



Mark Scheme

Mock Set 2

Pearson Edexcel GCE Mathematics

Advanced Subsidiary Level in Mathematics

Paper 01 8MA0/01 Pure Mathematics

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

April 2023

Publications Code 8MA0_01_MS2_MS

All the material in this publication is copyright

© Pearson Education Ltd 2023

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

 - bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$(6, -5)$ and $(2, 1)$		
	Attempts to find the gradient of l_1 : $m = \frac{1-5}{2-6} = -\frac{3}{2}$	M1	1.1b
	$y-1 = -\frac{3}{2}(x-2)$ or $y-5 = -\frac{3}{2}(x-6)$	dM1	1.1b
	$y = -\frac{3}{2}x + 4$	A1	1.1b
		(3)	
(b)	$l_2: y = \frac{2}{3}x$	B1ft	2.2a
	Attempts to solve $y = -\frac{3}{2}x + 4$ and $y = \frac{2}{3}x$ simultaneously $\frac{2}{3}x = -\frac{3}{2}x + 4$ leading to $x = \dots$	M1	1.1a
	$x = \frac{24}{13}$	A1	1.1b
	$\left(\frac{24}{13}, \frac{16}{13}\right)$	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a) M1: Attempts to find the gradient of l_1 using $\frac{\Delta y}{\Delta x}$. Condone one sign error e.g., $\frac{6}{4}$ dM1: $y - y_1 = m(x - x_1)$ with either $(6, -5)$ or $(2, 1)$ and their $m = -\frac{3}{2}$ If $y = mx + c$ is used they must proceed as far as $c = \dots$ A1: $y = -\frac{3}{2}x + 4$ or $y = 4 - \frac{3}{2}x$ only.			
(b) B1ft: Deduces the equation of l_2 is $y = \frac{-1}{-\frac{3}{2}}x$ M1: Attempts to solve their $y = -\frac{3}{2}x + 4$ and their $y = \frac{2}{3}x$ simultaneously "$\frac{2}{3}x = -\frac{3}{2}x + 4$" leading to $x = \dots$ May be implied by their values A1: $x = \frac{24}{13}$ or $y = \frac{16}{13}$ Condone non-recurring decimals given, e.g. $x = \text{awrt } 1.85$ or $y = \text{awrt } 1.23$ here. A1: $\left(\frac{24}{13}, \frac{16}{13}\right)$. Accept $x = \frac{24}{13}, y = \frac{16}{13}$.			

Question	Scheme	Marks	AOs
2(a)	$x^2 + y^2 - 12x + 10y = 0$		
(i)	Centre = $(6, -5)$	B1	1.1b
(ii)	$(x - 6)^2 + (y + 5)^2 = 61$	M1	1.1b
	Radius = $\sqrt{61}$	A1	1.1b
		(3)	
(b)	$k = -5 + \sqrt{61}$	B1ft	2.2a
		(1)	
(c)	At Q, $y = -10$	B1	1.1b
	Area = $\frac{1}{2} \times "10" \times "6"$	M1	3.1a
	Area = 30	A1	1.1b
		(3)	

(7 marks)

Notes:

(a)(i)

B1: Centre = $(6, -5)$

(a)(ii)

M1: Attempts to complete the square to achieve $(x \pm 6)^2 + (y \pm 5)^2 \pm \dots = \dots$

A1: Radius = $\sqrt{61}$

(b)

B1ft: Deduces that $k = "-5" + "\sqrt{61}"$ only, where -5 is the y coordinate of their centre and $\sqrt{61}$ is their radius. The y coordinate of the centre must be negative for the follow through.

(c)

B1: y coordinate of Q is -10 seen or implied

M1: A complete method to find the area using the x coordinate of their centre and their y coordinate for Q. Look for $\frac{1}{2} \times "10" \times "6"$.

A1: 30 only.

Question	Scheme	Marks	AOs
3(a)	$y = 4x - 3x^{\frac{3}{2}} + 13, \quad y = 13 - 2x, \quad A(4, 5)$		
	$y = 4(4) - 3(4)^{\frac{3}{2}} + 13 = 5 \text{ and } y = 13 - 2(4) = 5$	M1	1.1b
	The curve and the line intersect at $(4, 5)$ *	A1*	2.1
		(2)	
(b)	$\int 4x - 3x^{\frac{3}{2}} + 13 dx = 2x^2 - \frac{6}{5}x^{\frac{5}{2}} + 13x (+c)$	M1 A1	1.1b 1.1b
	$\left[2x^2 - \frac{6}{5}x^{\frac{5}{2}} + 13x \right]_0^4 - \frac{1}{2} \times 4(13 + 5)$ $= \frac{228}{5} - 36$	dM1	3.1a
	Area of $R = 9.6$	A1	1.1b
		(4)	

(b) Alternative			
	$\int 4x - 3x^{\frac{3}{2}} + 13 - (13 - 2x) dx = 3x^2 - \frac{6}{5}x^{\frac{5}{2}} (+c)$	M1 A1	1.1b 1.1b
	$\left[3x^2 - \frac{6}{5}x^{\frac{5}{2}} \right]_0^4 = 3(4)^2 - \frac{6}{5}(4)^{\frac{5}{2}} (-0)$	dM1	3.1a
	Area of $R = 9.6$	A1	1.1b

(6 marks)

Notes:

(a) M1: Attempts to substitute $x = 4$ into both $y = 4x - 3x^{\frac{3}{2}} + 13$ and $y = 13 - 2x$.

Alternatively, sets $4x - 3x^{\frac{3}{2}} + 13 = 13 - 2x$, solves, and substitutes $x = 4$ into either equation for y

A1*: Obtains $y = 5$ for both and concludes that the curve and line intersect at $(4, 5)$.

In the alternative, solves $4x - 3x^{\frac{3}{2}} + 13 = 13 - 2x$ using correct algebra to achieve $x = 4$ and substitutes into either equation for y to achieve $y = 5$ and concludes that the curve and line intersect at $(4, 5)$.

(b)

M1: For an attempt to integrate $x^n \rightarrow x^{n+1}$ for C in at least one term.

A1: Correct integration.

dM1: For the key step in achieving a fully correct strategy for the area, e.g., attempts the trapezium and subtracts from the area enclosed between the curve, the x -axis, the y -axis and $x = 4$.

(Condone the omission of the “ $- 0$ ”)

A1: 9.6 o.e. e.g., $\frac{48}{5}$

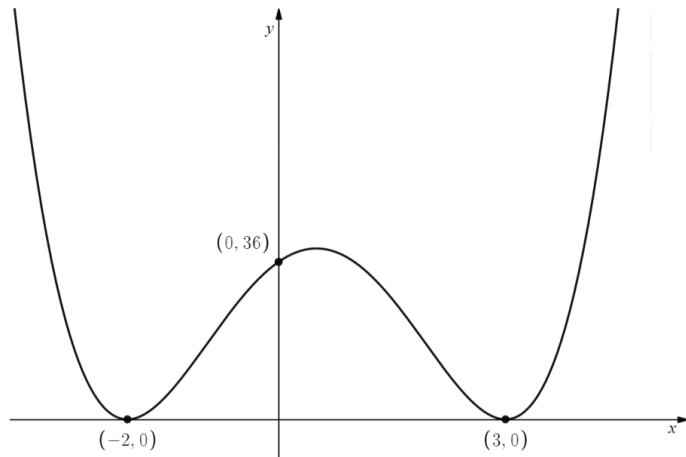
(b) Alternative

M1: For an attempt to integrate $x^n \rightarrow x^{n+1}$ for “ $C - l$ ” in at least one term.

A1: Correct integration.

dM1: For the key step in achieving a fully correct strategy for the area. (Condone the omission of the “ $- 0$ ”)

A1: 9.6 o.e. e.g., $\frac{48}{5}$

Question	Scheme	Marks	AOs	
4(a)	$f(x) = x^4 - 2x^3 - 11x^2 + 12x + 36$			
	$f(3) = (3)^4 - 2(3)^3 - 11(3)^2 + 12(3) + 36$ $= 81 - 54 - 99 + 36 + 36 = \dots$	M1	1.1b	
	$f(3) = 0$ hence $(x - 3)$ is a factor of $f(x)$ (by the factor theorem). *	A1*	2.4	
		(2)		
(b)	Deduces $a = 2$	B1	2.2a	
		(1)		
(c)		Shape (positive quartic with two minima).	B1	1.1b
		$(-2, 0)$ and $(3, 0)$	B1ft	1.1b
		$(0, 36)$	B1	1.1b
		Maximum in 1st quadrant.	B1	2.2a
		(4)		

(7 marks)

Notes:

(a)

M1: Attempts to calculate $f(3)$. Attempted division of $f(x)$ by $(x-3)$ is M0.

Either line in the main scheme is acceptable.

A1*: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$f(3) = 0$ hence $(x-3)$ is a factor of $f(x)$ (by the factor theorem).

$f(3) = 0$ hence $(x-3)$ is a factor.

(b)

B1: Deduces that $a = 2$

(c)

B1: Shape (positive quartic with two minima).

B1ft: $(-2, 0)$ and $(3, 0)$ labelled in the correct place at the minima. Condone -2 and 3 .

Follow through on their a .

B1: $(0, 36)$ labelled as the y intercept. Condone 36 .

B1: Local maximum in the first quadrant is the only other turning point.

Question	Scheme	Marks	AOs
5	$2x - y + 6 = 0$ and $y = 2x^2 + kx + 9$		
	Rearranges to $y = 2x + 6$ and substitutes into $y = 2x^2 + kx + 9$ $2x + 6 = 2x^2 + kx + 9$	M1	1.1b
	$2x^2 + (k - 2)x + 3 = 0$	A1	1.1b
	Uses the discriminant $(k - 2)^2 - 4(2)(3) > 0$ proceeding to $k \dots$	dM1	2.1
	$k \dots 2 - 2\sqrt{6}$ or $k \dots 2 + 2\sqrt{6}$	A1	1.1b
	$\{k : k < 2 - 2\sqrt{6}\} \cup \{k : k > 2 + 2\sqrt{6}\}$	A1	2.5
		(5)	

(5 marks)

Notes:

M1: For an attempt to rearrange the linear equation to make y the subject and substitute into the quadratic equation.

A1: For a correct 3TQ with like terms collected, set $= 0$.

May be implied by correct use of the discriminant with $a = 2$, $b = (k - 2)$, $c = 3$.

dM1: For the key step in using the discriminant with their a , b and c which must include k , proceeding to at least one critical value for k

A1: One correct critical value. Allow any inequality/equality here. Condone $\sqrt{24}$ for $2\sqrt{6}$

A1: $\{k : k < 2 - 2\sqrt{6}\} \cup \{k : k > 2 + 2\sqrt{6}\}$ cso. Set notation required. Condone $\sqrt{24}$ for $2\sqrt{6}$.

Alternative

M1: For an attempt to rearrange the linear equation to make x the subject and substitute into the quadratic equation for both instances of x .

A1: For a correct 3TQ with like terms collected, i.e., $y^2 + (k - 14)y + (54 - 6k) = 0$.

May be implied by correct use of the discriminant with $a = 1$, $b = (k - 14)$, $c = (54 - 6k)$.

dM1A1A1: As in the main scheme.

Question	Scheme	Marks	AOs
6	$3^x = 3\sqrt{3}(9^{y+1})$		
	$3^x = 3^{1.5} (3^2)^{y+1}$	B1	1.1b
	$3^x = 3^{1.5+2y+2} \Rightarrow x = 1.5 + 2y + 2$	M1	2.1
	$y = \frac{1}{2}x - \frac{7}{4}$	A1	1.1b
		(3)	
(3 marks)			
Notes:			
B1: Writes a correct equation in powers of 3 only. M1: Complete process of writing a correct equation in powers of 3 only and using correct index laws to obtain an equation linking x with y without indices. A1: $y = \frac{1}{2}x - \frac{7}{4}$ o.e. in required form.			

Question	Scheme	Marks	AOs
7	$4x^2 > 20x - 27$		
	$4x^2 - 20x + 27 > 0$ $4x^2 - 20x + 27 = 4\left(x - \frac{5}{2}\right)^2 - 25 + 27 \quad \text{or} \quad (2x - 5)^2 + 2$	M1	3.1a
	$= 4\left(x - \frac{5}{2}\right)^2 + 2 \quad \text{with comment (see notes)}$	A1	1.1b
	As $4\left(x - \frac{5}{2}\right)^2 \geq 0$, $4\left(x - \frac{5}{2}\right)^2 + 2 \geq 2 > 0$ hence $4x^2 > 20x - 27$ for all x	A1	2.4
		(3)	
(3 marks)			
Notes:			
Method One: Completing the square			
M1: For an attempt to rearrange and complete the square. Accept $4(x - 2.5)^2 \dots$ or $(2x - 5)^2 \dots$ A1: For either $4(x - 2.5)^2 + 2$ or $(2x - 5)^2 + 2$ with either e.g., $4(x - 2.5)^2 \geq 0$ or $(2x - 5)^2 + 2 \geq 2$ or minimum at $(2.5, 2)$. Accept the inequality statement in words. Condone e.g., $4(x - 2.5)^2 > 0$ or $(x - 2.5)^2$ is always positive for this mark. A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and conclusion.			
Method Two: Discriminant			
M1: For an attempt to rearrange and find the discriminant $b^2 - 4ac$ with a correct a, b and c which may be within a quadratic formula. You may condone missing brackets. A1: Correct value of $b^2 - 4ac = -32$ and states that the curve is U shaped (or intercept is $(0, 27)$) or equivalent such as positive x^2 etc. A1: Explains that as $b^2 - 4ac$ is negative there are no roots, and since the curve is U shaped then $4x^2 - 20x + 27 > 0$ hence $4x^2 > 20x - 27$			
Method Three: Differentiation			
M1: For an attempt to rearrange, differentiate and find the turning point. This would require an attempt to find $\frac{dy}{dx}$, setting it equal to 0 and solving to find the x value and y value. A1: For differentiating $\frac{dy}{dx} = 8x - 20 = 0 \Rightarrow (2.5, 2)$ is the turning point. A1: Shows that $(2.5, 2)$ is the minimum using either the second derivative or stating that the curve is U shaped etc. and $4x^2 - 20x + 27 \geq 2 > 0$ hence $4x^2 > 20x - 27$			

Question	Scheme	Marks	AOs
8(a)	$(2+3x)^6$		
	$64+\dots$	B1	1.1b
	$\dots+\binom{6}{1}2^5.(3x)+\binom{6}{2}2^4.(3x)^2+\binom{6}{3}2^3.(3x)^3+\dots$	M1	1.1b
	Two of $\dots+576x+2160x^2+4320x^3+\dots$	A1	1.1b
	$64+576x+2160x^2+4320x^3+\dots$	A1	1.1b
		(4)	
(b)	$(2-3x)^6$		
	$64-576x+2160x^2-4320x^3+\dots$	B1ft	2.2a
		(1)	
(c)	$\left[(2+3x)^6+(2-3x)^6\right]^2$		
	$(64+576x+2160x^2+4320x^3+\dots)+(64-576x+2160x^2-4320x^3+\dots)$ $=128+4320x^2+\dots$	M1	2.2a
	$(128+4320x^2+\dots)^2=16384+1105920x^2$	A1	1.1b
		(2)	
(d)	$\left[(2+ax)^n+(2-ax)^n\right]^p$		
	$(2^n+2^n)^p$	M1	2.1
	$= (2 \times 2^n)^p = 2^{p(n+1)}$	A1	1.1b
		(2)	

(9 marks)

Notes:

(a)
B1: For 64
M1: Attempts the binomial expansion. May be awarded on either term two and/or term three.
 Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $(3x)$
A1: For two out of three simplified terms correct from $\dots+576x+2160x^2+4320x^3+\dots$
A1: For all remaining terms correct $\dots+576x+2160x^2+4320x^3+\dots$ ignore any extra terms.
 Listing is acceptable for all 4 marks.

(b)
B1ft: Deduces that the signs of the second and fourth terms should be negative.
 Follow through on their terms "64"- "576"x+ "2160"x² - "4320"x³ +...

(c)

M1: Deduces that the second terms will cancel and adds their two answers to arrive at an expression of the form $2 \times 64 + 2 \times 2160x^2 + \dots$ Dependent on part (b).

A1: $16384 + 1105920x^2 + \dots$

(d)

M1: For the key step in realising that the term independent of x will be $(2^n + 2^n)^p$ or $(2 \times 2^n)^p$

A1: $2^{p(n+1)}$ or 2^{np+p} o.e.

Question	Scheme	Marks	AOs
9(a)	$5\cos\theta = 24\tan\theta$		
	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$ and arrives at a quadratic equation in $\sin\theta$	M1	3.1a
	$5\cos\theta = 24\tan\theta \Rightarrow 5\cos^2\theta = 24\sin\theta$	B1	1.1b
	$5(1 - \sin^2\theta) = 24\sin\theta \Rightarrow 5 - 5\sin^2\theta = 24\sin\theta$	M1	1.1b
	Arrives at $5\sin^2\theta + 24\sin\theta - 5 = 0$ with no errors. *	A1*	2.1
		(4)	
(b)	$(5\sin x - 1)(\sin x + 5) = 0 \Rightarrow \sin x = \dots \Rightarrow x = \dots$	M1	1.1b
	Any one of $x = 11.5^\circ, 168.5^\circ, 371.5^\circ, 528.5^\circ$	A1	1.1b
	$x = 11.5^\circ, 168.5^\circ, 371.5^\circ, 528.5^\circ$ only	A1	2.2a
		(3)	
(c)	Deduces that there are 8 times as many solutions in the interval. $8 \times "4" = 32$	B1ft	2.2a
		(1)	
(8 marks)			
Notes:			
(a) M1: An overall problem-solving mark, condoning slips, for an attempt to - Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ - Use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ - Arrive at a quadratic equation in $\sin\theta$ B1: Uses the correct identity and multiplies across to give $5\cos\theta = 24\tan\theta \Rightarrow 5\cos^2\theta = 24\sin\theta$ M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a quadratic in $\sin\theta$ A1*: Arrives at the given answer $5\sin^2\theta + 24\sin\theta - 5 = 0$ with no errors.			
(b) M1: Attempts to solve the given quadratic in $\sin x$ using an appropriate method (it is acceptable to use a calculator to solve this) and proceeds to at least one value of x A1: At least one correct value of x A1: $x = 11.5^\circ, 168.5^\circ, 371.5^\circ, 528.5^\circ$ only in the given interval. Ignore solutions outside the interval. Do not penalise missing degree symbols.			
(c) B1ft: $8 \times$ their number of solutions to part (b). Allow a restart – so 32 is accepted regardless of their answer in (b).			

Question	Scheme	Marks	AOs
10(a)	$H = A(x-3)^2 + 1$	M1	3.3
	$x = 0, H = 0 \Rightarrow 0 = A(0-3)^2 + 1 \Rightarrow A = -\frac{1}{9}$	dM1	3.1b
	$H = -\frac{1}{9}(x-3)^2 + 1$	A1	1.1b
		(3)	
(b)	$\{H = 0.5\} \Rightarrow 0.5 = -\frac{1}{9}(x-3)^2 + 1 \Rightarrow (x-3)^2 = 4.5$	M1	3.1b
	$x = 3 \pm \frac{3}{2}\sqrt{2}$	dM1	1.1b
	$3 + \frac{3}{2}\sqrt{2} - \left(3 - \frac{3}{2}\sqrt{2}\right)$ or $2 \times \frac{3}{2}\sqrt{2}$	ddM1	1.1b
	$\Rightarrow$ Greatest horizontal length $= 3\sqrt{2} =$ awrt 4.24 m	A1	3.2a
		(4)	
(c)	Gives a limitation of the model. Accept e.g., • The model is not valid before take-off (or after landing). • The take-off and landing might not be at the same height. • The ground might not be horizontal. • The snowboarder is modelled as a particle. • The poles may not be vertical. • The trajectory of the snowboarder is a perfect parabola. • There is no spin accounted for. • There is no wind resistance in the model.	B1	3.5b
		(1)	

(8 marks)

Notes:

(a)

M1: Translates the situation given into a suitable equation for the model.

e.g., uses the turning point (3,1) to write $H = A(x-3)^2 + 1$

dM1: Applies a complete strategy with appropriate constraints to find all constants in their model.

e.g., uses (0,0) or (6,0) on their model and finds $A = \dots$

A1: Finds a correct equation linking H with x , i.e., $H = -\frac{1}{9}(x-3)^2 + 1$ or equivalent.

Condone use of y in place of H for both the M1 and dM1 marks, but not for A1.

(b)

M1: Substitutes $H = 0.5$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x-a)^2 = b$; $a \neq 0, b > 0$

dM1: Correct method of solving their quadratic equation to give at least one solution.

ddM1: Subtracts their two solutions either way round. Alternatively, multiplies the surd part of their expression by 2.

A1: Either $3\sqrt{2}$ or awrt 4.24 m. Correct units required. Must now be positive.

(c)

B1: See main scheme but accept any suitable comment.

Question	Scheme	Marks	AOs
11	$2\log_6(x+3) = 2 - \log_6(4-x)$		
	Uses the power law $\log_6(x+3)^2 = 2 - \log_6(4-x)$	M1	1.1b
	Uses the addition law $\log_6((x+3)^2(4-x)) = 2$	M1	1.1b
	Removes the log $(x+3)^2(4-x) = 36$	M1	1.1b
	Expands to a cubic in x $-x^3 - 2x^2 + 15x + 36 = 36$	dddM1	3.1a
	Correct cubic expression = 0 $x^3 + 2x^2 - 15x = 0$	A1	1.1b
	Factorises and solves $x(x+5)(x-3) = 0 \Rightarrow x = \dots$	M1	1.1b
	$x = 0, x = 3$ only	A1	2.3
		(7)	

(7 marks)

Notes:

M1: Uses the power law of logs $2\log_6(x+3) = \log_6(x+3)^2$

M1: Uses the addition law of logs following the above $\log_6(x+3)^2 + \log_6(4-x) = \log_6((x+3)^2(4-x))$

Alternatively uses the subtraction law following use of $2 = \log_6 36$, i.e., $2 - \log_6(4-x) = \log_6 \frac{36}{4-x}$

M1: Removes the log or converts 2 into $\log_6 36$. Look for 2 going to 36.

dddM1: For attempting to expand their three brackets to achieve a cubic in x

A1: For a correct cubic expression in x , set = 0

M1: For the correct method of solving their cubic = 0. May be implied by sight of two values for x from this cubic, i.e., two from $x = 0, x = 3, x = -5$

A1: $x = 0, x = 3$ only.

Question	Scheme			Marks	AOs
12(a)	$\theta = 22 + 64e^{-\frac{3}{32}t}, t \geq 0$				
	$t = 0 \Rightarrow \theta = 86^{\circ}\text{C}$			B1	3.4
				(1)	
(b)	Attempts to differentiate $\theta = 22 + 64e^{-\frac{3}{32}t}$ with respect to t $\frac{d\theta}{dt} = -6e^{-\frac{3}{32}t}$			M1	3.1b
	$\frac{d\theta}{dt} = -6e^{-\frac{3}{32}(10)}$			dM1	3.4
	2.35 ($^{\circ}\text{C}/\text{minute}$)			A1	1.1b
				(3)	
(c)	$22 + 64e^{-\frac{3}{32}t} = 40$ $e^{-\frac{3}{32}t} = \frac{9}{32}$	or	$22 + 64e^{-\frac{3}{32}t} = 40$ $e^{-\frac{3}{32}t} = \frac{19}{32}$	M1	1.1b
	$t_1 = 13.53$		$t_2 = 5.56$	A1	1.1b
	$T = t_1 - t_2 = 13.53 - 5.56$			dM1	1.1b
	7.97 minutes = 7 minutes 58 seconds			A1	1.1b
				(4)	
(d)	20.8 $^{\circ}\text{C}$ is below the lower limit 22 $^{\circ}\text{C}$ and so the model is inaccurate (for large values of t).			B1	3.5a
				(1)	
(e)	Increase the coefficient of t (but keep it below 0).			B1	3.5c
				(1)	
(10 marks)					
Notes:					
(a)					
B1: Uses the model to state that the initial temperature is 86 $^{\circ}\text{C}$. Units required.					
(b)					
M1: Attempts to differentiate $\theta = 22 + 64e^{-\frac{3}{32}t}$ with respect to t . Look for $64e^{-\frac{3}{32}t} \rightarrow ke^{-\frac{3}{32}t}$.					
dM1: Substitutes $t = 10$ into their $\frac{d\theta}{dt}$					
A1: awrt 2.35 (2.3496...)					

(c)

M1: Attempts to solve $22 + 64e^{-\frac{3}{32}t} = 40$ or $22 + 64e^{-\frac{3}{32}t} = 60$ as far as $e^{-\frac{3}{32}t} = k, k > 0$

A1: awrt 13.5 or 5.6

dM1: Solves both $22 + 64e^{-\frac{3}{32}t} = 40$ and $22 + 64e^{-\frac{3}{32}t} = 60$ with correct use of logarithms to arrive at two values for t and subtracts either way round.

A1: 7 minutes 58 seconds or 478 seconds.

(d)

B1: States that the model is inaccurate (for large values of t) and provides a valid justification.
e.g., 20.8°C is lower than the room temperature which is not possible.

Alternatively, attempt to solve $22 + 64e^{-\frac{3}{32}t} = 20.8$ as far as $64e^{-\frac{3}{32}t} = k, k < 0$ and state no solutions.

Do not allow simply “there is an asymptote at 22°C ” without explanation that the model will not drop lower than this.

Substituting $t = 120$ and suggesting that 22 is close or not close to 20.8 is not acceptable.

(e)

B1: Decrease the $\frac{3}{32}$ or increase $-\frac{3}{32}$ (or the coefficient of t).

There is no need to mention limiting the coefficient at 0.

Question	Scheme	Marks	AOs
13(a)	$(V =)\pi r^2 h = 400$	B1	1.1b
	$A = 2\pi r^2 + 2\pi r h$	B1	1.1b
	$h = \frac{400}{\pi r^2} \Rightarrow A = 2\pi r^2 + 2\pi r \left(\frac{400}{\pi r^2} \right)$	M1	1.1b
	$A = 2\pi r^2 + \frac{800}{r} *$	A1*	1.1b
		(4)	
(b)	Attempts to differentiate $A = 2\pi r^2 + \frac{800}{r}$ with respect to r $\frac{dA}{dr} = 4\pi r - 800r^{-2}$	M1 A1	3.1b 1.1b
	Sets $\frac{dA}{dr} = 0 \Rightarrow r^3 = \frac{200}{\pi}$	dM1	1.1b
	$\Rightarrow r = \sqrt[3]{\frac{200}{\pi}} \text{ (cm)}$	A1	1.1b
		(4)	
(c)	Finds $\frac{d^2 A}{dr^2} = 4\pi + 1600r^{-3}$ at $r = \sqrt[3]{\frac{200}{\pi}}$	M1	1.1b
	$\frac{d^2 A}{dr^2} = (+37.7) > 0$ hence minimum (surface area).	A1ft	2.4
		(2)	
(d)	Substitutes $r = \sqrt[3]{\frac{200}{\pi}}$ in $A = 2\pi r^2 + \frac{800}{r}$	M1	1.1b
	Minimum surface area = awrt 301 (cm ²)	A1ft	1.1b
		(2)	
(12 marks)			
Notes:			
(a) B1: Correct equation for volume: $\pi r^2 h = 400$ B1: Correct formula for surface area in terms of the radius and height: $A = 2\pi r^2 + 2\pi r h$ M1: Rearranges $\pi r^2 h = 400$ to $h = \frac{400}{\pi r^2}$ and substitutes in to h in their formula for the surface area A1*: cso.			

(b)

M1: Attempts to differentiate $A = 2\pi r^2 + \frac{800}{r}$ with respect to r . Look for $\left(\frac{dA}{dr} = \dots r \pm \dots r^{-2}\right)$

A1: $\left(\frac{dA}{dr} = \right) 4\pi r - 800r^{-2}$ Condone $\frac{dA}{dr}$ appearing as $\frac{dy}{dx}$ or being absent.

dM1: Sets their $\frac{dA}{dr} = 0$ and arrives at $r^3 = k, k > 0$. $\frac{dA}{dr}$ must have been of the form $\dots r \pm \dots r^{-2}$

A1: $r = \sqrt[3]{\frac{200}{\pi}}$ or exact equivalent. Condone omission of units or use of incorrect units. Note $r = 3.99$ to s.f.

(c)

M1: Finds $\frac{d^2 A}{dr^2}$ following on from their $\frac{dA}{dr}$ (which must be of equivalent difficulty) and attempts to find its value or sign at their r

A1ft: $\frac{d^2 A}{dr^2} = (+37.7) > 0$ hence minimum (surface area).

Alternatively, $\frac{d^2 A}{dr^2} = 4\pi + 1600r^{-3} > 0$ as +ve + +ve > 0 as $r > 0$.

Requires a correct calculation or expression, a correct statement, and a correct conclusion.

Follow through on their r ($r > 0$) and their $\frac{d^2 A}{dr^2}$.

$\frac{d^2 A}{dr^2}$ must be used for this mark to meet the demand of the question.

(d)

M1: For a correct method for finding $A =$ from their solution to $\frac{dA}{dr} = 0$

May be implied by correct final answer. Do not accept attempts using negative values of r .

A1ft: Minimum surface area = awrt 301 (cm²) Condone omission of units or use of incorrect units.

Question	Scheme	Marks	AOs
14(a)	Attempts to find $ \vec{OA} $ and uses the cosine rule in an attempt to find $ \vec{AB} $.	M1	3.1a
	$ \vec{OA} = \sqrt{3^2 + 4^2} = 5$	B1	1.2
	$ \vec{AB} ^2 = 5^2 + 10^2 - 2(5)(10) \cos 60^\circ \Rightarrow \vec{AB} = \dots$	M1	1.1b
	$ \vec{AB} = 5\sqrt{3} \quad *$	A1*	1.1b
		(4)	
(b)	$(5\sqrt{3})^2 = (4\sqrt{3})^2 + p^2$	M1	1.1b
	$p = 3\sqrt{3}$	A1	1.1b
	$\vec{OB} = \vec{OA} + \vec{AB} = 3\mathbf{i} - 4\mathbf{j} + (-4\sqrt{3})\mathbf{i} + (-3\sqrt{3})\mathbf{j}$	dM1	2.1
	$\vec{OB} = (3 - 4\sqrt{3})\mathbf{i} + (-4 - 3\sqrt{3})\mathbf{j}$	A1	2.5
		(4)	

(8 marks)

Notes:

(a)

M1: An overall problem-solving mark, condoning slips, for using the given information in an attempt to

- find $|\vec{OA}|$
- use the cosine rule to find $|\vec{AB}|$

B1: $|\vec{OA}| = 5$ seen.

M1: Attempts to use the cosine rule to find $|\vec{AB}|$.

A1*: Complete solution showing all steps. There is no need to see any working for $|\vec{OA}| = 5$ but it should be stated or seen on a diagram as a minimum.

(b)

M1: Attempts to find p using Pythagoras' Theorem with $|\vec{AB}| = 5\sqrt{3}$

A1: $p = 3\sqrt{3}$ or $\sqrt{27}$ or awrt 5.2

dM1: Attempts to use $\vec{OB} = \vec{OA} - \vec{BA}$ with their p .

Condone slips but there must be a clear attempt to subtract the two vectors the correct way round.

A1: $\vec{OB} = (3 - 4\sqrt{3})\mathbf{i} + (-4 - 3\sqrt{3})\mathbf{j}$ only.

(b) Alternative

M1: Attempts to find p using Pythagoras' Theorem with $|\vec{OB}| = 10$

For reference: $(3 - 4\sqrt{3})^2 + (-4 - p)^2 = 100$; $p^2 + 8p - 27 - 24\sqrt{3} = 0$

A1: $p = 3\sqrt{3}$ or $\sqrt{27}$ or awrt 5.2

dM1: Attempts to use $\overrightarrow{OB} = \overrightarrow{OA} - \overrightarrow{BA}$ with their p .

Condone slips but there must be a clear attempt to subtract the two vectors the correct way round.

A1: $\overrightarrow{OB} = (3 - 4\sqrt{3})\mathbf{i} + (-4 - 3\sqrt{3})\mathbf{j}$ only.

