute uncertainty is 0.5 degrees because in a single reading of an analogue instrument, the absolute uncertainty is half the width of the smallest gradations
- Similarly, $T_1$ is approximately (34.0 ± 0.5) degrees
- Therefore, propagating the uncertainties of two quantities that are subtracted from each other means summing the individual absolute uncertainties:
  - Hence, the absolute error in $\Delta T$ is 0.5 + 0.5 = 1 degree (to 1 significant figure)

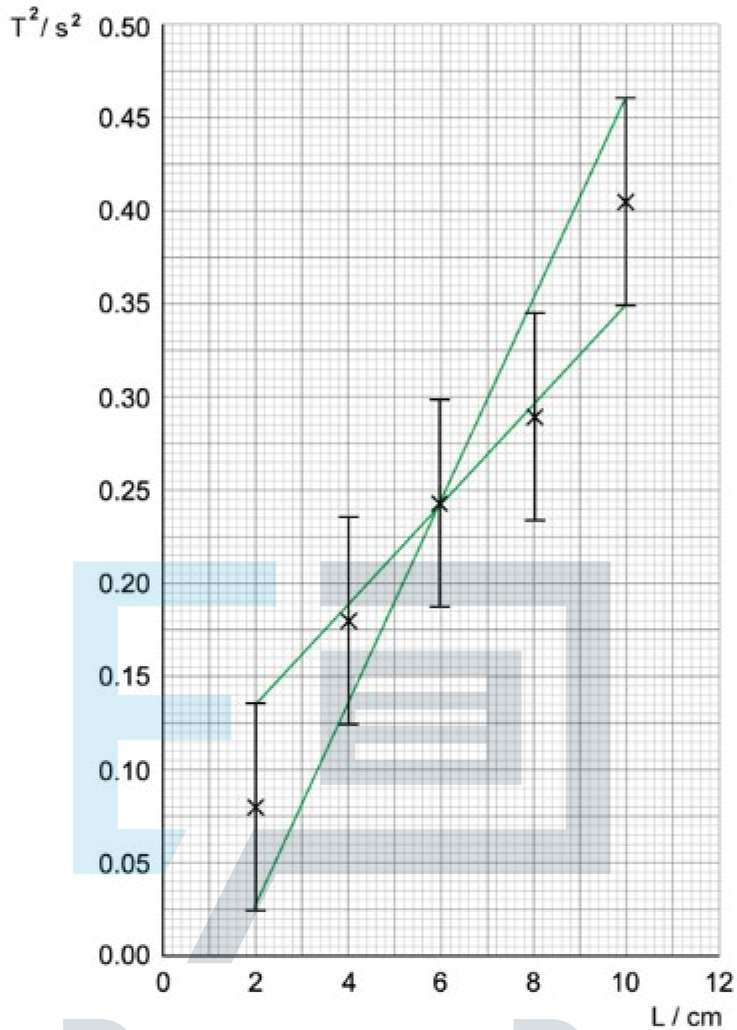
- Therefore, the best estimate for the temperature rise is (54 ± 1) degrees
  - This is as expressed in option **C**

You might be tempted to think that the answer is (54.0 ± 0.5) degrees, but in the **strictest** and most **consistent** application of the rules of propagating uncertainties, you must not neglect that in order to take a reading of temperature difference (and arguably, all things that require a reading comparing 'both ends' of a scale) you must take **both** absolute uncertainties into account. As long as you make your reasoning clear in your examination, you will receive full credit.

10

The correct answer is **A** because:

- $G_{max}$ = the maximum possible, or steepest, gradient that could be drawn through the data
  - $G_{max} = \frac{0.455 - 0.025}{10 - 2} = \frac{0.430}{8} = \frac{430}{8000}$
- $G_{min}$ = the minimum possible, or shallowest, gradient that could be drawn through the data
  - $G_{min} = \frac{0.350 - 0.135}{10 - 2} = \frac{0.215}{8} = \frac{215}{8000}$






- To find the uncertainty in the gradient of the graph, we can use
  - % uncertainty = $\frac{\frac{G_{max} - G_{min}}{2}}{} \times 100\%$
  - % uncertainty = $\frac{\frac{430}{8000} - \frac{215}{8000}}{2} \times 100\%$
  - % uncertainty = $\frac{\frac{215}{8000}}{2} \times 100\%$
  - % uncertainty = $\frac{215}{16000} \times 100\% = \frac{21500}{16000} = \frac{215}{160}$
  - Dividing the numerator and denominator by 5 gives: $\frac{43}{32}$ %

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| **B** is incorrect as | the percentage uncertainty is calculated by dividing the range by 2 and then x by 100 and not just finding the range of the values and then x by 100 |
| **C** is incorrect as | this is the value of $G_{max}$ and not the percentage uncertainty |
| **D** is incorrect as | this is the value of $G_{min}$ and not the percentage uncertainty |

Remember to calculate the maximum possible gradient by drawing a line from the highest point on the highest error bar to the lowest point on the lowest error bar. Then use the lowest point on the highest error bar and the highest point on the lowest error bar to find the minimum possible gradient.

Keep your answers in fractions and take lots of space for your workings, so you don't make an error when manipulating them.