

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Time 2 hours

Paper
reference

4PM1/01

Further Pure Mathematics PAPER 1



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r}$ $|r| < 1$

Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots$ for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 Given that $\frac{2\sqrt{3} - 4}{3\sqrt{3} + 5}$ can be written in the form $a + b\sqrt{3}$ where a and b are integers,
find, without using a calculator, the value of a and the value of b

Show your working clearly.

(3)

(Total for Question 1 is 3 marks)



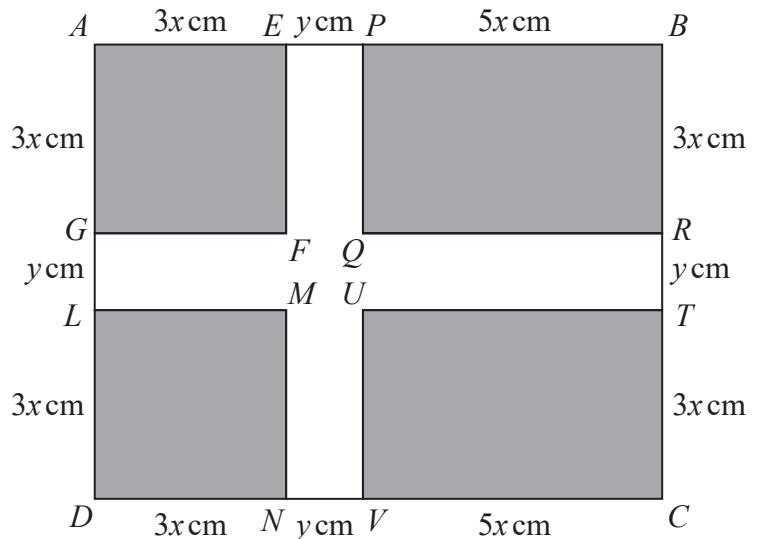


Figure 1

Figure 1 shows the design for a flag consisting of a white cross on a grey background.

$AEGF$ and $DLMN$ are squares with sides of length $3x$ cm.

$BPQR$ and $CTUV$ are rectangles with sides of length $5x$ cm and $3x$ cm.

The width of the cross is y cm.

The total area of the flag is H cm²

(a) Write down an expression, in terms of x and y , for H .

(1)

Given that the area of the cross is K cm²

(b) show that $K = 14xy + y^2$

(3)

The total area of the flag is to be 3432 cm² and the area of the cross is to be 1080 cm²

(c) Find the value of x and the value of y

(5)



Question 2 continued



Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)



3

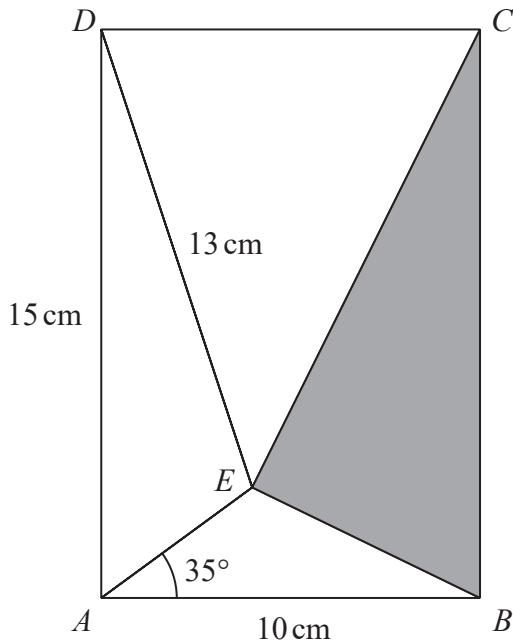


Diagram NOT
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Figure 2

Figure 2 shows a rectangle $ABCD$ with $AB = 10 \text{ cm}$ and $AD = 15 \text{ cm}$.
 E is the point inside the rectangle such that $DE = 13 \text{ cm}$ and angle $BAE = 35^\circ$

Given that angle AED is obtuse,

find the area, in cm^2 to one decimal place, of triangle BCE .

(7)



Question 3 continued

(Total for Question 3 is 7 marks)



- 4 The common ratio of a geometric series G is positive.

The sum of the first 4 terms of G is 80

The sum to infinity of G is 81

Show that the sum of the first 7 terms of G differs from the sum to infinity of G by $\frac{1}{27}$

(7)

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Question 4 continued

(Total for Question 4 is 7 marks)



- 5 Given that $(2 + 3x)^{-1}$ can be expressed in the form $p(1 + qx)^{-1}$ where p and q are constants,

(a) find the value of p and the value of q

(2)

(b) Hence expand $(2 + 3x)^{-1}$ in ascending powers of x up to and including the term in x^3 , expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1+x}{2+3x}$$

(c) Obtain a series expansion for $f(x)$, in ascending powers of x up to and including the term in x^3 , expressing each coefficient as an exact fraction in its lowest terms.

(2)

(d) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x) \, dx$$

(4)



Question 5 continued



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Question 5 continued

(Total for Question 5 is 11 marks)



- 6 (a) Complete the table of values for $y = 1 + 3e^{-x}$ giving your answers to 2 decimal places where appropriate.

(2)

x	0	0.25	0.5	1	1.5	2	3
y		3.34	2.82		1.67		1.15

- (b) On the grid opposite, draw the graph of $y = 1 + 3e^{-x}$ for $0 \leq x \leq 3$

(2)

- (c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$x = e^{-x} \quad \text{in the interval } 0 \leq x \leq 3$$

(3)

- (d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

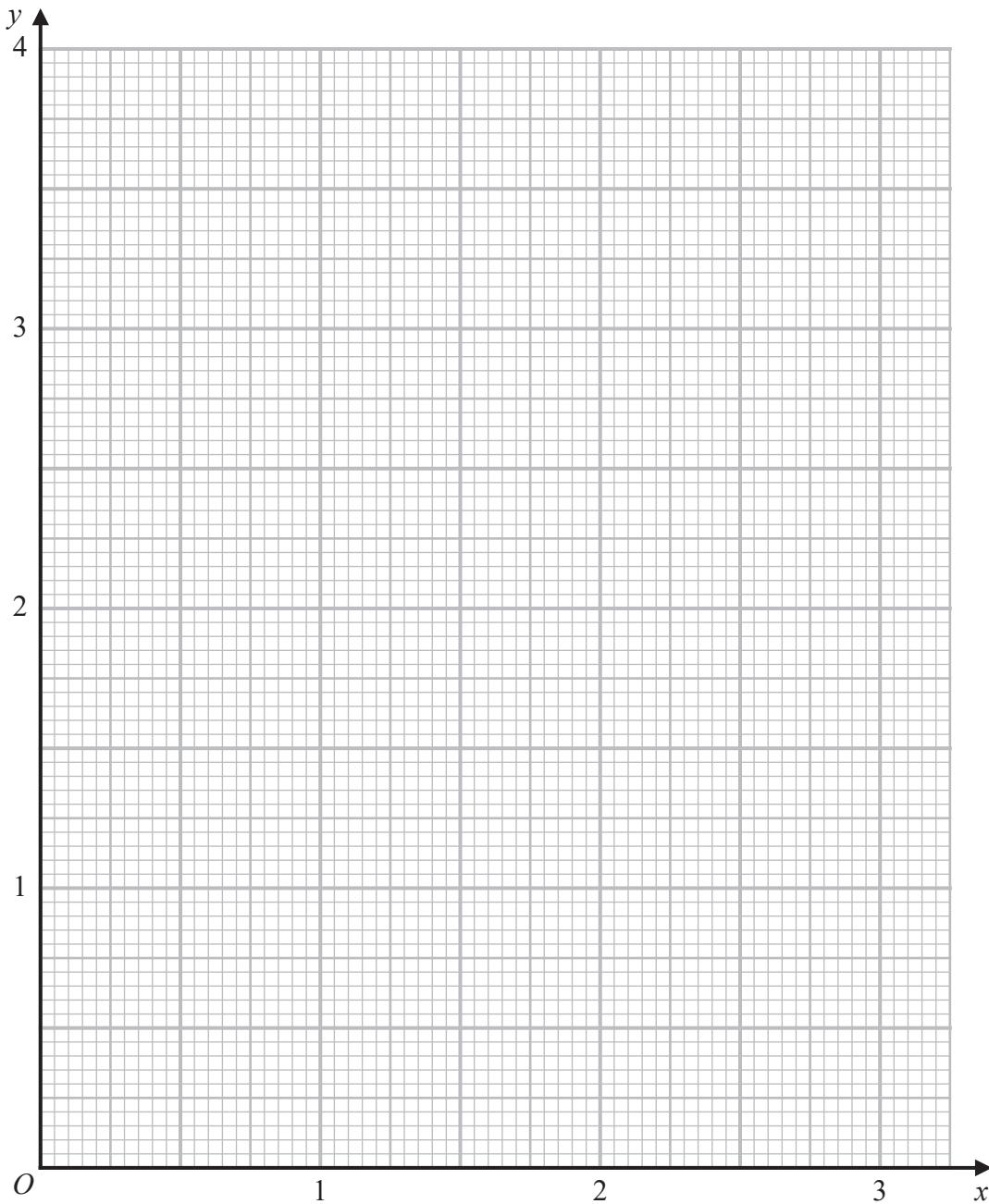
$$\ln(x-1)^3 = -3x \quad \text{in the interval } 0 \leq x \leq 3$$

(4)



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Question 6 continued



Turn over for a spare grid if you need to redraw your graph.



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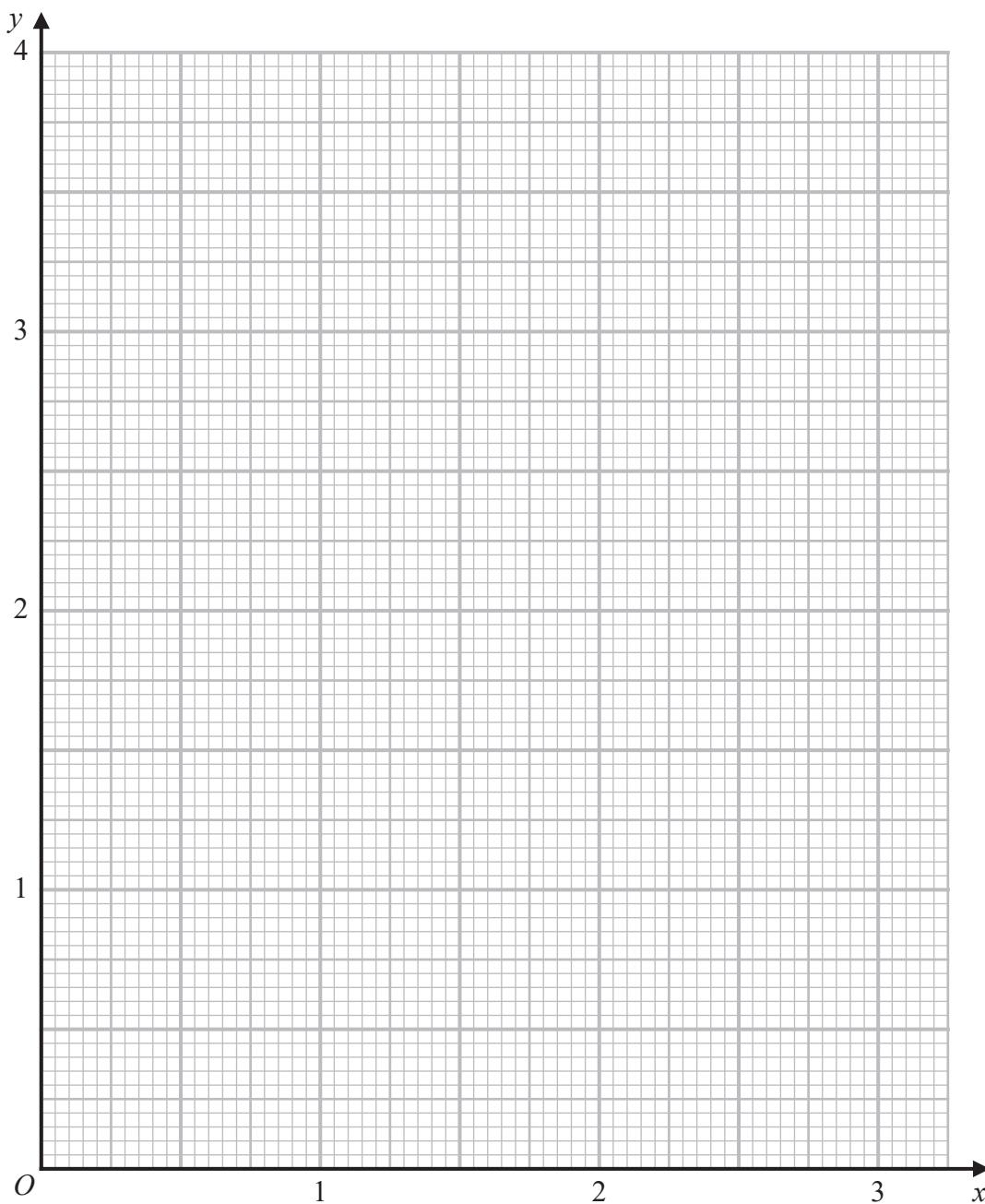
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Question 6 continued

Only use this grid if you need to redraw your graph.



(Total for Question 6 is 11 marks)



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- 7 The point with coordinates $(4, -104)$ lies on the curve C with equation $y = f(x)$

Given that $f'(x) = 4x^3 - 12x^2 - 19x + 12$

- (a) (i) show that C passes through the origin,

(4)

- (ii) show that C has a maximum at the point on the curve where $x = 0.5$

(3)

The curve C has another turning point at A and another turning point at B .

Given that the x coordinate of A is negative,

- (b) (i) find the coordinates of A and the coordinates of B ,

(5)

- (ii) determine the nature of these turning points.

(3)



Question 7 continued



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Question 7 continued

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Question 7 continued

(Total for Question 7 is 15 marks)



- 8** The volume of a sphere is 500 cm^3

- (a) Calculate the radius, in cm to 3 significant figures, of the sphere.

(2)

The surface area of the sphere is increased by 20 cm^2

- (b) Using calculus, find an estimate for the increase in the radius, in cm to 2 significant figures, of the sphere.

(5)



Question 8 continued

(Total for Question 8 is 7 marks)



9

$$f(x) = 3x^4 + 4x^3 - 36x^2 + 64$$

Given that $f(x)$ can be written in the form $(x - 2)^2(ax^2 + bx + c)$

- (a) find the value of a , the value of b and the value of c

(4)

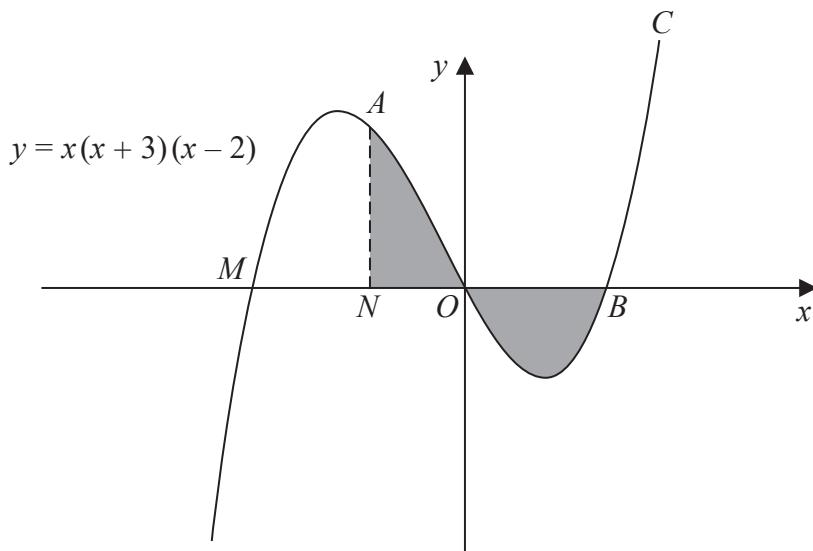


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accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = x(x + 3)(x - 2)$

The curve C crosses the x -axis at the point M , the origin and the point B .

The point N lies on the x -axis between M and O .

The point A lies on C such that AN is parallel to the y -axis.

The area of the shaded region bounded by the curve and OB is numerically equal to the area of the shaded region bounded by the curve, ON and NA .

Given that the coordinates of N are $(n, 0)$,

- (b) use algebraic integration to show that n satisfies the equation

$$(x - 2)^2(3x^2 + 16x + 16) = 0$$

(7)

- (c) Hence find the exact coordinates of A .

(5)



Question 9 continued



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Question 9 continued

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Question 9 continued

(Total for Question 9 is 16 marks)



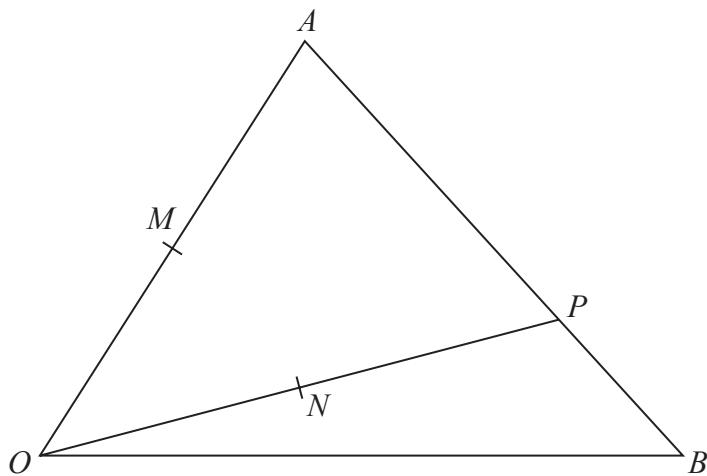


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Figure 4

Figure 4 shows triangle OAB in which

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

The point P lies on AB such that $AP:PB = 3:1$

The point M is the midpoint of OA and the point N is the midpoint of OP .

(a) Find, as simplified expressions in terms of \mathbf{a} and \mathbf{b} , the vector

- (i) \overrightarrow{OP} (ii) \overrightarrow{MN} (4)

The point C lies on OB such that ANC is a straight line.

(b) Using a vector method, find the vector \overrightarrow{OC} as a simplified expression in terms of \mathbf{b} (6)

Given that $\frac{\text{area of quadrilateral } AMNP}{\text{area of triangle } OAB} = K$

(c) find the exact value of K (4)



Question 10 continued



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(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS

