1.1 Number Toolkit

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Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.1 Number Toolkit
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) Sub a and b into Q.

$$Q = \frac{30 \sin 2(45^\circ)}{8(2)}$$

$$\boxed{Q = 1.875}$$

b) i) $\boxed{Q = 1.88 \quad (2dp)}$

ii) $\boxed{Q = 1.9 \quad (2sf)}$

c) Percentage error formula.

$$\epsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad \text{(in formula booklet)}$$

where V_A is the approximated value and V_E is the exact value.

$V_A = 2 \qquad V_E = 1.875$

Sub V_A and V_E into formula.

$$\epsilon = \left| \frac{2 - 1.875}{1.875} \right| \times 100\%$$

$$\epsilon = 6.666\ldots\%$$

$$\boxed{\epsilon = 6.67\% \quad (3sf)}$$

Question 2

a) Sub x and y into R.

$$R = \frac{4(1.25)}{6\cos 5(36°)}$$

$$R = -\frac{5}{6} \text{ (fraction)}$$

b) $R = -0.83333...$

i) $R = -0.8$ (1dp)

ii) $R = -0.833$ (3sf)

c) Percentage error formula.

$$\epsilon = \left|\frac{V_A - V_E}{V_E}\right| \times 100\%$$ (in formula booklet)

where V_E is the exact value and V_A is the approximated value.

$V_A = -1$ $V_E = -\frac{5}{6}$

Sub V_A and V_E into formula.

$$\epsilon = \left|\frac{-1 - (-\frac{5}{6})}{(-\frac{5}{6})}\right| \times 100\%$$

$\epsilon = 20\%$

Question 3

a) Sub a and b into C.

$$C = \sqrt[10]{\left(\frac{4.14 \times 10^{6}}{2.54 \times 10^{-7}}\right)^{3}}$$

$$C = 9197.0804...$$

$$C = 9197 \text{ (nearest integer)}$$

b) $C = 9.197 \times 10^{3}$

c) Percentage error formula.

$$\epsilon = \left|\frac{V_A - V_E}{V_E}\right| \times 100\% \quad \text{(in formula booklet)}$$

where V_E is the exact value and V_A is the approximated value.

$V_A = 9000$ $\quad V_E = \sqrt[10]{\left(\frac{4.14 \times 10^{6}}{2.54 \times 10^{-7}}\right)^{3}}$ (exact answer for C.)

Sub V_A and V_E into formula.

$$\epsilon = \left|\frac{9000 - \sqrt[10]{\left(\frac{4.14 \times 10^{6}}{2.54 \times 10^{-7}}\right)^{3}}}{\sqrt[10]{\left(\frac{4.14 \times 10^{6}}{2.54 \times 10^{-7}}\right)^{3}}}\right|$$

$$\epsilon = 2.1428...\ \%$$

$\epsilon = 2.14\ \%$

Question 4

a) Volume of a cylinder formula

$V = \pi r^2 h$ (in formula booklet)

$r = 12.7$ $h = 14.4$

Sub r and h into formula.

$V = \pi (12.7)^2 (14.4)$

$V = 7296.58...$

i) $V = 7296.6\ cm^3$ (1dp)

ii) $V = 7300\ cm^3$ (3sf)

iii) $V = 7297$ (nearest integer)

b) $V = 7.3 \times 10^3\ cm^3$

Question 5

a) For L

Any value equal to or more than 25.15cm will be rounded up to 25.2 cm (1dp).

Any value less than 25.25cm will be rounded down to 25.2 cm (1dp).

For W

Any value equal to or more than 21.35cm will be rounded up to 21.4 cm (1dp).

Any value less than 21.45cm will be rounded down to 21.4 cm (1dp).

Write bounds as an inequality.

i) $25.15\text{m} \leq L < 25.25\text{m}$

ii) $21.35\text{m} \leq W < 21.45\text{m}$

b) For lower bound use

$L = 25.15$ $W = 21.35$

$P = 2(25.15) + 2(21.35)$ $A = (25.15)(21.35)$

$P = 93\text{ m}$ $A = 536.9525\text{ m}^2$

For upper bound use

$L = 25.25$ $W = 21.45$

$P = 2(25.25) + 2(21.45)$ $A = (25.25)(21.45)$

$P = 93.4\text{ m}$ $A = 541.6125\text{ m}^2$

i) $93\text{ m} \leq P < 93.4\text{ m}$

ii) $537\text{m}^2 \leq A < 541\text{m}^2$ (3sf)

Question 6

i) Input equation into calculator.

$4 \times (6.2 \times 10^{-5}) = 0.00024$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10$...

2.48×10^{-4}

ii) Input equation into calculator.

$(4 \times 10^5) - (5 \times 10^4) = 350\ 000$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10$...

3.5×10^5

iii) Input equation into calculator.

$(4321^{-1})(1.2 \times 10^{-1}) = 0.00002777...$

Rewrite into form $a \times 10^k$, where $1 \leq a \leq 10$...

2.78×10^{-5} (3sf)

Question 7

a) i) $d = 2.72 \times 10^2$

$a = 0.272 \quad b = 2720 \quad c = 0.1 \quad d = 272$

ii) $b = 0.0272 \times 10^5$

b) Sub a, b, c and d into equation.

$0.272 + 0.0272 \times 10^5 - e(10e)^{-1} + 2.72 \times 10^2$

$0.272 + 2720 + 0.1 + 272$

$= 2992.172$

i) **2990 (3sf)**

ii) **2.99×10^3**

Question 8

a) i) Mean, $\bar{x}$, of a set of data

$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ and $n = \sum_{i=1}^{k} f_i$ (in formula booklet)

$\bar{x} = \frac{2.18 + 2.21 + 2.23 + 2.19 + 2.24}{5}$

$\bar{x} = 2.21$ m

Alternatively you could input the values into your GDC.

ii) Percentage error formula.

$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\%$ (in formula booklet)

$V_A = 2.21$ $V_E = 2.2$

Sub V_A and V_E into formula.

$E = \left| \frac{2.21 - 2.2}{2.2} \right| \times 100\%$

$E = 0.45454...$

$E = 0.455\%$ (3sf)

b) i) Mean, $\bar{x}$, of a set of data

$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ and $n = \sum_{i=1}^{k} f_i$ (in formula booklet)

$\bar{x} = \frac{20.3 + 19.9 + 20.3 + 20.4 + 20.1}{5}$

$\bar{x} = 20.2$ kg

Alternatively you could input the values into your GDC.

ii) Percentage error formula.

$E = \left| \frac{V_A - V_E}{V_E} \right| \times 100\%$ (in formula booklet)

$V_A = 20.2$ $V_E = 20$

Sub V_A and V_E into formula.

$E = \left| \frac{20.2 - 20}{20} \right| \times 100\%$

$E = 1\%$

Question 9

a) Volume of a cuboid formula.

$V = lwh$ (in formula booklet)

where l is the length, w is the width and h is the height.

$l = 80$ $w = 60$ $h = 20$

Sub l, w and h into formula.

$V = 80 \times 60 \times 20$

$V = 96\,000$ cm^3

b) $N = \frac{\text{box volume}}{\text{cube volume}}$

$N = \frac{96000}{300}$

N = 320 cubes

c) Percentage error formula.

$\epsilon = \left|\frac{V_A - V_E}{V_E}\right| \times 100\%$ (in formula booklet)

where V_E is the exact value and V_A is the approximated value.

$V_A = 320$ $V_E = 280$

Sub V_A and V_E into formula.

$\epsilon = \left|\frac{320 - 280}{280}\right| \times 100\%$

$\epsilon = 14.2857\%$

ϵ = 14.3%

Question 10`

(i) Using your GDC

$x = 2$
$y = -4$
$z = 5$

(ii) Using your GDC

$x = 6$
$y = -7$
$z = -9$

Question 11

(i) Using your GDC

$x = -1$
$y = -6$
$z = 7$

(ii) Using your GDC

$x = 3$
$y = 5$
$z = -3$