EXAM PAPERS PRACTICE

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Detailed mark scheme

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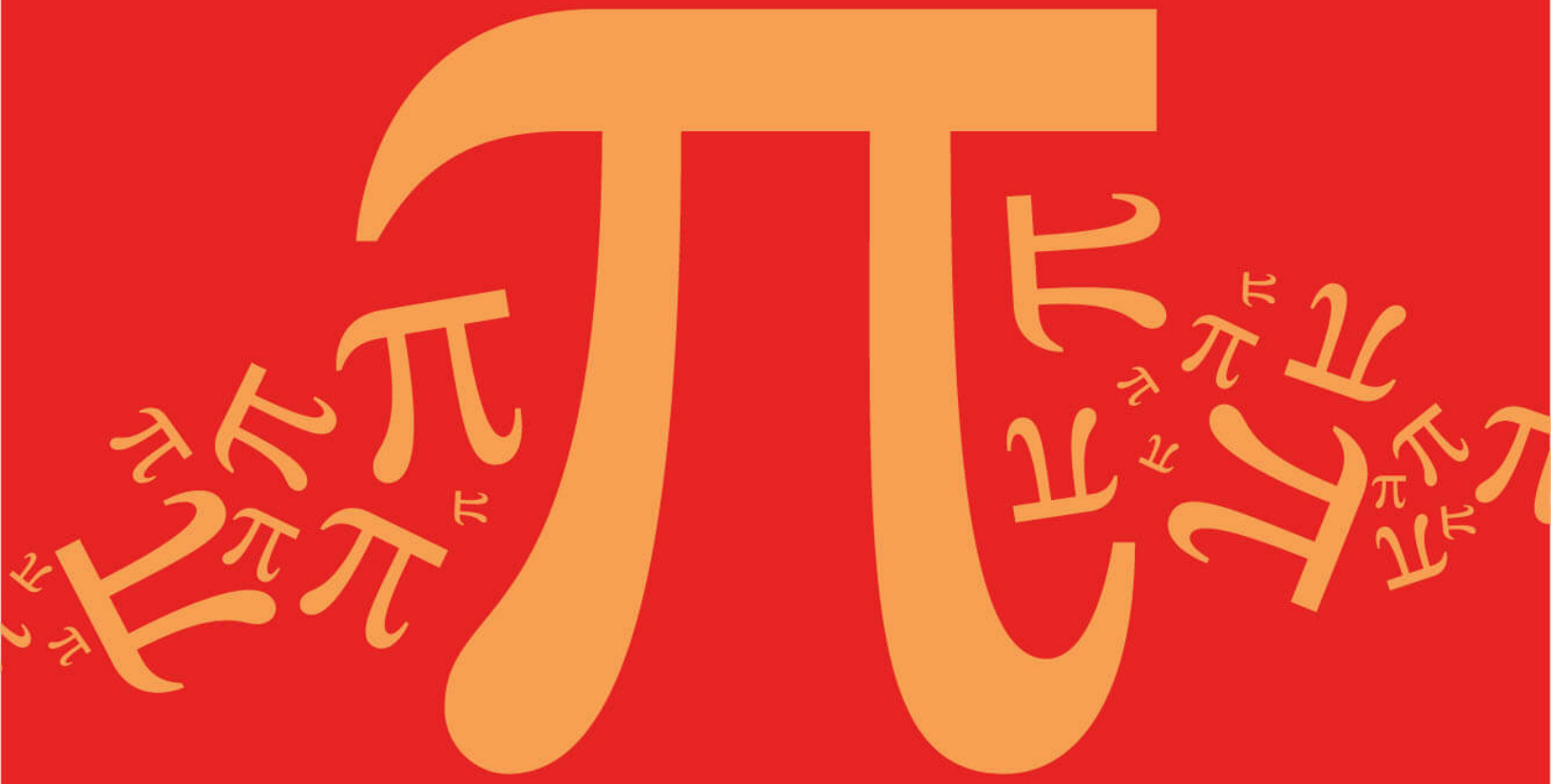
1.1 Number Toolkit



IB Maths - Revision Notes

AA SL

1.1.1 Standard Form

Standard Form

Standard form (sometimes called **scientific notation** or **standard index form**) gives us a way of writing very big and very small numbers using powers of 10.

Why use standard form?

- Some numbers are too big or too small to write easily or for your calculator to display at all
  - Imagine the number 50^{50}, the answer would take 84 digits to write out
  - Try typing 50^{50} into your calculator, you will see it displayed in **standard form**
- Writing very big or very small numbers in standard form allows us to:
  - Write them more neatly
  - Compare them more easily
  - Carry out calculations more easily
- Exam questions could ask for your answer to be written in standard form

How is standard form written?

- In standard form numbers are always written in the form $a \times 10^k$ where a and k satisfy the following conditions:
  - $1 \leq a < 10$
    - So there is one non-zero digit before the decimal point
  - $k \in \mathbb{Z}$
    - So k must be an integer
  - $k > 0$ for large numbers
    - How many times a is multiplied by 10
  - $k < 0$ for small numbers
    - How many times a is divided by 10

How are calculations carried out with standard form?

- Your GDC will display large and small numbers in standard form when it is in normal mode
  - Your GDC may display standard form as *a*E*n*
    - For example, 2.1×10^{-5} will be displayed as $2.1\text{E}-5$
    - If so, be careful to **rewrite the answer given in the correct form**, you will not get marks for copying directly from your GDC
- Your GDC will be able to carry out calculations in standard form

- If you put your GDC into scientific mode it will automatically convert numbers into standard form
  - Beware that your GDC may have more than one mode when in scientific mode
  - This relates to the number of significant figures the answer will be displayed in
  - Your GDC may add extra zeros to fill spaces if working with a high number of significant figures, you do not need to write these in your answer
- To add or subtract numbers written in the form $a \times 10^k$ without your GDC you will need to write them in full form first
  - Alternatively you can use 'matching powers of 10', because if the powers of 10 are the same, then the 'number parts' at the start can just be added or subtracted normally
    - For example
      $(6.3 \times 10^{14}) + (4.9 \times 10^{13}) = (6.3 \times 10^{14}) + (0.49 \times 10^{14}) = 6.79 \times 10^{14}$
    - Or
      $(7.93 \times 10^{-11}) - (5.2 \times 10^{-12}) = (7.93 \times 10^{-11}) - (0.52 \times 10^{-11}) = 7.41 \times 10^{-11}$
- To multiply or divide numbers written in the form $a \times 10^k$ without your GDC you can either write them in full form first or use the laws of indices

Exam Tip

- Your GDC will give very big or very small answers in standard form and will have a setting which will allow you to carry out calculations in scientific notation
- Make sure you are familiar with the form that your GDC gives answers in as it may be different to the form you are required to use in the exam

Worked example

Calculate the following, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

i) 3780×200

Using GDC: Choose scientific mode.

Input directly into GDC as ordinary numbers.

$3780 \times 200 = 7.56 \times 10^5$

GDC will automatically give answer in standard form.

Without GDC:

Calculate the value:

$3780 \times 200 = 756000$

Convert to standard form:

$756\,000 = 7.56 \times 10^5$

7.56×10^5

ii) $(7 \times 10^5) - (5 \times 10^4)$

Using GDC: Choose scientific mode.

Input directly into GDC

$7 \times 10^5 - 5 \times 10^4 = 6.5 \times 10^5$

This may be displayed as 6.5E5

Without GDC:

Convert to ordinary numbers:

$7 \times 10^5 = 700\,000$

$5 \times 10^4 = 50\,000$

Carry out the calculation:

$700\,000 - 50\,000 = 650\,000$

Convert to standard form:

$650\,000 = 6.5 \times 10^5$

6.5×10^5

iii) $(3.6 \times 10^{-3})(1.1 \times 10^{-5})$

Input directly into GDC:

(Choose scientific mode).

$(3.6 \times 10^{-3})(1.1 \times 10^{-5}) = 3.96 \times 10^{-8}$

3.96×10^{-8}

Note:

$10^{-3} \times 10^{-5} = 10^{-8}$

$(3.6 \times 10^{-3})(1.1 \times 10^{-5}) = 3.96 \times 10^{-8}$

$3.6 \times 1.1 = 3.96$

1.1.2 Laws of Indices

Laws of Indices

What are the laws of indices?

- Laws of indices (or index laws) allow you to simplify and manipulate expressions involving exponents
  - An exponent is a power that a number (called the base) is raised to
  - Laws of indices can be used when the numbers are written with the same base
- The index laws you need to know are:
  - $(xy)^m = x^m y^m$
  - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
  - $x^m \times x^n = x^{m+n}$
  - $x^m \div x^n = x^{m-n}$
  - $(x^m)^n = x^{mn}$
  - $x^1 = x$
  - $x^0 = 1$
  - $\frac{1}{x^m} = x^{-m}$
  - $x^{\frac{1}{n}} = \sqrt[n]{x}$
  - $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
- These laws are **not in the formula booklet** so you must remember them

How are laws of indices used?

- You will need to be able to carry out multiple calculations with the laws of indices
  - Take your time and apply each law individually
  - Work with numbers first and then with algebra
- Index laws only work with terms that have the same base, make sure you **change the base** of the term before using any of the index laws
  - Changing the base means rewriting the number as an exponent with the base you need
  - For example, $9^4 = (3^2)^4 = 3^{2\times4} = 3^8$
  - Using the above can them help with problems like $9^4 \div 3^7 = 3^8 \div 3^7 = 3^1 = 3$

Exam Tip

- Index laws are rarely a question on their own in the exam but are often needed to help you solve other problems, especially when working with logarithms or polynomials
- Look out for times when the laws of indices can be applied to help you solve a problem algebraically

Worked example

Simplify the following equations:

i) $\dfrac{(3x^2)(2x^3y^2)}{(6x^2y)}$.

Apply each law separately:

$3 \times 2 = 6$

$$\frac{(3x^2)(2x^3y^2)}{6x^2y}$$

expand numerator

$x^2 \times x^3 = x^5$

$$\frac{(6x^2)(x^3y^2)}{6x^2y}$$

cancelling

$$\frac{6x^5y^2}{6x^2y}$$

$x^5 \div x^2 = x^{5-2} = x^3$

$y^2 \div y = y^{2-1} = y$

$$x^3y$$

$$\frac{(3x^2)(2x^3y^2)}{6x^2y} = x^3y$$

ii) $(4x^2y^{-4})^3(2x^3y^{-1})^{-2}$.

$(4x^2y^{-4})^3(2x^3y^{-1})^{-2}$

Rewrite as a fraction

$$\frac{(4x^2y^{-4})^3}{(2x^3y^{-1})^2}$$

expand numerator and denominator

$$\frac{64x^6y^{-12}}{4x^6y^{-2}}$$

Cancelling

$$\frac{\cancel{64}\ \cancel{x^6}\ y^2}{\cancel{4}\ \cancel{x^6}\ y^{12}}$$

The negative exponents can be rewritten as their reciprocals

$$16y^{-10}$$

$$\boxed{\frac{16}{y^{10}}}$$